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No

THE MATHEMATICAL PRINCIPLES OF NATURAL PHILOSOPHY.

BY

SIR ISAAC NEWTON.

Translated into English

BY ANDREW MOTTE.

TO WHICH ARE ADDED,

Newton's System of the World;

A SHORT

Comment on, and Defence of, the Principia,

BY W. EMERSON.

WITH

THE LAWS OF THE MOON'S MOTION

According to Gravity.

BY JOHN MACHIN,

Astron., Prof. at Gresh., and Sec. to the Roy. Soc.

A new Edition,

(With the LIFE of the AUTHOR; and a PORTRAIT, taken from the Bust in the Royal Observatory at Greenwich)

CAREFULLY REVISED AND CORRECTED BY

W. DAVIS,

Author of the "Treatise on Land Surveying," the "Use of the Globes," Editor of the "Mathematical Companion," &c. &c. &c.

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THE

MATHEMATICAL PRINCIPLES

OF

NATURAL PHILOSOPHY.

OF THE MOTION OF BODIES.

BOOK II.

SECTION I.

Of the motion of bodies that are reſiſted in the ratio of the velocity.

PROPOSITION I. THEOREM I.

If a body is reſiſted in the ratio of its velocity, the motion loſt by reſiſtance is as the ſpace gone over in its motion.

FOR ſince the motion loſt in each equal particle of time is as the velocity, that is, as the particle of ſpace gone over, then, by compoſition, the motion loſt in the whole time will be as the whole ſpace gone over. Q.E.D.

COR. Therefore if the body, deſtitute of all gravity, move by its innate force only in free ſpaces, and there be given both its whole motion at the beginning, and alſo the motion remaining after ſome part of the way is gone over, there will be given alſo the whole ſpace which the body can deſcribe in an infinite time. For that ſpace will be to the ſpace now deſcribed as the whole motion at the beginning is to the part loſt of that motion.

LEMMA I.

Quantities proportional to their differences are continually proportional.

Let A be to A — B as B to B — C and C to C — D, &c. and, by converſion, A will be to B as B to C and C to D, &c. Q.E.D.

PROPOSITION II. THEOREM II.

If a body is reſiſted in the ratio of its velocity, and moves, by its vis inſita *only, through a ſimilar medium, and the times be taken equal, the velocities in the beginning of each of the times are in a geometrical progreſſion, and the ſpaces deſcribed in each of the times are as the velocities.* (Pl. 1, Fig. 1.)

Case 1. Let the time be divided into equal particles; and if at the very beginning of each particle we ſuppoſe the reſiſtance to act with one ſingle impulſe which is as the velocity, the decrement of the velocity in each of the particles of time will be as the ſame velocity. Therefore the velocities are proportional to their differences, and therefore (by lem. 1, book 2) continually proportional. Therefore if out of an equal number of particles there be compounded any equal portions of time, the velocities at the beginning of thoſe times will be as terms in a continued progreſſion, which are taken by intervals, omitting every where an equal number of intermediate terms. But the ratios of theſe terms are compounded of the equal ratios of the intermediate terms equally repeated, and therefore are equal. Therefore the velocities, being proportional to thoſe terms, are in geometrical progreſſion. Let thoſe equal particles of time be diminiſhed, and their number increaſed *in infinitum*, ſo that the impulſe of reſiſtance may become continual; and the velocities at the beginnings of equal times, always continually proportional, will be alſo in this caſe continually proportional. Q.E.D.

Case 2. And, by diviſion, the differences of the velocities, that is, the parts of the velocities loſt in each of the times, are as the wholes; but the ſpaces deſcribed in each of the times are as the loſt parts of the velocities (by prop. 1, book 2), and therefore are alſo as the wholes. Q.E.D.

Corol. Hence if to the rectangular aſymptotes AC, CH, the hyperbola BG is deſcribed, and AB, DG be drawn perpendicular to the aſymptote AC, and both the velocity of the body, and the reſiſtance of the medium, at the very beginning of the motion, be expreſſed by any given line AC, and, after ſome time is elapſed, by the indefinite line DC; the time may be expreſſed by the area ABGD, and the ſpace deſcribed in that time by the line AD. For if that area, by the motion of the point D, be uniformly increaſed in the ſame manner as the time, the right line DC will decreaſe in a geometrical ratio in the ſame manner as the velocity; and the parts of the right line AC, deſcribed in equal times, will decreaſe in the ſame ratio.

PROPOSITION III. PROBLEM I.

To define the motion of a body which, in a ſimilar medium, aſcends or deſcends in a right line, and is reſiſted in the ratio of its velocity, and acted upon by an uniform force of gravity. (Pl. 1, Fig. 2.)

The body aſcending, let the gravity be expounded by any given rectangle BACH; and the reſiſtance of the medium, at the beginning of the aſcent, by the rectangle BADE, taken on the contrary ſide of the right line AB. Through the point B, with the rectangular aſymptotes AC, CH, deſcribe an hyperbola, cutting the perpendiculars DE, de, in G, g; and the body aſcending will in the time DGgd deſcribe the ſpace EGge; in the time DGBA, the ſpace of the whole aſcent EGB; in the time ABKI, the ſpace of deſcent BFK; and in the time IKki the ſpace of deſcent KFfk; and the velocities of the bodies (proportional to the reſiſtance of the medium) in theſe periods of time will be ABED, ABed, 0, ABFI, ABfi reſpectively; and the greateſt velocity which the body can acquire by deſcending will be BACH.

For let the rectangle BACH (Pl. 1, Fig. 3) be reſolved into innumerable rectangles Ak, Kl, Lm, Mn, &c. which ſhall be as the increments of the velocities produced in ſo many equal times; then will 0, Ak, Al, Am, An, &c. be as the whole velocities, and therefore (by ſuppoſition) as the reſiſtances of the medium in the beginning of each of the equal

times. Make AC to AK, or ABHC to ABkK, as the force of gravity to the reſiſtance in the beginning of the ſecond time; then from the force of gravity ſubduct the reſiſtances, and ABHC, KkHC, LlHC, MmHC, &c. will be as the abſolute forces with which the body is acted upon in the beginning of each of the times, and therefore (by law 2) as the increments of the velocities, that is, as the rectangles Ak, Kl, Lm, Mn, &c. and therefore (by lem. 1, book 2) in a geometrical progreſſion. Therefore, if the right lines Kk, Ll, Mm, Nn, &c. are produced ſo as to meet the hyperbola in q, r, s, t, &c. the areas ABqK, KqrL, LrsM, MstN, &c. will be equal, and therefore analogous to the equal times and equal gravitating forces. But the area ABqK (by corol. 3, lem. 7 and 8, book 1) is to the area Bkq as Kq to $\frac{1}{2}$kq, or AC to $\frac{1}{2}$AK, that is, as the force of gravity to the reſiſtance in the middle of the firſt time. And by the like reaſoning the areas qKLr, rLMs, sMNt, &c. are to the areas qklr, rlms, smnt, &c. as the gravitating forces to the reſiſtances in the middle of the ſecond, third, fourth time, and ſo on. Therefore ſince the equal areas BAKq, qKLr, rLMs, sMNt, &c. are analogous to the gravitating forces, the areas Bkq, qklr, rlms, smnt, &c. will be analogous to the reſiſtances in the middle of each of the times, that is (by ſuppoſition), to the velocities, and ſo to the ſpaces deſcribed. Take the ſums of the analogous quantities, and the areas Bkq, Blr, Bms, Bnt, &c. will be analogous to the whole ſpaces deſcribed; and alſo the areas ABqK, ABrL, ABsM, ABtN, &c. to the times. Therefore the body, in deſcending, will in any time ABrL deſcribe the ſpace Blr, and in the time LrtN the ſpace rlnt. Q.E.D. And the like demonſtration holds in aſcending motion.

Corol. 1. Therefore the greateſt velocity that the body can acquire by falling is to the velocity acquired in any given time as the given force of gravity which perpetually acts upon it to the reſiſting force which oppoſes it at the end of that time.

Corol. 2. But the time being augmented in an arithmetical progreſſion, the ſum of that greateſt velocity and the

velocity in the aſcent, and alſo their difference in the deſcent, decreaſes in a geometrical progreſſion.

Corol. 3. Alſo the differences of the ſpaces, which are deſcribed in equal differences of the times, decreaſe in the ſame geometrical progreſſion.

Corol. 4. The ſpace deſcribed by the body is the difference of two ſpaces, whereof one is as the time taken from the beginning of the deſcent, and the other as the velocity; which [ſpaces] alſo at the beginning of the deſcent are equal among themſelves.

PROPOSITION IV. PROBLEM II.

Suppoſing the force of gravity in any ſimilar medium to be uniform, and to tend perpendicularly to the plane of the horizon; to define the motion of a projectile therein, which ſuffers reſiſtance proportional to its velocity. (Pl. 1, Fig. 4.)

Let the projectile go from any place D in the direction of any right line DP, and let its velocity at the beginning of the motion be expounded by the length DP. From the point P let fall the perpendicular PC on the horizontal line DC, and cut DC in A, ſo that DA may be to AC as the reſiſtance of the medium ariſing from the motion upwards at the beginning to the force of gravity; or (which comes to the ſame) ſo that the rectangle under DA and DP may be to that under AC and CP as the whole reſiſtance at the beginning of the motion to the force of gravity. With the aſymptotes DC, CP deſcribe any hyperbola GTBS cutting the perpendiculars DG, AB in G and B; complete the parallelogram DGKC, and let its ſide GK cut AB in Q. Take a line N in the ſame ratio to QB as DC is in to CP; and from any point R of the right line DC erect RT perpendicular to it, meeting the hyperbola in T, and the right lines EH, GK, DP in I, t, and V; in that perpendicular take Vr equal to $\frac{tGT}{N}$, or, which is the ſame thing, take Rr equal to $\frac{GTIE}{N}$; and the projectile in the time DRTG will arrive at the point r, deſcribing the curve line DraF, the locus of the point r; thence it will come to its greateſt height a in the perpendicular AB; and after-

wards ever approach to the afymptote PC. And its velocity in any point r will be as the tangent rL to the curve. Q.E.I.

For N is to QB as DC to CP or DR to RV, and therefore RV is equal to $\frac{DR \times QB}{N}$, and Rr (that is, RV — Vr, or $\frac{DR \times QB - tGT}{N}$) is equal to $\frac{DR \times AB - RDGT}{N}$. Now let the time be expounded by the area RDGT and (by laws, cor. 2,) diftinguifh the motion of the body into two others, one of afcent, the other lateral. And fince the refiftance is as the motion, let that alfo be diftinguifhed into two parts proportional and contrary to the parts of the motion: and therefore the length defcribed by the lateral motion will be (by prop. 2, book 2) as the line DR, and the height (by prop. 3, book 2) as the area DR × AB — RDGT, that is, as the line Rr. But in the very beginning of the motion the area RDGT is equal to the rectangle DR × AQ, and therefore that line Rr (or $\frac{DR \times AB - DR \times AQ}{N}$) will then be to DR as AB — AQ or QB to N, that is, as CP to DC; and therefore as the motion upwards to the motion lengthwife at the beginning. Since, therefore, Rr is always as the height, and DR always as the length, and Rr is to DR at the beginning as the height to the length, it follows, that Rr is always to DR as the height to the length; and therefore that the body will move in the line DraF, which is the locus of the point r. Q.E.D.

Cor. 1. Therefore Rr is equal to $\frac{DR \times AB}{N} - \frac{RDGT}{N}$; and therefore if RT be produced to X fo that RX may be equal to $\frac{DR \times AB}{N}$, that is, if the parallelogram ACPY be completed, and DY cutting CP in Z be drawn, and RT be produced till it meets DY in X; Xr will be equal to $\frac{RDGT}{N}$, and therefore proportional to the time.

Cor. 2. Whence if innumerable lines CR, or, which is the fame, innumerable lines ZX, be taken in a geometrical

progreſſion, there will be as many lines Xr in an arithmetical progreſſion. And hence the curve DraF is eaſily delineated by the table of logarithms.

Cor. 3. If a parabola be conſtructed to the vertex D (Pl. 1. Fig. 5), and the diameter DG produced downwards, and its latus rectum is to 2 DP as the whole reſiſtance at the beginning of the motion to the gravitating force, the velocity with which the body ought to go from the place D, in the direction of the right line DP, ſo as in an uniform reſiſting medium to deſcribe the curve DraF, will be the ſame as that with which it ought to go from the ſame place D, in the direction of the ſame right line DP, ſo as to deſcribe a parabola in a non-reſiſting medium. For the latus rectum of this parabola, at the very beginning of the motion, is $\frac{DV^2}{Vr}$; and Vr is $\frac{tGT}{N}$ or $\frac{DR \times Tt}{2N}$. But a right line, which, if drawn, would touch the hyperbola GTS in G, is parallel to DK, and therefore Tt is $\frac{CK \times DR}{DC}$, and N is $\frac{QB \times DC}{CP}$. And therefore Vr is equal to $\frac{DR^2 \times CK \times CP}{2DC^2 \times QB}$, that is (becauſe DR and DC, DV and DP are proportionals), to $\frac{DV^2 \times CK \times CP}{2DP^2 \times QB}$; and the latus rectum $\frac{DV^2}{Vr}$ comes out $\frac{2DP^2 \times QB}{CK \times CP}$, that is (becauſe QB and CK, DA and AC are proportional), $\frac{2DP^2 \times DA}{AC \times CP}$, and therefore is to 2DP as DP × DA to CP × AC; that is, as the reſiſtance to the gravity. Q.E.D.

Cor. 4. Hence if a body be projected from any place D (Pl. 1, Fig. 4) with a given velocity, in the direction of a right line DP given by poſition, and the reſiſtance of the medium, at the beginning of the motion, be given, the curve DraF, which that body will deſcribe, may be found. For the velocity being given, the latus rectum of the parabola is given, as is well known. And taking 2DP to that latus rectum, as the force of gravity to the reſiſting force, DP is alſo given.

Then cutting DC in A, ſo that CP × AC may be to DP × DA in the ſame ratio of the gravity to the reſiſtance, the point A will be given. And hence the curve DraF is alſo given.

Cor. 5. And, on the contrary, if the curve DraF be given, there will be given both the velocity of the body and the reſiſtance of the medium in each of the places r. For the ratio of CP × AC to DP × DA being given, there is given both the reſiſtance of the medium at the beginning of the motion, and the latus rectum of the parabola; and thence the velocity at the beginning of the motion is given alſo. Then from the length of the tangent rL there is given both the velocity proportional to it, and the reſiſtance proportional to the velocity in any place r.

Cor. 6. But ſince the length 2DP is to the latus rectum of the parabola as the gravity to the reſiſtance in D; and, from the velocity augmented, the reſiſtance is augmented in the ſame ratio, but the latus rectum of the parabola is augmented in the duplicate of that ratio, it is plain that the length 2DP is augmented in that ſimple ratio only; and is therefore always proportional to the velocity; nor will it be augmented or diminiſhed by the change of the angle CDP, unleſs the velocity be alſo changed.

Cor. 7. Hence appears the method of determining the curve DraF (Pl. 1, Fig. 6) nearly, from the phænomena, and thence collecting the reſiſtance and velocity with which the body is projected. Let two ſimilar and equal bodies be projected with the ſame velocity, from the place D, in different angles CDP, CDp; and let the places F, f, where they fall upon the horizontal plane DC, be known. Then taking any length for DP or Dp, ſuppoſe the reſiſtance in D to be to the gravity in any ratio whatſoever, and let that ratio be expounded by any length SM. Then, by computation, from that aſſumed length DP, find the lengths DF, Df; and from the ratio $\frac{Ff}{DF}$, found by calculation, ſubduct the ſame ratio as found by experiment; and let the difference be expounded by the perpendicular MN. Repeat the ſame a ſecond and a third time, by aſſuming always a new ratio SM of the reſiſtance to the gravity, and collecting a new difference MN. Draw the

affirmative differences on one ſide of the right line SM, and the negative on the other ſide; and through the points N,N,N, draw a regular curve NNN, cutting the right line SMMM in X, and SX will be the true ratio of the reſiſtance to the gravity, which was to be found. From this ratio the length DF is to be collected by calculation; and a length, which is to the aſſumed length DP as the length DF known by experiment to the length DF juſt now found, will be the true length DP. This being known, you will have both the curve line DraF which the body deſcribes, and alſo the velocity and reſiſtance of the body in each place.

SCHOLIUM.

But, yet, that the reſiſtance of bodies is in the ratio of the velocity, is more a mathematical hypotheſis than a phyſical one. In mediums void of all tenacity, the reſiſtances made to bodies are in the duplicate ratio of the velocities. For by the action of a ſwifter body, a greater motion in proportion to a greater velocity is communicated to the ſame quantity of the medium in a leſs time; and in an equal time, by reaſon of a greater quantity of the diſturbed medium, a motion is communicated in the duplicate ratio greater; and the reſiſtance (by law 2 and 3) is as the motion communicated. Let us, therefore, ſee what motions ariſe from this law of reſiſtance.

SECTION II.

Of the motion of bodies that are reſiſted in the duplicate ratio of their velocities.

PROPOSITION V. THEOREM III.

If a body is reſiſted in the duplicate ratio of its velocity, and moves by its innate force only through a ſimilar medium; and the times be taken in a geometrical progreſſion, proceeding from leſs to greater terms: I ſay, that the velocities at the beginning of each of the times are in the ſame geometrical progreſſion inverſely; and that the ſpaces are equal, which are deſcribed in each of the times. (Pl. 1, Fig. 7.)

For ſince the reſiſtance of the medium is proportional to the ſquare of the velocity, and the decrement of the velocity is proportional to the reſiſtance: if the time be divided into in-

numerable equal particles, the ſquares of the velocities at the beginning of each of the times will be proportional to the differences of the ſame velocities. Let thoſe particles of time be AK, KL, LM, &c. taken in the right line CD; and erect the perpendiculars AB, Kk, Ll, Mm, &c. meeting the hyperbola BklmG, deſcribed with the centre C, and the rectangular aſymptotes CD, CH, in B, k, l, m, &c.; then AB will be to Kk as CK to CA, and, by diviſion, AB — Kk to Kk as AK to CA, and, alternately, AB — Kk to AK as Kk to CA; and therefore as AB × Kk to AB × CA. Therefore ſince AK and AB × CA are given, AB — Kk will be as AB × Kk; and, laſtly, when AB and Kk coincide, as AB^2. And, by the like reaſoning, Kk — Ll, Ll — Mm, &c. will be as Kk^2, Ll^2, &c. Therefore the ſquares of the lines AB, Kk, Ll, Mm, &c. are as their differences; and, therefore, ſince the ſquares of the velocities were ſhewn above to be as their differences, the progreſſion of both will be alike. This being demonſtrated it follows alſo that the areas deſcribed by theſe lines are in a like progreſſion with the ſpaces deſcribed by theſe velocities. Therefore if the velocity at the beginning of the firſt time AK be expounded by the line AB, and the velocity at the beginning of the ſecond time KL by the line Kk, and the length deſcribed in the firſt time by the area AKkB, all the following velocities will be expounded by the following lines Ll, Mm, &c. and the lengths deſcribed, by the areas Kl, Lm, &c. And, by compoſition, if the whole time be expounded by AM, the ſum of its parts, the whole length deſcribed will be expounded by AMmB the ſum of its parts. Now conceive the time AM to be divided into the parts AK, KL, LM, &c. ſo that CA, CK, CL, CM, &c. may be in a geometrical progreſſion; and thoſe parts will be in the ſame progreſſion, and the velocities AB, Kk, Ll, Mm, &c. will be in the ſame progreſſion inverſely, and the ſpaces deſcribed Ak, Kl, Lm, &c. will be equal. Q.E.D.

Cor. 1. Hence it appears, that if the time be expounded by any part AD of the aſymptote, and the velocity in the beginning of the time by the ordinate AB, the velocity at the end of the time will be expounded by the ordinate DG; and the whole ſpace deſcribed by the adjacent hyperbolic

area ABGD; and the ſpace which any body can deſcribe in the ſame time AD, with the firſt velocity AB, in a non-reſiſting medium, by the rectangle AB × AD.

Cor. 2. Hence the ſpace deſcribed in a reſiſting medium is given, by taking it to the ſpace deſcribed with the uniform velocity AB in a non-reſiſting medium, as the hyperbolic area ABGD to the rectangle AB × AD.

Cor. 3. The reſiſtance of the medium is alſo given, by making it equal, in the very beginning of the motion, to an uniform centripetal force, which could generate, in a body falling through a non-reſiſting medium, the velocity AB in the time AC. For if BT be drawn touching the hyperbola in B, and meeting the aſymptote in T, the right line AT will be equal to AC, and will expreſs the time in which the firſt reſiſtance, uniformly continued, may take away the whole velocity AB.

Cor. 4. And thence is alſo given the proportion of this reſiſtance to the force of gravity, or any other given centripetal force.

Cor. 5. And, *viceverſa*, if there is given the proportion of the reſiſtance to any given centripetal force, the time AC is alſo given, in which a centripetal force equal to the reſiſtance may generate any velocity as AB; and thence is given the point B, through which the hyperbola, having CH, CD for its aſymptotes, is to be deſcribed; as alſo the ſpace ABGD, which a body, by beginning its motion with that velocity AB, can deſcribe in any time AD, in a ſimilar reſiſting medium.

PROPOSITION VI. THEOREM IV.

Homogeneous and equal ſpherical bodies, oppoſed by reſiſtances that are in the duplicate ratio of the velocities, and moving on by their innate force only, will, in times which are reciprocally as the velocities at the beginning, deſcribe equal ſpaces, and loſe parts of their velocities proportional to the wholes. (Pl. 1, Fig. 8.)

To the rectangular aſymptotes CD, CH deſcribe any hyperbola BbEe, cutting the perpendiculars AB, ab, DE, de in B, b, E, e; let the initial velocities be expounded by the perpendiculars AB, DE, and the times by the lines Aa, Dd.

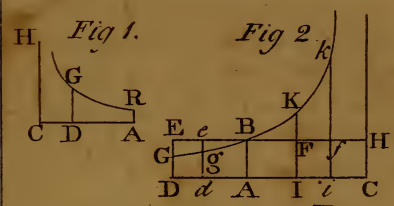
Therefore as Aa is to Dd, ſo (by the hypotheſis) is DE to AB, and ſo (from the nature of the hyperbola) is CA to CD; and, by compoſition, ſo is Ca to Cd. Therefore the areas ABba, DEed, that is, the ſpaces deſcribed, are equal among themſelves, and the firſt velocities AB, DE are proportional to the laſt ab, de; and therefore, by diviſion, proportional to the parts of the velocities loſt, AB — ab, DE — de. Q.E.D.

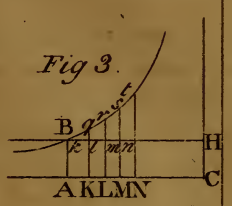
PROPOSITION VII. THEOREM V.

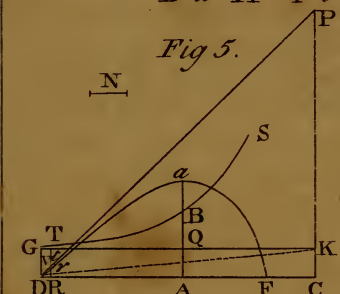
If ſpherical bodies are reſiſted in the duplicate ratio of their velocities, in times which are as the firſt motions directly, and the firſt reſiſtances inverſely, they will loſe parts of their motions proportional to the wholes, and will deſcribe ſpaces proportional to thoſe times and the firſt velocities conjunctly.

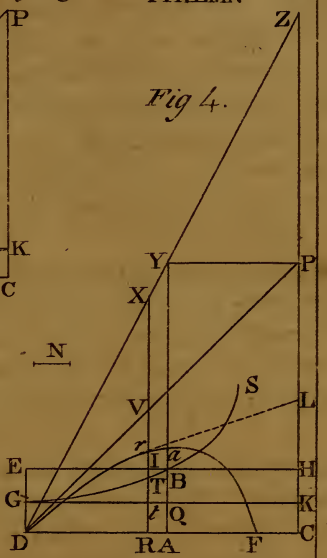
For the parts of the motions loſt are as the reſiſtances and times conjunctly. Therefore, that thoſe parts may be proportional to the wholes, the reſiſtance and time conjunctly ought to be as the motion. Therefore the time will be as the motion directly and the reſiſtance inverſely. Wherefore the particles of the times being taken in that ratio, the bodies will always loſe parts of their motions proportional to the wholes, and therefore will retain velocities always proportional to their firſt velocities. And becauſe of the given ratio of the velocities, they will always deſcribe ſpaces which are as the firſt velocities and the times conjunctly. Q.E.D.

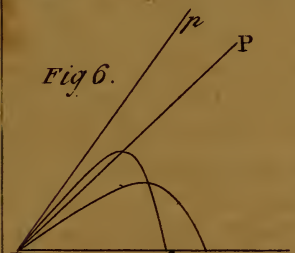
Cor. 1. Therefore if bodies equally ſwift are reſiſted in a duplicate ratio of their diameters, homogeneous globes moving with any velocities whatſoever, by deſcribing ſpaces proportional to their diameters, will loſe parts of their motions proportional to the wholes. For the motion of each globe will be as its velocity and maſs conjunctly, that is, as the velocity and the cube of its diameter; the reſiſtance (by ſuppoſition) will be as the ſquare of the diameter and the ſquare of the velocity conjunctly; and the time (by this propoſition) is in the former ratio directly, and in the latter inverſely, that is, as the diameter directly and the velocity inverſely; and therefore the ſpace, which is proportional to the time and velocity, is as the diameter.

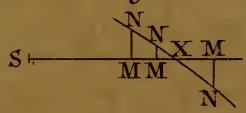


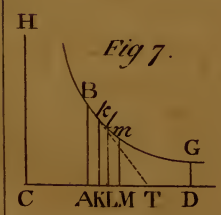


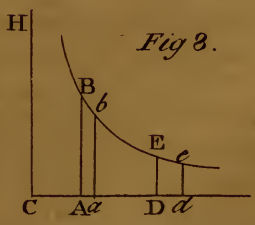












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Cor. 2. If bodies equally ſwift are reſiſted in a ſeſquiplicate ratio of their diameters, homogeneous globes, moving with any velocities whatſoever, by deſcribing ſpaces that are in a ſeſquiplicate ratio of the diameters, will loſe parts of their motions proportional to the wholes.

Cor. 3. And univerſally, if equally ſwift bodies are reſiſted in the ratio of any power of the diameters, the ſpaces, in which homogeneous globes moving with any velocity whatſoever, will loſe parts of their motions proportional to the wholes, will be as the cubes of the diameters applied to that power. Let thoſe diameters be D and E; and if the reſiſtances, where the velocities are ſuppoſed equal, are as D^n and E^n; the ſpaces in which the globes, moving with any velocities whatſoever, will loſe parts of their motions proportional to the wholes will be as D^{3-n} and E^{3-n}. And therefore homogeneous globes, in deſcribing ſpaces proportional to D^{3-n} and E^{3-n}, will retain their velocities in the ſame ratio to one another as at the beginning.

Cor. 4. Now if the globes are not homogeneous, the ſpace deſcribed by the denſer globe muſt be augmented in the ratio of the denſity. For the motion, with an equal velocity, is greater in the ratio of the denſity, and the time (by this prop.) is augmented in the ratio of motion directly, and the ſpace deſcribed in the ratio of the time.

Cor. 5. And if the globes move in different mediums, the ſpace, in a medium which, *cæteris paribus*, reſiſts the moſt, muſt be diminiſhed in the ratio of the greater reſiſtance. For the time (by this prop.) will be diminiſhed in the ratio of the augmented reſiſtance, and the ſpace in the ratio of the time.

LEMMA II.

The moment of any genitum *is equal to the moments of each of the generating ſides drawn into the indices of the powers of thoſe ſides, and into their co-efficients continually.*

I call any quantity a *genitum* which is not made by addition or ſubduction of divers parts, but is generated or produced in arithmetic by the multiplication, diviſion, or extraction of the root of any terms whatſoever; in geometry by the

invention of contents and ſides, or of the extremes and means of proportionals. Quantities of this kind are products, quotients, roots, rectangles, ſquares, cubes, ſquare and cubic ſides, and the like. Theſe quantities I here conſider as variable and indetermined, and increaſing or decreaſing, as it were, by a perpetual motion or flux; and I underſtand their momentaneous increments or decrements by the name of moments; ſo that the increments may be eſteemed as added or affirmative moments; and the decrements as ſubducted or negative ones. But take care not to look upon finite particles as ſuch. Finite particles are not moments, but the very quantities generated by the moments. We are to conceive them as the juſt naſcent principles of finite magnitudes. Nor do we in this lemma regard the magnitude of the moments, but their firſt proportion, as naſcent. It will be the ſame thing, if, inſtead of moments, we uſe either the velocities of the increments and decrements (which may alſo be called the motions, mutations, and fluxions of quantities), or any finite quantities proportional to thoſe velocities. The co-efficient of any generating ſide is the quantity which ariſes by applying the genitum to that ſide.

Wherefore the ſenſe of the lemma is, that if the moments of any quantities A, B, C, &c. increaſing or decreaſing by a perpetual flux, or the velocities of the mutations which are proportional to them, be called a, b, c, &c. the moment or mutation of the generated rectangle AB will be $aB + bA$; the moment of the generated content ABC will be $aBC + bAC + cAB$; and the moments of the generated powers A^2, A^3, A^4, $A^{\frac{1}{2}}$, $A^{\frac{3}{2}}$, $A^{\frac{1}{3}}$, $A^{\frac{2}{3}}$, A^{-1}, A^{-2}, $A^{-\frac{1}{2}}$ will be $2aA$, $3aA^2$, $4aA^3$, $\frac{1}{2}aA^{-\frac{1}{2}}$, $\frac{3}{2}aA^{\frac{1}{2}}$, $\frac{1}{3}aA^{-\frac{2}{3}}$, $\frac{2}{3}aA^{-\frac{1}{3}}$, $-aA^{-2}$, $-2aA^{-3}$, $-\frac{1}{2}aA^{-\frac{3}{2}}$ reſpectively; and, in general, that the moment of any power $A^{\frac{n}{m}}$, will be $\frac{n}{m}aA^{\frac{n-m}{m}}$. Alſo, that the moment of the generated quantity A^2B will be $2aAB + bA^2$; the moment of the generated quantity $A^3B^4C^2$ will be $3aA^2B^4C^2 + 4bA^3B^3C^2 + 2cA^3B^4C$; and the moment of the

generated quantity $\frac{A^3}{B^2}$ or A^3B^{-2} will be $3aA^2B^{-2} - 2bA^3B^{-3}$; and so on. The lemma is thus demonstrated.

Case 1. Any rectangle, as AB, augmented by a perpetual flux, when, as yet, there wanted of the sides A and B half their moments $\frac{1}{2}a$ and $\frac{1}{2}b$, was $A - \frac{1}{2}a$ into $B - \frac{1}{2}b$, or $AB - \frac{1}{2}aB - \frac{1}{2}bA + \frac{1}{4}ab$; but as soon as the sides A and B are augmented by the other half moments, the rectangle becomes $A + \frac{1}{2}a$ into $B + \frac{1}{2}b$, or $AB + \frac{1}{2}aB + \frac{1}{2}bA + \frac{1}{4}ab$. From this rectangle subduct the former rectangle, and there will remain the excess $aB + bA$. Therefore with the whole increments a and b of the sides, the increment $aB + bA$ of the rectangle is generated. Q.E.D.

Case 2. Suppose AB always equal to G, and then the moment of the content ABC or GC (by case 1) will be $gC + cG$, that is (putting AB and $aB + bA$ for G and g), $aBC + bAC + cAB$. And the reasoning is the same for contents under ever so many sides. Q.E.D.

Case 3. Suppose the sides A, B, and C, to be always equal among themselves; and the moment $aB + bA$, of A^2, that is, of the rectangle AB, will be $2aA$; and the moment $aBC + bAC + cAB$ of A^3, that is, of the content ABC, will be $3aA^2$. And by the same reasoning the moment of any power A^n is naA^{n-1}. Q.E.D.

Case 4. Therefore since $\frac{1}{A}$ into A is 1, the moment of $\frac{1}{A}$ drawn into A, together with $\frac{1}{A}$ drawn into a, will be the moment of 1, that is, nothing. Therefore the moment of $\frac{1}{A}$, or of A^{-1}, is $\frac{-a}{A^2}$. And generally, since $\frac{1}{A^n}$ into A^n is 1, the moment of $\frac{1}{A^n}$ drawn into A^n together with $\frac{1}{A^n}$ into naA^{n-1} will be nothing. And, therefore, the moment of $\frac{1}{A^n}$ or A^{-n} will be $-\frac{na}{A^{n+1}}$. Q.E.D.

CASE 5. And since A^1 into $A^{\frac{1}{2}}$ is A, the moment of $A^{\frac{1}{2}}$ drawn into $2A^{\frac{1}{2}}$ will be a (by case 3); and, therefore, the moment of $A^{\frac{1}{2}}$ will be $\frac{a}{2A^{\frac{1}{2}}}$ or $\frac{1}{2}aA^{-\frac{1}{2}}$. And, generally, putting $A^{\frac{m}{n}}$ equal to B, then A^m will be equal to B^n, and therefore maA^{m-1} equal to nbB^{n-1}, and maA^{-1} equal to nbB^{-1}, or $nbA^{-\frac{m}{n}}$; and therefore $\frac{m}{n}aA^{\frac{m-n}{n}}$ is equal to b, that is, equal to the moment of $A^{\frac{m}{n}}$. Q.E.D.

CASE 6. Therefore the moment of any generated quantity A^mB^n is the moment of A^m drawn into B^n, together with the moment of B^n drawn into A^m, that is, $maA^{m-1}B^n + nbB^{n-1}A^m$; and that whether the indices m and n of the powers be whole numbers or fractions, affirmative or negative. And the reasoning is the same for contents under more powers. Q.E.D.

COR. 1. Hence in quantities continually proportional, if one term is given, the moments of the rest of the terms will be as the same terms multiplied by the number of intervals between them and the given term. Let A, B, C, D, E, F, be continually proportional; then if the term C is given, the moments of the rest of the terms will be among themselves as — 2A, — B, D, 2E, 3F.

COR. 2. And if in four proportionals the two means are given, the moments of the extremes will be as those extremes. The same is to be understood of the sides of any given rectangle.

COR. 3. And if the sum or difference of two squares is given, the moments of the sides will be reciprocally as the sides.

SCHOLIUM.

In a letter of mine to Mr. *J. Collins*, dated *December* 10, 1672, having described a method of tangents, which I suspected to be the same with *Slusius*'s method, which at that time was not made public, I subjoined these words: *This is one particular, or rather a corollary, of a general method, which extends itself, without any troublesome calculation, not*

only to the drawing of tangents to any curve lines, whether geometrical or mechanical, or any how reſpecting right lines or other curves, but alſo to the reſolving other abſtruſer kinds of problems about the crookedneſs, areas, lengths, centres of gravity of curves &c.; *nor is it (as* Hudden's *method* de Maximis & Minimis) *limited to equations which are free from ſurd quantities. This method I have interwoven with that other of working in equations, by reducing them to infinite ſeries.* So far that letter. And theſe laſt words relate to a treatiſe I compoſed on that ſubject in the year 1671. The foundation of that general method is contained in the preceding lemma.

PROPOSITION VIII. THEOREM VI.

If a body in an uniform medium, being uniformly acted upon by the force of gravity, aſcends or deſcends in a right line; and the whole ſpace deſcribed be diſtinguiſhed into equal parts, and in the beginning of each of the parts (by adding or ſubducting the reſiſting force of the medium to or from the force of gravity, when the body aſcends or deſcends) you collect the abſolute forces; I ſay, that thoſe abſolute forces are in a geometrical progreſſion. (Pl. 2, Fig. 1.)

For let the force of gravity be expounded by the given line AC; the force of reſiſtance by the indefinite line AK; the abſolute force in the deſcent of the body by the difference KC; the velocity of the body by a line AP, which ſhall be a mean proportional between AK and AC, and therefore in a ſubduplicate ratio of the reſiſtance; the increment of the reſiſtance made in a given particle of time by the lineolæ KL, and the contemporaneous increment of the velocity by the lineolæ PQ; and with the centre C, and rectangular aſymptotes CA, CH, deſcribe any hyperbola BNS meeting the erected perpendiculars AB, KN, LO in B, N, and O. Becauſe AK is as AP^2, the moment KL of the one will be as the moment 2APQ of the other, that is, as AP × KC; for the increment PQ of the velocity is (by law 2.) proportional to the generating force KC. Let the ratio of KL be compounded with the ratio of KN, and the rectangle KL × KN will become as AP × KC × KN; that is (becauſe the rectangle KC × KN is given), as AP. But the ultimate

ratio of the hyperbolic area KNOL to the rectangle KL × KN becomes, when the points K and L coincide, the ratio of equality. Therefore that hyperbolic evanescent area is as AP. Therefore the whole hyperbolic area ABOL is compoſed of particles KNOL which are always proportional to the velocity AP; and therefore is itſelf proportional to the ſpace deſcribed with that velocity. Let that area be now divided into equal parts, as ABMI, IMNK, KNOL, &c. and the abſolute forces AC, IC, KC, LC, &c. will be in a geometrical progreſſion. Q.E.D. And by a like reaſoning, in the aſcent of the body, taking, on the contrary ſide of the point A, the equal areas ABmi, imnk, knol, &c. it will appear that the abſolute forces AC, iC, kC, lC, &c. are continually proportional. Therefore if all the ſpaces in the aſcent and deſcent are taken equal, all the abſolute forces lC, kC, iC, AC, IC, KC, LC, &c. will be continually proportional. Q.E.D.

Cor. 1. Hence if the ſpace deſcribed be expounded by the hyperbolic area ABNK, the force of gravity, the velocity of the body, and the reſiſtance of the medium, may be expounded by the lines AC, AP, and AK reſpectively; and *vice verſa*.

Cor. 2. And the greateſt velocity which the body can ever acquire in an infinite deſcent will be expounded by the line AC.

Cor. 3. Therefore if the reſiſtance of the medium anſwering to any given velocity be known, the greateſt velocity will be found, by taking it to that given velocity in a ratio ſubduplicate of the ratio which the force of gravity bears to that known reſiſtance of the medium.

PROPOSITION IX. THEOREM VII.

Suppoſing what is above demonſtrated, I ſay, that if the tangents of the angles of the ſector of a circle, and of an hyperbola, be taken proportional to the velocities, the radius being of a fit magnitude, all the time of the aſcent to the higheſt place will be as the ſector of the circle, and all the time of deſcending from the higheſt place as the ſector of the hyperbola. (Pl. 2, Fig. 2.)

To the right line AC, which expreſſes the force of gravity, let AD be drawn perpendicular and equal. From the centre D with the ſemi-diameter AD deſcribe as well the quadrant

AtE of a circle, as the rectangular hyperbola AVZ, whose axis is AK, principal vertex A, and asymptote DC. Let Dp, DP be drawn; and the circular sector AtD will be as all the time of the ascent to the highest place; and the hyperbolic sector ATD as all the time of descent from the highest place; if so be that the tangents Ap, AP of those sectors be as the velocities. (Pl. 2, Fig. 2.)

Case 1. Draw Dvq cutting off the moments or least particles tDv and qDp, described in the same time, of the sector ADt and of the triangle ADp. Since those particles (because of the common angle D) are in a duplicate ratio of the sides, the particle tDv will be as $\frac{qDp \times tD^2}{pD^2}$, that is (because tD is given), as $\frac{qDp}{pD^2}$. But pD^2 is $AD^2 + Ap^2$, that is, $AD^2 + AD \times Ak$, or $AD \times Ck$; and qDp is $\frac{1}{2} AD \times pq$. Therefore tDv, the particle of the sector, is as $\frac{pq}{Ck}$; that is, as the least decrement pq of the velocity directly, and the force Ck which diminishes the velocity, inversely; and therefore as the particle of time answering to the decrement of the velocity And, by composition, the sum of all the particles tDv in the sector ADt will be as the sum of the particles of time answering to each of the lost particles pq of the decreasing velocity Ap, till that velocity, being diminished into nothing, vanishes; that is the whole sector ADt is as the whole time of ascent to the highest place. Q.E.D.

Case 2. Draw DQV cutting off the least particles TDV and PDQ of the sector DAV, and of the triangle DAQ; and these particles will be to each other as DT^2 to DP^2, that is (if TX and AP are parallel), as DX^2 to DA^2 or TX^2 to AP^2; and, by division, as $DX^2 - TX^2$ to $DA^2 - AP^2$. But, from the nature of the hyperbola, $DX^2 - TX^2$ is AD^2; and, by the supposition, AP^2 is $AD \times AK$. Therefore the particles are to each other as AD^2 to $AD^2 - AD \times AK$; that is, as AD to AD — AK or AC to CK: and therefore the particle TDV of the sector is $\frac{PDQ \times AC}{CK}$; and therefore (because AC

and AD are given) as $\frac{PQ}{CK}$; that is, as the increment of the velocity directly, and as the force generating the increment inverſely; and therefore as the particle of the time anſwering to the increment. And, by compoſition, the ſum of the particles of time, in which all the particles PQ of the velocity AP are generated, will be as the ſum of the particles of the ſector ATD; that is, the whole time will be as the whole ſector. Q.E.D.

Cor. 1. Hence if AB be equal to a fourth part of AC, the ſpace which a body will deſcribe by falling in any time will be to the ſpace which the body could deſcribe, by moving uniformly on in the ſame time with its greateſt velocity AC, as the area ABNK, which expreſſes the ſpace deſcribed in falling to the area ATD, which expreſſes the time. For ſince AC is to AP as AP to AK, then (by cor. 1, lem. 2, of this book) LK is to PQ as 2AK to AP, that is, as 2AP to AC, and thence LK is to $\frac{1}{2}$PQ as AP to $\frac{1}{4}$AC or AB; and KN is to AC or AD as AB to CK; and, therefore, *ex æquo*, LKNO to DPQ as AP to CK. But DPQ was to DTV as CK to AC. Therefore, *ex æquo*, LKNO is to DTV as AP to AC; that is, as the velocity of the falling body to the greateſt velocity whieh the body by falling can acquire. Since, therefore, the moments LKNO and DTV of the areas ABNK and ATD are as the velocities, all the parts of thoſe areas generated in the ſame time will be as the ſpaces deſcribed in the ſame time; and therefore the whole areas ABNK and ADT, generated from the beginning, will be as the whole ſpaces deſcribed from the beginning of the deſcent. Q.E.D.

Cor. 2. The ſame is true alſo of the ſpace deſcribed in the aſcent. That is to ſay, that all that ſpace is to the ſpace deſcribed in the ſame time, with the uniform velocity AC, as the area ABnk is to the ſector ADt.

Cor. 3. The velocity of the body, falling in the time ATD, is to the velocity which it would acquire in the ſame time in a non-reſiſting ſpace, as the triangle APD to the hyperbolic ſector ATD. For the velocity in a non-reſiſting medium would be as the time ATD, and in a reſiſting medium is as

AP, that is, as the triangle APD. And thoſe velocities, at the beginning of the deſcent, are equal among themſelves, as well as thoſe areas ATD, APD.

Cor. 4. By the ſame argument, the velocity in the aſcent is to the velocity with which the body in the ſame time, in a non-reſiſting ſpace, would loſe all its motion of aſcent, as the triangle ApD to the circular ſector AtD; or as the right line Ap to the arc At.

Cor. 5. Therefore the time in which a body, by falling in a reſiſting medium, would acquire the velocity AP, is to the time in which it would acquire its greateſt velocity AC, by falling in a non-reſiſting ſpace, as the ſector ADT to the triangle ADC: and the time in which it would loſe its velocity Ap, by aſcending in a reſiſting medium, is to the time in which it would loſe the ſame velocity by aſcending in a non-reſiſting ſpace, as the arc At to its tangent Ap.

Cor. 6. Hence from the given time there is given the ſpace deſcribed in the aſcent or deſcent. For the greateſt velocity of a body deſcending *in infinitum* is given (by corol. 2 and 3, theor. 6, of this book); and thence the time is given in which a body would acquire that velocity by falling in a non-reſiſting ſpace. And taking the ſector ADT or ADt to the triangle ADC in the ratio of the given time to the time juſt now found, there will be given both the velocity AP or Ap, and the area ABNK or ABnk, which is to the ſector ADT, or ADt, as the ſpace ſought to the ſpace which would, in the given time, be uniformly deſcribed with that greateſt velocity found juſt before.

Cor. 7. And by going backward, from the given ſpace of aſcent or deſcent ABnk or ABNK, there will be given the time ADt or ADT.

PROPOSITION X. PROBLEM III.

Suppoſe the uniform force of gravity to tend directly to the plane of the horizon, and the reſiſtance to be as the denſity of the medium and the ſquare of the velocity conjunctly: it is propoſed to find the denſity of the medium in each place, which ſhall make the body move in any given curve line; the velocity of the body and the reſiſtance of the medium in each place. (Pl. 2, Fig. 3.)

Let PQ be a plane perpendicular to the plane of the ſcheme itſelf; PFHQ a curve line meeting that plane in the points P and Q; G, H, I, K four places of the body going on in this curve from F to Q; and GB, HC, ID, KE four parallel ordinates let fall from theſe points to the horizon, and ſtanding on the horizontal line PQ at the points B, C, D, E; and let the diſtances BC, CD, DE, of the ordinates be equal among themſelves. From the points G and H let the right lines GL, HN, be drawn touching the curve in G and H, and meeting the ordinates CH, DI, produced upwards, in L and N; and complete the parallelogram HCDM. And the times in which the body deſcribes the arcs GH, HI, will be in a ſubduplicate ratio of the altitudes LH, NI, which the bodies would deſcribe in thoſe times, by falling from the tangents; and the velocities will be as the lengths deſcribed GH, HI directly, and the times inverſely. Let the times be expounded by T and t, and the velocities by $\frac{GH}{T}$ and $\frac{HI}{t}$; and the decrement of the velocity produced in the time t will be expounded by $\frac{GH}{T} - \frac{HI}{t}$. This decrement ariſes from the reſiſtance which retards the body, and from the gravity which accelerates it. Gravity, in a falling body, which in its fall deſcribes the ſpace NI, produces a velocity with which it would be able to deſcribe twice that ſpace in the ſame time, as *Galileo* has demonſtrated; that is, the velocity $\frac{2NI}{t}$: but if the body deſcribes the arc HI, it augments that arc only by the length HI — HN or $\frac{MI \times NI}{HI}$; and therefore generates only the velocity $\frac{2MI \times NI}{t \times HI}$. Let this velocity be added to the before-mentioned decrement, and we ſhall have the decrement of the velocity ariſing from the reſiſtance alone, that is, $\frac{GH}{T} - \frac{HI}{t} + \frac{2MI \times NI}{t \times HI}$. Therefore ſince, in the ſame time, the action of gravity generates, in a falling

body, the velocity $\frac{2NI}{t}$, the reſiſtance will be to the gravity as $\frac{GH}{T} - \frac{HI}{t} + \frac{2MI \times NI}{t \times HI}$ to $\frac{2NI}{t}$ or as $\frac{t \times GH}{T} - HI + \frac{2MI \times NI}{HI}$ to 2NI.

Now for the abſciſſas CB, CD, CE, put — o, o, 2o. For the ordinate CH put P; and for MI put any ſeries $Qo + Ro^2 + So^3$, &c. And all the terms of the ſeries after the firſt, that is, $Ro^2 + So^3 +$, &c. will be NI; and the ordinates DI, EK, and BG will be $P - Qo - Ro^2 - So^3 -$, &c. $P - 2Qo - 4Ro^2 - 8So^3 -$, &c. and $P + Qo - Ro^2 + So^3 -$, &c. reſpectively. And by ſquaring the differences of the ordinates BG — CH and CH — DI, and to the ſquares thence produced adding the ſquares of BC and CD themſelves, you will have $oo + QQoo - 2QRo^3 +$, &c. and $oo + QQoo + 2QRo^3 +$, &c. the ſquares of the arcs GH, HI; whoſe roots $o\sqrt{1 + QQ} - \frac{QRoo}{\sqrt{1 + QQ}}$, and $o\sqrt{1 + QQ} + \frac{QRoo}{\sqrt{1 + QQ}}$ are the arcs GH and HI. Moreover, if from the ordinate CH there be ſubducted half the ſum of the ordinates BG and DI, and from the ordinate DI there be ſubducted half the ſum of the ordinates CH and EK, there will remain Roo and $Roo + 3So^3$, the verſed ſines of the arcs GI and HK. And theſe are proportional to the lineolæ LH and NI, and therefore in the duplicate ratio of the infinitely ſmall times T and t: and thence the ratio $\frac{t}{T}$ is $\sqrt{\frac{R + 3So}{R}}$ or $\frac{R + \frac{3}{2}So}{R}$; and $\frac{t \times GH}{T} - HI + \frac{2MI \times NI}{HI}$, by ſubſtituting the values of $\frac{t}{T}$, GH, HI, MI and NI juſt found, becomes $\frac{3Soo}{2R}\sqrt{1 + QQ}$. And ſince 2NI is 2Roo, the reſiſtance will be now to the gravity as $\frac{3Soo}{2R}\sqrt{1 + QQ}$ to 2Roo, that is, as $3S\sqrt{1 + QQ}$ to 4RR.

And the velocity will be such, that a body going off therewith from any place H, in the direction of the tangent HN, would describe, in vacuo, a parabola, whose diameter is HC, and its latus rectum $\frac{HN^2}{NI}$ or $\frac{1+QQ}{R}$.

And the resistance is as the density of the medium and the square of the velocity conjunctly; and therefore the density of the medium is as the resistance directly, and the square of the velocity inversely; that is, as $\frac{3S\sqrt{1+QQ}}{4RR}$ directly and $\frac{1+QQ}{R}$ inversely; that is, as $\frac{S}{R\sqrt{1+QQ}}$. Q.E.I.

Cor. 1. If the tangent HN be produced both ways, so as to meet any ordinate AF in T, $\frac{HT}{AC}$ will be equal to $\sqrt{1+QQ}$, and therefore in what has gone before may be put for $\sqrt{1+QQ}$. By this means the resistance will be to the gravity as $3S \times HT$ to $4RR \times AC$; the velocity will be as $\frac{HT}{AC\sqrt{R}}$, and the density of the medium will be as $\frac{S \times AC}{R \times HT}$.

Cor. 2. And hence, if the curve line PFHQ be defined by the relation between the base or abscissa AC and the ordinate CH, as is usual, and the value of the ordinate be resolved into a converging series, the problem will be expeditiously solved by the first terms of the series; as in the following examples.

Example 1. Let the line PFHQ be a semi-circle described upon the diameter PQ, to find the density of the medium that shall make a projectile move in that line.

Bisect the diameter PQ in A; and call AQ, n; AC, a; CH, e; and CD, o: then DI^2 or $AQ^2 - AD^2 = nn - aa - 2ao - oo$, or $ee - 2ao - oo$; and the root being extracted by our method, will give $DI = e - \frac{ao}{e} - \frac{oo}{2e} - \frac{aaoo}{2e^3} - \frac{ao^3}{2e^3} - \frac{a^3o^3}{2e^5}$, &c. Here put nn for $ee + aa$, and

DI will become $= e - \frac{ao}{e} - \frac{nnoo}{2e^3} - \frac{anno^3}{2e^5} -$, &c.

Such series I distinguish into successive terms after this manner: I call that the first term in which the infinitely small quantity o is not found; the second, in which that quantity is of one dimension only; the third, in which it arises to two dimensions; the fourth, in which it is of three; and so *ad infinitum*. And the first term, which here is e, will always denote the length of the ordinate CH, standing at the beginning of the indefinite quantity o. The second term, which here is $\frac{ao}{e}$, will denote the difference between CH and DN; that is, the lineolæ MN which is cut off by completing the parallelogram HCDM; and therefore always determines the position of the tangent HN; as, in this case, by taking MN to HM as $\frac{ao}{e}$ to o, or a to e. The third term, which here is $\frac{nnoo}{2e^3}$, will represent the lineolæ IN, which lies between the tangent and the curve; and therefore determines the angle of contact IHN, or the curvature which the curve line has in H. If that lineolæ IN is of a finite magnitude, it will be expressed by the third term, together with those that follow *in infinitum*. But if that lineolæ be diminished *in infinitum*, the terms following become infinitely less than the third term, and therefore may be neglected. The fourth term determines the variation of the curvature; the fifth, the variation of the variation; and so on. Whence, by the way, appears no contemptible use of these series in the solution of problems that depend upon tangents, and the curvature of curves.

Now compare the series $e - \frac{ao}{e} - \frac{nnoo}{2e^3} - \frac{anno^3}{2e^5}$ — &c. with the series $P - Qo - Roo - So^3$ — &c. and for P, Q, R and S, put e, $\frac{a}{e}$, $\frac{nn}{2e^3}$ and $\frac{ann}{2e^5}$, and for $\sqrt{1+QQ}$ put $\sqrt{1+\frac{aa}{ee}}$ or $\frac{n}{e}$; and the density of the medium will come out as $\frac{a}{ne}$;

that is (becauſe n is given), as $\frac{a}{e}$, or $\frac{AC}{CH}$, that is, as that length of the tangent HT, which is terminated at the ſemi-diameter AF ſtanding perpendicularly on PQ: and the reſiſtance will be to the gravity as 3a to 2n, that is, as 3AC to the diameter PQ of the circle; and the velocity will be as $\sqrt{CH}$. Therefore if the body goes from the place F, with a due velocity, in the direction of a line parallel to PQ, and the denſity of the medium in each of the places H is as the length of the tangent HT, and the reſiſtance alſo in any place H is to the force of gravity as 3AC to PQ, that body will deſcribe the quadrant FHQ of a circle. Q.E.I.

But if the ſame body ſhould go from the place P, in the direction of a line perpendicular to PQ, and ſhould begin to move in an arc of the ſemi-circle PFQ, we muſt take AC or a on the contrary ſide of the centre A; and therefore its ſign muſt be changed, and we muſt put $-a$ for $+a$. Then the denſity of the medium would come out as $-\frac{a}{e}$. But nature does not admit of a negative denſity, that is, a denſity which accelerates the motion of bodies; and therefore it cannot naturally come to paſs that a body by aſcending from P ſhould deſcribe the quadrant PF of a circle. To produce ſuch an effect, a body ought to be accelerated by an impelling medium, and not impeded by a reſiſting one.

Example 2. Let the line PFQ be a parabola, having its axis AF perpendicular to the horizon PQ, to find the denſity of the medium, which will make a projectile move in that line. (Pl. 2, Fig. 4).

From the nature of the parabola, the rectangle PDQ is equal to the rectangle under the ordinate DI and ſome given right line: that is, if that right line be called b; PC, a; PQ, c; CH, e; and CD, o; the rectangle $a + o$ into $c - a - o$ or $ac - aa - 2ao + co - oo$, is equal to the rectangle b into DI, and therefore DI is equal to $\frac{ac - aa}{b} + \frac{c - 2a}{b}o - \frac{oo}{b}$.

Now the ſecond term $\frac{c - 2a}{b}o$ of this ſeries is to be put for

Qo, and the third term $\frac{oo}{b}$ for Roo. But since there are no more terms, the coefficient S of the fourth term will vanish; and therefore the quantity $\frac{S}{R\sqrt{1+QQ}}$, to which the density of the medium is proportional, will be nothing. Therefore, where the medium is of no density, the projectile will move in a parabola; as *Galileo* hath heretofore demonstrated. Q.E.I.

Example 3. Let the line AGK be an hyperbola, having its asymptote NX perpendicular to the horizontal plane AK, to find the density of the medium that will make a projectile move in that line. (Pl. 2, Fig. 5.)

Let MX be the other asymptote, meeting the ordinate DG produced in V; and, from the nature of the hyperbola, the rectangle of XV into VG will be given. There is also given the ratio of DN to VX, and therefore the rectangle of DN into VG is given. Let that be bb: and, completing the parallelogram DNXZ, let BN be called a; BD, o; NX, c; and let the given ratio of VZ to ZX or DN be $\frac{m}{n}$. Then DN will be equal to $a-o$, VG equal to $\frac{bb}{a-o}$, VZ equal to $\frac{m}{n}\times a-o$, and GD or NX — VZ — VG equal to $c-\frac{m}{n}a+\frac{m}{n}o-\frac{bb}{a-o}$. Let the term $\frac{bb}{a-o}$ be resolved into the converging series $\frac{bb}{a}+\frac{bb}{aa}o+\frac{bb}{a^3}oo+\frac{bb}{a^4}o^3$, &c. and GD will become equal to $c-\frac{m}{n}a-\frac{bb}{a}+\frac{m}{n}o-\frac{bb}{aa}o-\frac{bb}{a^3}o^2-\frac{bb}{a^4}o^3$, &c. The second term $\frac{m}{n}o-\frac{bb}{aa}o$ of this series is to be used for Qo; the third $\frac{bb}{a^3}o^2$, with its sign changed for Ro^2; and the fourth $\frac{bb}{a^4}o^3$, with its sign changed also for So^3,

and their coefficients $\frac{m}{n} - \frac{bb}{aa}$, $\frac{bb}{a^3}$ and $\frac{bb}{a^4}$ are to be put for Q, R, and S in the former rule. Which being done, the denfity of the medium will come out as $\frac{\frac{bb}{a^4}}{\frac{bb}{a^3}\sqrt{1+\frac{mm}{nn}-\frac{2mbb}{naa}+\frac{b^4}{a^4}}}$ or $\frac{1}{\sqrt{aa+\frac{mm}{nn}aa-\frac{2mbb}{n}+\frac{b^4}{aa}}}$, that is, if in VZ you take VY equal to VG, as $\frac{1}{XY}$. For aa and $\frac{m^2}{n^2}a^2 - \frac{2mbb}{n} + \frac{b^4}{aa}$ are the fquares of XZ and ZY. But the ratio of the refiftance to gravity is found to be that of 3XY to 2YG; and the velocity is that with which the body would defcribe a parabola, whofe vertex is G, diameter DG, latus rectum $\frac{XY^2}{VG}$. Suppofe, therefore, that the denfities of the medium in each of the places G are reciprocally as the diftances XY, and that the refiftance in any place G is to the gravity as 3XY to 2YG; and a body let go from the place A, with a due velocity, will defcribe that hyperbola AGK. Q.E.I.

Example 4. Suppofe, indefinitely, the line AGK to be an hyperbola defcribed with the centre X, and the afymptotes MX, NX, fo that, having conftructed the rectangle XZDN, whofe fide ZD cuts the hyperbola in G and its afymptote in V, VG may be reciprocally as any power DN^n of the line ZX or DN, whofe index is the number n: to find the denfity of the medium in which a projected body will defcribe this curve. (Pl. 2, Fig. 5.)

For BN, BD, NX, put A, O, C refpectively, and let VZ be to XZ or DN as d to e, and VG be equal to $\frac{bb}{DN^n}$; then DN will be equal to $A - O$, $VG = \frac{bb}{\overline{A-O}^n}$, $VZ = \frac{d}{e}\overline{A-O}$, and GD or $NX - VZ - VG$ equal to $C - \frac{d}{e}A + \frac{d}{e}O -$

$\frac{bb}{\overline{A-O|^n}}$. Let the term $\frac{bb}{\overline{A-O|^n}}$ be reſolved into an infinite ſeries $\frac{bb}{A^n} + \frac{nbb}{A^{n+1}} \times O + \frac{nn+n}{2A^{n+2}} \times bb\,O^2 + \frac{n^3+3nn+2n}{6A^{n+3}} \times bb\,O^3$, &c. and GD will be equal to $C - \frac{d}{e}A - \frac{bb}{A^n} + \frac{d}{e}O - \frac{nbb}{A^{n+1}}O - \frac{+nn+n}{2A^{n+2}}bb\,O^2 - \frac{+n^3+3nn+2n}{6A^{n+3}}bbO^3$, &c. The ſecond term $\frac{d}{e}O - \frac{nbb}{A^{n+1}}O$ of this ſeries is to be uſed for Qo, the third $\frac{nn+n}{2A^{n+2}}$ bbO^2 for Roo, the fourth $\frac{n^3+3nn+2n}{6A^{n+3}}$ bbO^3 for So^3. And thence the denſity of the medium $\frac{S}{R\sqrt{1+QQ}}$, in any place G, will be $\frac{n+2}{3\sqrt{A^2 + \frac{dd}{ee}A^2 - \frac{2dnbb}{eA^n}A + \frac{nnb^4}{A^{2n}}}}$, and therefore if in VZ you take VY equal to $n \times VG$, that denſity is reciprocally as XY. For A^2 and $\frac{dd}{ee}A^2 - \frac{2dnbb}{eA^n}A + \frac{nnb^4}{A^{2n}}$ are the ſquares of XZ and ZY. But the reſiſtance in the ſame place G is to the force of gravity as $3S \times \frac{XY}{A}$ to 4RR, that is, as XY to $\frac{2nn+2n}{n+2}VG$. And the velocity there is the ſame where-with the projected body would move in a parabola, whoſe ver-tex is G, diameter GD, and latus rectum $\frac{1+QQ}{R}$ or $\frac{2XY^2}{nn+n \times VG^2}$. Q.E.I.

SCHOLIUM.

In the ſame manner that the denſity of the medium comes out to be as $\frac{S \times AC}{R \times HT}$, in cor. 1, if the reſiſtance is put as any power V^n of the velocity V, the denſity of the medium

will come out to be as $\frac{S}{R^{\frac{4-n}{2}}} \times \left|\frac{AC}{HT}\right|^{n-1}$. (Fig. 3.)

And therefore if a curve can be found, such that the ratio of $\frac{S}{R^{\frac{4-n}{2}}}$ to $\left|\frac{HT}{AC}\right|^{n-1}$, or of $\frac{S^2}{R^{4-n}}$ to $\left|1+QQ\right|^{n-1}$ may be given; the body, in an uniform medium, whose resistance is as the power V^n of the velocity V, will move in this curve. But let us return to more simple curves.

Because there can be no motion in a parabola except in a non-resisting medium, but in the hyperbolas here described it is produced by a perpetual resistance; it is evident that the line which a projectile describes in an uniformly resisting medium approaches nearer to these hyperbolas than to a parabola. That line is certainly of the hyperbolic kind, but about the vertex it is more distant from the asymptotes, and in the parts remote from the vertex draws nearer to them than these hyperbolas here described. The difference, however, is not so great between the one and the other but that these latter may be commodiously enough used in practice instead of the former. And perhaps these may prove more useful than an hyperbola that is more accurate, and at the same time more compounded. They may be made use of, then, in this manner. (Pl. 2, Fig. 5.)

Complete the parallelogram XYGT, and the right line GT will touch the hyperbola in G, and therefore the density of the medium in G is reciprocally as the tangent GT, and the velocity there as $\sqrt{\frac{GT^2}{GV}}$; and the resistance is to the force of gravity as GT to $\frac{2nn+2n}{n+2} \times GV$.

Therefore if a body projected from the place A, in the direction of the right line AH (Pl. 2, Fig. 6), describes the hyperbola AGK, and AH produced meets the asymptote NX in H, and AI drawn parallel to it meets the other asymptote MX in I; the density of the medium in A will be reciprocally as AH, and the velocity of the body as $\sqrt{\frac{AH^2}{AI}}$, and the resistance

there to the force of gravity as AH to $\frac{2nn + 2n}{n + 2} \times$ AI.

Hence the following rules are deduced.

RULE 1. If the denſity of the medium at A, and the velocity with which the body is projected remain the ſame, and the angle NAH be changed; the lengths AH, AI, HX will remain. Therefore if thoſe lengths, in any one caſe, are found, the hyperbola may afterwards be eaſily determined from any given angle NAH.

RULE 2. If the angle NAH, and the denſity of the medium at A, remain the ſame, and the velocity with which the body is projected be changed, the length AH will continue the ſame; and AI will be changed in a duplicate ratio of the velocity reciprocally.

RULE 3. If the angle NAH, the velocity of the body at A, and the accelerative gravity remain the ſame, and the proportion of the reſiſtance at A to the motive gravity be augmented in any ratio; the proportion of AH to AI will be augmented in the ſame ratio, the latus rectum of the above mentioned parabola remaining the ſame, and alſo the length $\frac{AH^2}{AI}$ proportional to it; and therefore AH will be diminiſhed in the ſame ratio, and AI will be diminiſhed in the duplicate of that ratio. But the proportion of the reſiſtance to the weight is augmented, when either the ſpecific gravity is made leſs, the magnitude remaining equal, or when the denſity of the medium is made greater, or when, by diminiſhing the magnitude, the reſiſtance becomes diminiſhed in a leſs ratio than the weight.

RULE 4. Becauſe the denſity of the medium is greater near the vertex of the hyperbola than it is in the place A, that a mean denſity may be preſerved, the ratio of the leaſt of the tangents GT to the tangent AH ought to be found, and the denſity in A augmented in a ratio a little greater than that of half the ſum of thoſe tangents to the leaſt of the tangents GT.

RULE 5. If the lengths AH, AI are given, and the figure AGK is to be deſcribed, produce HN to X, ſo that HX may

be to AI as n + 1 to 1; and with the centre X, and the afymptotes MX, NX, defcribe an hyperbola through the point A, fuch that AI may be to any of the lines VG as XV^n to XI^n.

Rule 6. By how much the greater the number n is, fo much the more accurate are thefe hyperbolas in the afcent of the body from A, and lefs accurate in its defcent to K; and the contrary. The conic hyperbola keeps a mean ratio between thefe, and is more fimple than the reft. Therefore if the hyperbola be of this kind, and you are to find the point K, where the projected body falls upon any right line AN paffing through the point A, let AN produced meet the afymptotes MX, NX in M and N, and take NK equal to AM.

Rule 7. And hence appears an expeditious method of determining this hyperbola from the phænomena. Let two fimilar and equal bodies be projected with the fame velocity, in different angles HAK, hAk (Pl. 2, Fig. 6), and let them fall upon the plane of the horizon in K and k; and note the proportion of AK to Ak. Let it be as d to e. Then erecting a perpendicular AI of any length, affume any how the length AH or Ah, and thence graphically, or by fcale and compafs, collect the lengths AK, Ak (by rule 6). If the ratio of AK to Ak be the fame with that of d to e, the length of AH was rightly affumed. If not, take on the indefinite right line SM (Pl. 2, Fig. 7), the length SM equal to the affumed AH; and erect a perpendicular MN equal to the difference $\frac{AK}{Ak} - \frac{d}{e}$ of the ratios drawn into any given right line. By the like method, from feveral affumed lengths AH, you may find feveral points N; and draw through them all a regular curve NNXN, cutting the right line SMMM in X. Laftly, affume AH equal to the abfciffa SX, and thence find again the length AK; and the lengths, which are to the affumed length AI, and this laft AH, as the length AK known by experiment, to the length AK laft found, will be the true lengths AI and AH, which were to be found. But thefe being given, there will be given alfo the refifting force of the medium in the place A, it being to

the force of gravity as AH to 2AI. Let the denſity of the medium be increaſed by rule 4, and if the reſiſting force juſt found be increaſed in the ſame ratio, it will become ſtill more accurate.

Rule 8. The lengths AH, HX being found; let there be now required the poſition of the line AH, according to which a projectile thrown with that given velocity ſhall fall upon any point K. At the points A and K (Pl. 2, Fig. 6) erect the lines AC, KF perpendicular to the horizon; whereof let AC be drawn downwards, and be equal to AI or $\frac{1}{2}$HX. With the aſymptotes AK, KF, deſcribe an hyperbola, whoſe conjugate ſhall paſs through the point C; and from the centre A, with the interval AH, deſcribe a circle cutting that hyperbola in the point H; then the projectile thrown in the direction of the right line AH will fall upon the point K. Q.E.I. For the point H, becauſe of the given length AH, muſt be ſomewhere in the circumference of the deſcribed circle. Draw CH meeting AK and KF in E and F; and becauſe CH, MX are parallel, and AC, AI equal, AE will be equal to AM, and therefore alſo equal to KN. But CE is to AE as FH to KN, and therefore CE and FH are equal. Therefore the point H falls upon the hyperbolic curve deſcribed with the aſymptotes AK, KF, whoſe conjugate paſſes through the point C; and is therefore found in the common interſection of this hyperbolic curve and the circumference of the deſcribed circle. Q.E.D. It is to be obſerved that this operation is the ſame, whether the right line AKN be parallel to the horizon, or inclined thereto in any angle; and that from two interſections H, H, there ariſe two angles NAH, NAH; and that in mechanical practice it is ſufficient once to deſcribe a circle, then to apply a ruler CH, of an indeterminate length, ſo to the point C, that its part FH, intercepted between the circle and the right line FK, may be equal to its part CE placed between the point C and the right line AK.

What has been ſaid of hyperbolas may be eaſily applied to parabolas. For if (Pl. 2, Fig. 8) a parabola be repreſented by XAGK, touched by a right line XV in the vertex X, and the ordinates IA, VG be as any powers XI^n, XV^n, of the ab-

ſciſſas XI, XV; draw XT, GT, AH, whereof let XT be parallel to VG, and let GT, AH touch the parabola in G and A: and a body projected from any place A, in the direction of the right line AH, with a due velocity, will deſcribe this parabola, if the denſity of the medium in each of the places G be reciprocally as the tangent GT. In that caſe the velocity in G will be the ſame as would cauſe a body, moving in a non-reſiſting ſpace, to deſcribe a conic parabola, having G for its vertex, VG produced downwards for its diameter, and $\frac{2GT^2}{nn - n \times VG}$ for its latus rectum. And the reſiſting force in G will be to the force of gravity as GT to $\frac{2nn - 2n}{n - 2}$ VG.

Therefore if NAK repreſent an horizontal line, and both the denſity of the medium at A, and the velocity with which the body is projected, remaining the ſame, the angle NAH be any how altered, the lengths AH, AI, HX will remain; and thence will be given the vertex X of the parabola, and the poſition of the right line XI; and by taking VG to IA as XV^n to XI^n, there will be given all the points G of the parabola, through which the projectile will paſs.

SECTION III.

Of the motions of bodies which are reſiſted partly in the ratio of the velocities, and partly in the duplicate of the ſame ratio.

PROPOSITION XI. THEOREM VIII.

If a body be reſiſted partly in the ratio and partly in the duplicate ratio of its velocity, and moves in a ſimilar medium by its innate force only; and the times be taken in arithmetical progreſſion; then quantities reciprocally proportional to the velocities, increaſed by a certain given quantity, will be in geometrical progreſſion. (Pl. 3, Fig. 1.)

With the centre C, and the rectangular aſymptotes CADd and CH, deſcribe an hyperbola BEe, and let AB, DE, de, be parallel to the aſymptote CH. In the aſymptote CD let A, G be given points; and if the time be expounded by the hyperbolic area ABED uniformly increaſing, I ſay, that the velocity may be expreſſed by the length DF, whoſe reciprocal

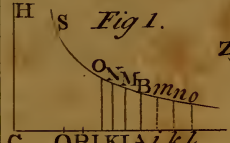
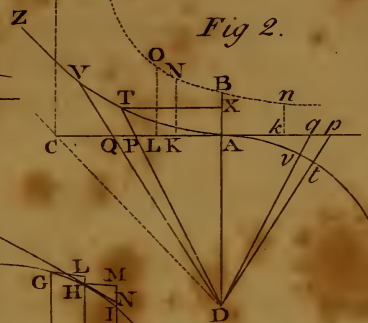


Fig 1.



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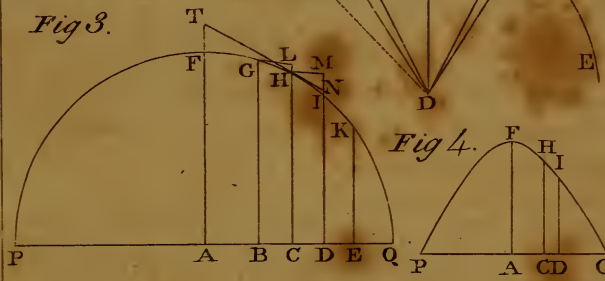

Fig 3.

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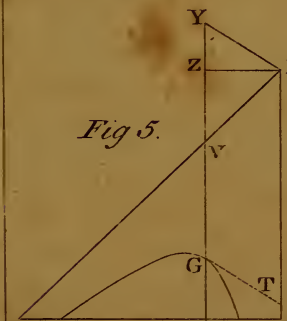

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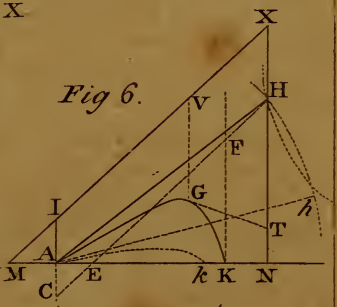

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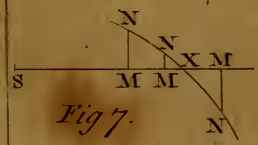

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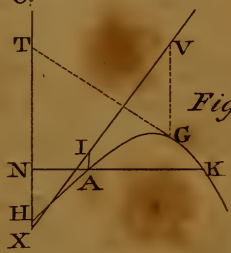

Fig 8.

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GD, together with the given line CG, compoſe the length CD increaſing in a geometrical progreſſion.

For let the areola DEed be the leaſt given increment of the time, and Dd will be reciprocally as DE, and therefore directly as CD. Therefore the decrement of $\frac{1}{GD}$, which (by lem. 2, book 2) is $\frac{Dd}{GD^2}$, will be alſo as $\frac{CD}{GD^2}$ or $\frac{CG+GD}{GD^2}$, that is, as $\frac{1}{GD}+\frac{CG}{GD^2}$. Therefore the time ABED uniformly increaſing by the addition of the given particles EDde, it follows that $\frac{1}{GD}$ decreaſes in the ſame ratio with the velocity. For the decrement of the velocity is as the reſiſtance, that is (by the ſuppoſition), as the ſum of two quantities, whereof one is as the velocity, and the other as the ſquare of the velocity; and the decrement of $\frac{1}{GD}$ is as the ſum of the quantities $\frac{1}{GD}$ and $\frac{CG}{GD^2}$, whereof the firſt is $\frac{1}{GD}$ itſelf, and the laſt $\frac{CG}{GD^2}$ is as $\frac{1}{GD^2}$: therefore $\frac{1}{GD}$ is as the velocity, the decrements of both being analogous. And if the quantity GD, reciprocally proportional to $\frac{1}{GD}$, be augmented by the given quantity CG; the ſum CD, the time ABED uniformly increaſing, will increaſe in a geometrical progreſſion. Q.E.D.

Cor. 1. Therefore, if, having the points A and G given, the time be expounded by the hyperbolic area ABED, the velocity may be expounded by $\frac{1}{GD}$ the reciprocal of GD.

Cor. 2. And by taking GA to GD as the reciprocal of the velocity at the beginning to the reciprocal of the velocity at the end of any time ABED, the point G will be found. And that point being found, the velocity may be found from any other time given.

PROPOSITION XII. THEOREM IX.

The ſame things being ſuppoſed, I ſay, that if the ſpaces deſcribed are taken in arithmetical progreſſion, the velocities

augmented by a certain given quantity will be in geometrical progreſſion. (Pl. 3, Fig. 2.)

In the aſymptote CD let there be given the point R, and, erecting the perpendicular RS meeting the hyperbola in S, let the ſpace deſcribed be expounded by the hyperbolic area RSED; and the velocity will be as the length GD, which, together with the given line CG, compoſes a length CD decreaſing in a geometrical progreſſion, while the ſpace RSED increaſes in an arithmetical progreſſion.

For, becauſe the increment EDde of the ſpace is given, the lineolæ Dd, which is the decrement of GD, will be reciprocally as ED, and therefore directly as CD; that is, as the ſum of the ſame GD and the given length CG. But the decrement of the velocity, in a time reciprocally proportional thereto, in which the given particle of ſpace DdeE is deſcribed, is as the reſiſtance and the time conjunctly, that is, directly as the ſum of two quantities, whereof one is as the velocity, the other as the ſquare of the velocity, and inverſely as the velocity; and therefore directly as the ſum of two quantities, one of which is given, the other is as the velocity. Therefore the decrement both of the velocity and the line GD is as a given quantity and a decreaſing quantity conjunctly; and, becauſe the decrements are analogous, the decreaſing quantities will always be analogous; viz. the velocity, and the line GD. Q.E.D.

Cor. 1. If the velocity be expounded by the length GD, the ſpace deſcribed will be as the hyperbolic area DESR.

Cor. 2. And if the point R be aſſumed any how, the point G will be found, by taking GR to GD as the velocity at the beginning to the velocity after any ſpace RSED is deſcribed. The point G being given, the ſpace is given from the given velocity: and the contrary.

Cor. 3. Whence ſince (by prop. 11) the velocity is given from the given time, and (by this prop.) the ſpace is given from the given velocity; the ſpace will be given from the given time: and the contrary.

PROPOSITION XIII. THEOREM X.

Suppoſing that a body attracted downwards by an uniform gravity aſcends or deſcends in a right line; and that the ſame is reſiſted partly in the ratio of its velocity, and partly in the duplicate ratio thereof; I ſay, that, if right lines parallel to the diameters of a circle and an hyperbola be drawn through the ends of the conjugate diameters, and the velocities be as ſome ſegments of thoſe parallels drawn from a given point, the times will be as the ſectors of the areas cut off by right lines drawn from the centre to the ends of the ſegments; and the contrary. (Pl. 3, Fig. 3.)

Case 1. Suppoſe firſt that the body is aſcending, and from the centre D, with any ſemi-diameter DB, deſcribe a quadrant BETF of a circle, and through the end B of the ſemi-diameter DB draw the indefinite line BAP, parallel to the ſemi-diameter DF. In that line let there be given the point A, and take the ſegment AP proportional to the velocity. And ſince one part of the reſiſtance is as the velocity, and another part as the ſquare of the velocity, let the whole reſiſtance be as $AP^2 + 2BAP$. Join DA, DP, cutting the circle in E and T, and let the gravity be expounded by DA^2, ſo that the gravity ſhall be to the reſiſtance in P as DA^2 to $AP^2 + 2BAP$; and the time of the whole aſcent will be as the ſector EDT of the circle.

For draw DVQ, cutting off the moment PQ of the velocity AP, and the moment DTV of the ſector DET anſwering to a given moment of time; and that decrement PQ of the velocity will be as the ſum of the forces of gravity DA^2 and of reſiſtance $AP^2 + 2BAP$, that is (by 12 prop. 2 book, elem.), as DP^2. Then the area DPQ, which is proportional to PQ, is as DP^2, and the area DTV, which is to the area DPQ as DT^2 to DP^2, is as the given quantity DT^2. Therefore the area EDT decreaſes uniformly according to the rate of the future time, by ſubduction of given particles DTV, and is therefore proportional to the time of the whole aſcent. Q.E.D.

Case 2. If the velocity in the aſcent of the body be expounded by the length AP as before, and the reſiſtance be made as $AP^2 + 2BAP$, and if the force of gravity be leſs

than can be expreſſed by DA^2; take BD (Fig. 4) of ſuch a length, that $AB^2 - BD^2$ may be proportional to the gravity, and let DF be perpendicular and equal to DB, and through the vertex F deſcribe the hyperbola FTVE, whoſe conjugate ſemi-diameters are DB and DF, and which cuts DA in E, and DP, DQ in T and V; and the time of the whole aſcent will be as the hyperbolic ſector TDE.

For the decrement PQ of the velocity, produced in a given particle of time, is as the ſum of the reſiſtance $AP^2 + 2BAP$ and of the gravity $AB^2 - BD^2$, that is, as $BP^2 - BD^2$. But the area DTV is to the area DPQ as DT^2 to DP^2; and, therefore, if GT be drawn perpendicular to DF, as GT^2 or $GD^2 - DF^2$ to BD^2, and as GD^2 to BP^2, and, by diviſion, as DF^2 to $BP^2 - BD^2$. Therefore ſince the area DPQ is as PQ, that is, as $BP^2 - BD^2$, the area DTV will be as the given quantity DF^2. Therefore the area EDT decreaſes uniformly in each of the equal particles of time, by the ſubduction of ſo many given particles DTV, and therefore is proportional to the time. Q.E.D.

Case 3. Let AP be the velocity in the deſcent of the body, and $AP^2 + 2BAP$ the force of reſiſtance, and $BD^2 - AB^2$ the force of gravity, the angle DBA being a right one. And if with the centre D, and the principal vertex B, there be deſcribed a rectangular hyperbola BETV (Pl. 3, Fig. 5) cutting DA, DP, and DQ produced in E, T, and V; the ſector DET of this hyperbola will be as the whole time of deſcent.

For the increment PQ of the velocity, and the area DPQ proportional to it, is as the exceſs of the gravity above the reſiſtance, that is, as $BD^2 - AB^2 - 2BAP - AP^2$ or $BD^2 - BP^2$. And the area DTV is to the area DPQ as DT^2 to DP^2; and therefore as GT^2 or $GD^2 - BD^2$ to BP^2, and as GD^2 to BD^2, and, by diviſion, as BD^2 to $BD^2 - BP^2$. Therefore ſince the area DPQ is as $BD^2 - BP^2$, the area DTV will be as the given quantity BD^2. Therefore the area EDT increaſes uniformly in the ſeveral equal particles of time by the addition of as many given particles DTV, and therefore is proportional to the time of the deſcent. Q.E.D.

Cor. If with the centre D and the ſemi-diameter DA there be drawn through the vertex A an arc At ſimilar to the arc ET, and ſimilarly ſubtending the angle ADT, the velocity AP will be to the velocity which the body in the time EDT, in a non-reſiſting ſpace, can loſe in its aſcent, or acquire in its deſcent, as the area of the triangle DAP to the area of the ſector DAt; and therefore is given from the time given. For the velocity in a non-reſiſting medium is proportional to the time, and therefore to this ſector; in a reſiſting medium, it is as the triangle; and in both mediums, where it is leaſt, it approaches to the ratio of equality, as the ſector and triangle do.

SCHOLIUM.

One may demonſtrate alſo that caſe in the aſcent of the body, where the force of gravity is leſs than can be expreſſed by DA^2 or $AB^2 + BD^2$, and greater than can be expreſſed by $AB^2 - DB^2$, and muſt be expreſſed by AB^2. But I haſten to other things.

PROPOSITION XIV. THEOREM XI.

The ſame things being ſuppoſed, I ſay, that the ſpace deſcribed in the aſcent or deſcent is as the difference of the area by which the time is expreſſed, and of ſome other area which is augmented or diminiſhed in an arithmetical progreſſion; if the forces compounded of the reſiſtance and the gravity be taken in a geometrical progreſſion. (Pl. 3, Fig. 6, 7, 8.)

Take AC (in the three laſt figures) proportional to the gravity, and AK to the reſiſtance; but take them on the ſame ſide of the point A, if the body is deſcending, otherwiſe on the contrary. Erect Ab, which make to DB as DB^2 to 4BAC: and to the rectangular aſymptotes CK, CH, deſcribe the hyperbola bN; and, erecting KN perpendicular to CK, the area AbNK will be augmented or diminiſhed in an arithmetical progreſſion, while the forces CK are taken in a geometrical progreſſion. I ſay, therefore, that the diſtance of the body from its greateſt altitude is as the exceſs of the area AbNK above the area DET.

For ſince AK is as the reſiſtance, that is, as $AP^2 \times 2BAP$; aſſume any given quantity Z, and put AK equal to

$\frac{AP^2 + 2BAP}{Z}$; then (by lem. 2 of this book) the moment KL of AK will be equal to $\frac{2APQ + 2BA \times PQ}{Z}$ or $\frac{2BPQ}{Z}$, and the moment KLON of the area AbNK will be equal to $\frac{2BPQ \times LO}{Z}$ or $\frac{BPQ \times BD^3}{2Z \times CK \times AB}$.

Case 1. Now if the body afcends, and the gravity be as $AB^2 + BD^2$, BET (in Fig. 6), being a circle, the line AC, which is proportional to the gravity, will be $\frac{AB^2 + BD^2}{Z}$, and DP^2 or $AP^2 + 2BAP + AB^2 + BD^2$ will be $AK \times Z + AC \times Z$ or $CK \times Z$; and therefore the area DTV will be to the area DPQ as DT^2 or DB^2 to $CK \times Z$.

Case 2. If the body afcends, and the gravity be as $AB^2 - BD^2$, the line AC (in Fig. 7) will be $\frac{AB^2 - BD^2}{Z}$, and DT^2 will be to DP^2 as DF^2 or DB^2 to $BP^2 - BD^2$ or $AP^2 + 2BAP + AB^2 - BD^2$, that is, to $AK \times Z + AC \times Z$ or $CK \times Z$. And therefore the area DTV will be to the area DPQ as DB^2 to $CK \times Z$.

Case 3. And by the fame reafoning, if the body defcends, and therefore the gravity is as $BD^2 - AB^2$, and the line AC (in Fig. 8) becomes equal to $\frac{BD^2 - AB^2}{Z}$; the area DTV will be to the area DPQ as DB^2 to $CK \times Z$: as above.

Since, therefore, thefe areas are always in this ratio, if for the area DTV, by which the moment of the time, always equal to itfelf, is expreffed, there be put any determinate rect angle, as $BD \times m$, the area DPQ, that is, $\frac{1}{2}BD \times PQ$, will be to $BD \times m$ as $CK \times Z$ to BD^2. And thence $PQ \times BD^3$ becomes equal to $2BD^2 \times m \times CK \times Z$, and the moment KLON of the area AbNK, found before, becomes $\frac{BP \times BD \times m}{AB}$. From the area DET fubduct its moment DTV or $BD \times m$, and there will remain $\frac{AP \times BD \times m}{AB}$. There-

fore the difference of the moments, that is, the moment of the difference of the areas, is equal to $\frac{AP \times BD \times m}{AB}$; and therefore (becauſe of the given quantity $\frac{BD \times m}{AB}$) as the velocity AP; that is, as the moment of the ſpace which the body deſcribes in its aſcent or deſcent. And therefore the difference of the areas, and that ſpace, increaſing or decreaſing by proportional moments, and beginning together or vaniſhing together, are proportional. Q.E.D.

Cor. If the length, which ariſes by applying the area DET to the line BD, be called M; and another length V be taken in that ratio to the length M, which the line DA has to the line DE; the ſpace which a body, in a reſiſting medium, deſcribes in its whole aſcent or deſcent, will be to the ſpace which a body, in a non-reſiſting medium, falling from reſt, can deſcribe in the ſame time, as the difference of the aforeſaid areas to $\frac{BD \times V^2}{AB}$; and therefore is given from the time given. For the ſpace in a non-reſiſting medium is in a duplicate ratio of the time, or as V^2; and, becauſe BD and AB are given, as $\frac{BD \times V^2}{AB}$. This area is equal to the area $\frac{DA^2 \times BD \times M^2}{DE^2 \times AB}$ and the moment of M is m; and therefore the moment of this area is $\frac{DA^2 \times BD \times 2M \times m}{DE^2 \times AB}$. But this moment is to the moment of the difference of the aforeſaid areas DET and AbNK, viz. to $\frac{AP \times BD \times m}{AB}$, as $\frac{DA^2 \times BD \times M}{DE^2}$ to $\frac{1}{2}BD \times AP$, or as $\frac{DA^2}{DE^2}$ into DET to DAP; and, therefore, when the areas DET and DAP are leaſt, in the ratio of equality. Therefore the area $\frac{BD \times V^2}{AB}$ and the difference of the areas DET and AbNK, when all theſe areas are leaſt, have equal moments; and are therefore equal. Therefore ſince the velocities, and

therefore alſo the ſpaces in both mediums deſcribed together, in the beginning of the deſcent, or the end of the aſcent, approach to equality, and therefore are then one to another as the area $\frac{BD \times V^2}{AB}$, and the difference of the areas DET and AbNK; and moreover ſince the ſpace, in a non-reſiſting medium, is perpetually as $\frac{BD \times V^2}{AB}$, and the ſpace, in a reſiſting medium, is perpetually as the difference of the areas DET and AbNK; it neceſſarily follows, that the ſpaces, in both mediums, deſcribed in any equal times, are one to another as that area $\frac{BD \times V^2}{AB}$, and the difference of the areas DET and AbNK. Q.E.D.

SCHOLIUM.

The reſiſtance of ſpherical bodies in fluids ariſes partly from the tenacity, partly from the attrition, and partly from the denſity of the medium. And that part of the reſiſtance which ariſes from the denſity of the fluid, is, as I ſaid, in a duplicate ratio of the velocity; the other part, which ariſes from the tenacity of the fluid, is uniform, or as the moment of the time; and, therefore, we might now proceed to the motion of bodies, which are reſiſted partly by an uniform force, or in the ratio of the moments of the time, and partly in the duplicate ratio of the velocity. But it is ſufficient to have cleared the way to this ſpeculation in the 8th and 9th prop. foregoing, and their corollaries. For in thoſe propoſitions, inſtead of the uniform reſiſtance made to an aſcending body ariſing from its gravity, one may ſubſtitute the uniform reſiſtance which ariſes from the tenacity of the medium, when the body moves by its *vis inſita* alone; and when the body aſcends in a right line, add this uniform reſiſtance to the force of gravity, and ſubduct it when the body deſcends in a right line. One might alſo go on to the motion of bodies which are reſiſted in part uniformly, in part in the ratio of the velocity, and in part in the duplicate ratio of the ſame velo-

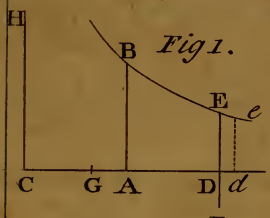


Fig 1.

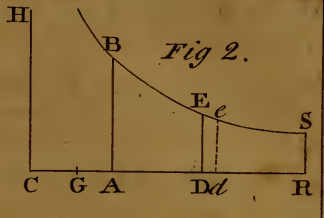
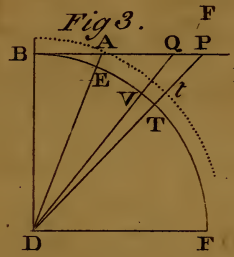


Fig 2.



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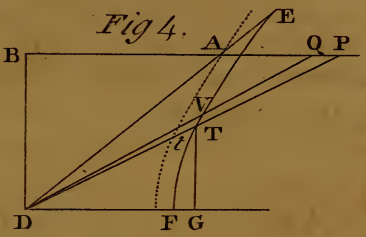

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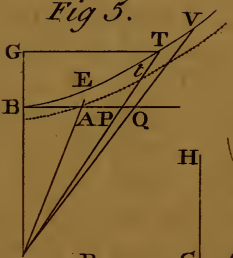

Fig 5.

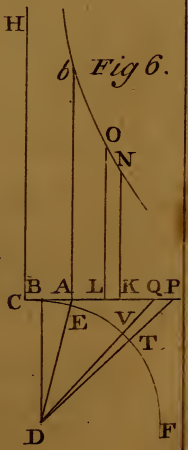

Fig 6.

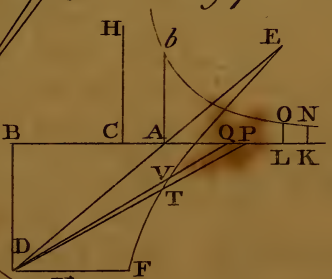

Fig 7.

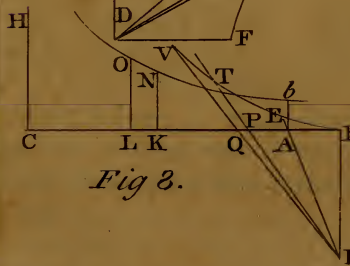

Fig 8.

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city. And I have opened a way to this in the 13th and 14th prop. foregoing, in which the uniform reſiſtance ariſing from the tenacity of the medium may be ſubſtituted for the force of gravity, or be compounded with it as before. But I haſten to other things.

SECTION IV.

Of the circular motion of bodies in reſiſting mediums.

LEMMA III.

Let PQR *be a ſpiral cutting all the radii* SP, SQ, SR, *&c. in equal angles. Draw the right line* PT *touching the ſpiral in any point* P, *and cutting the radius* SQ *in* T; *draw* PO, QO *perpendicular to the ſpiral, and meeting in* O, *and join* SO. *I ſay, that if the points* P *and* Q *approach and coincide, the angle* PSO *will become a right angle, and the ultimate ratio of the rectangle* TQ × 2PS *to* PQ^2 *will be the ratio of equality.* (Pl. 4, Fig. 1.)

For from the right angles OPQ, OQR, ſubduct the equal angles SPQ, SQR, and there will remain the equal angles OPS, OQS. Therefore a circle which paſſes through the points OSP will paſs alſo through the point Q. Let the points P and Q coincide, and this circle will touch the ſpiral in the place of coincidence PQ, and will therefore cut the right line OP perpendicularly. Therefore OP will become a diameter of this circle, and the angle OSP, being in a ſemicircle, becomes a right one. Q.E.D.

Draw QD, SE perpendicular to OP, and the ultimate ratios of the lines will be as follows: TQ to PD as TS or PS to PE, or 2PO to 2PS; and PD to PQ as PQ to 2PO; and, *ex æquo perturbatè*, TQ to PQ as PQ to 2PS. Whence PQ^2 becomes equal to TQ × 2PS. Q.E.D.

PROPOSITION XV. THEOREM XII.

If the denſity of a medium in each place thereof be reciprocally as the diſtance of the places from an immovable centre, and the centripetal force be in the duplicate ratio of the denſity; I ſay, that a body may revolve in a ſpiral which cuts all the radii drawn from that centre in a given angle. (Pl. 4, Fig. 2.)

Suppoſe every thing to be as in the foregoing lemma, and produce SQ to V ſo that SV may be equal to SP. In any time let a body, in a reſiſting medium, deſcribe the leaſt arc PQ, and in double the time the leaſt arc PR; and the decrements of thoſe arcs ariſing from the reſiſtance, or their differences from the arcs which would be deſcribed in a non-reſiſting medium in the ſame times, will be to each other as the ſquares of the times in which they are generated; therefore the decrement of the arc PQ is the fourth part of the decrement of the arc PR. Whence alſo if the area QSr be taken equal to the area PSQ, the decrement of the arc PQ will be equal to half the lineolæ Rr; and therefore the force of reſiſtance and the centripetal force are to each other as the lineolæ $\frac{1}{2}$Rr and TQ which they generate in the ſame time. Becauſe the centripetal force with which the body is urged in P is reciprocally as SP^2, and (by lem. 10, book 1) the lineolæ TQ, which is generated by that force, is in a ratio compounded of the ratio of this force and the duplicate ratio of the time in which the arc PQ is deſcribed (for in this caſe I neglect the reſiſtance, as being infinitely leſs than the centripetal force), it follows that $TQ \times SP^2$, that is (by the laſt lemma), $\frac{1}{2}PQ^2 \times SP$, will be in a duplicate ratio of the time, and therefore the time is as $PQ \times \sqrt{SP}$; and the velocity of the body, with which the arc PQ is deſcribed in that time, as $\frac{PQ}{PQ \times \sqrt{SP}}$ or $\frac{1}{\sqrt{SP}}$, that is, in the ſubduplicate ratio of SP reciprocally. And, by a like reaſoning, the velocity with which the arc QR is deſcribed, is in the ſubduplicate ratio of SQ reciprocally. Now thoſe arcs PQ and QR are as the deſcribing velocities to each other; that is, in the ſubduplicate ratio of SQ to SP, or as SQ to $\sqrt{SP \times SQ}$; and, becauſe of the equal angles SPQ, SQr, and the equal areas PSQ, QSr, the arc PQ is to the arc Qr as SQ to SP. Take the differences of the proportional conſequents, and the arc PQ will be to the arc Rr as SQ to SP — $\sqrt{SP \times SQ}$, or $\frac{1}{2}$VQ. For the points P and Q coinciding, the ultimate ratio of SP — $\sqrt{SP \times SQ}$ to $\frac{1}{2}$VQ is the ratio of equality. Becauſe the decrement of the arc PQ ariſing from the reſiſtance, or its

double Rr, is as the reſiſtance and the ſquare of the time conjunctly, the reſiſtance will be as $\frac{Rr}{PQ^2 \times SP}$. But PQ was to Rr as SQ to $\frac{1}{2}$VQ, and thence $\frac{Rr}{PQ^2 \times SP}$ becomes as $\frac{\frac{1}{2}VQ}{PQ \times SP \times SQ}$, or as $\frac{\frac{1}{2}OS}{OP \times SP^2}$. For the points P and Q coinciding, SP and SQ coincide alſo, and the angle PVQ becomes a right one; and, becauſe of the ſimilar triangles PVQ, PSO, PQ becomes to $\frac{1}{2}$VQ as OP to $\frac{1}{2}$OS. Therefore $\frac{OS}{OP \times SP^2}$ is as the reſiſtance, that is, in the ratio of the denſity of the medium in P and the duplicate ratio of the velocity conjunctly. Subduct the duplicate ratio of the velocity, namely, the ratio $\frac{1}{SP^2}$, and there will remain the denſity of the medium in P, as $\frac{OS}{OP \times SP}$. Let the ſpiral be given, and, becauſe of the given ratio of OS to OP, the denſity of the medium in P will be as $\frac{1}{SP}$. Therefore in a medium whoſe denſity is reciprocally as SP the diſtance from the centre, a body will revolve in this ſpiral. Q.E.D.

Cor. 1. The velocity in any place P, is always the ſame wherewith a body in a non-reſiſting medium with the ſame centripetal force would revolve in a circle, at the ſame diſtance SP from the centre.

Cor. 2. The denſity of the medium, if the diſtance SP be given, is as $\frac{OS}{OP}$, but if that diſtance is not given, as $\frac{OS}{OP \times SP}$. And thence a ſpiral may be fitted to any denſity of the medium.

Cor. 3. The force of the reſiſtance in any place P is to the centripetal force in the ſame place as $\frac{1}{2}$OS to OP. For thoſe forces are to each other as $\frac{1}{2}$Rr and TQ, or as $\frac{\frac{1}{2}VQ \times PQ}{SQ}$

and $\frac{\frac{1}{2}PQ^2}{SP}$, that is, as $\frac{1}{2}VQ$ and PQ, or $\frac{1}{2}OS$ and OP. The ſpiral, therefore, being given, there is given the proportion of the reſiſtance to the centripetal force; and, *vice verſa*, from that proportion given the ſpiral is given.

Cor. 4. Therefore the body cannot revolve in this ſpiral, except where the force of reſiſtance is leſs than half the centripetal force. Let the reſiſtance be made equal to half the centripetal force, and the ſpiral will coincide with the right line PS, and in that right line the body will deſcend to the centre with a velocity that is to the velocity, with which it was proved before, in the caſe of the parabola (theor. 10, book 1), the deſcent would be made in a non-reſiſting medium, in the ſubduplicate ratio of unity to the number two. And the times of the deſcent will be here reciprocally as the velocities, and therefore given.

Cor. 5. And becauſe at equal diſtances from the centre the velocity is the ſame in the ſpiral PQR as it is in the right line SP, and the length of the ſpiral is to the length of the right line PS in a given ratio, namely, in the ratio of OP to OS; the time of the deſcent in the ſpiral will be to the time of the deſcent in the right line SP in the ſame given ratio, and therefore given.

Cor. 6. If from the centre S, with any two given intervals, two circles are deſcribed; and theſe circles remaining, the angle which the ſpiral makes with the radius PS be any how changed; the number of revolutions which the body can complete in the ſpace between the circumferences of thoſe circles, going round in the ſpiral from one circumference to another, will be as $\frac{PS}{OS}$, or as the tangent of the angle which the ſpiral makes with the radius PS; and the time of the ſame revolutions will be as $\frac{OP}{OS}$, that is, as the ſecant of the ſame angle, or reciprocally as the denſity of the medium.

Cor. 7. If a body, in a medium whoſe denſity is reciprocally as the diſtances of places from the centre, revolves in any

curve AEB (Pl.4, Fig. 3) about that centre, and cuts the firſt radius AS in the ſame angle in B as it did before in A, and that with a velocity that ſhall be to its firſt velocity in A reciprocally in a ſubduplicate ratio of the diſtances from the centre (that is, as AS to a mean proportional between AS and BS) that body will continue to deſcribe innumerable ſimilar revolutions BFC, CGD, &c. and by its interſections will diſtinguiſh the radius AS into parts AS, BS, CS, DS, &c. that are continually proportional. But the times of the revolutions will be as the perimeters of the orbits AEB, BFC, CGD, &c. directly, and the velocities at the beginnings A, B, C of thoſe orbits inverſely; that is, as $AS^{\frac{3}{2}}$, $BS^{\frac{3}{2}}$, $CS^{\frac{3}{2}}$. And the whole time in which the body will arrive at the centre, will be to the time of the firſt revolution as the ſum of all the continued proportionals $AS^{\frac{3}{2}}$, $BS^{\frac{3}{2}}$, $CS^{\frac{3}{2}}$, going on *ad infinitum*, to the firſt term $AS^{\frac{3}{2}}$; that is, as the firſt term $AS^{\frac{3}{2}}$ to the difference of the two firſt $AS^{\frac{3}{2}} - BS^{\frac{3}{2}}$, or as $\frac{2}{3}$ AS to AB very nearly. Whence the whole time may be eaſily found.

Cor. 8. From hence alſo may be deduced, near enough, the motions of bodies in mediums whoſe denſity is either uniform, or obſerves any other aſſigned law. From the centre S, with intervals SA, SB, SC, &c. continually proportional, deſcribe as many circles; and ſuppoſe the time of the revolutions between the perimeters of any two of thoſe circles, in the medium whereof we treated, to be to the time of the revolutions between the ſame in the medium propoſed as the mean denſity of the propoſed medium between thoſe circles to the mean denſity of the medium whereof we treated, between the ſame circles, nearly: and that the ſecant of the angle in which the ſpiral above determined, in the medium whereof we treated, cuts the radius AS, is in the ſame ratio to the ſecant of the angle in which the new ſpiral, in the propoſed medium, cuts the ſame radius: and alſo that the number of all the revolutions between the ſame two circles is nearly as the tangents of thoſe angles. If this be done every where

between every two circles, the motion will be continued through all the circles. And by this means one may without difficulty conceive at what rate and in what time bodies ought to revolve in any regular medium.

Cor. 9. And although these motions becoming eccentrical should be performed in spirals approaching to an oval figure, yet, conceiving the several revolutions of those spirals to be at the same distances from each other, and to approach to the centre by the same degrees as the spiral above described, we may also understand how the motions of bodies may be performed in spirals of that kind.

PROPOSITION XVI. THEOREM XIII.

If the density of the medium in each of the places be reciprocally as the distance of the places from the immovable centre, and the centripetal force be reciprocally as any power of the same distance, I say, that the body may revolve in a spiral intersecting all the radii drawn from that centre in a given angle. (Pl. 4, Fig. 2.)

This is demonstrated in the same manner as the foregoing proposition. For if the centripetal force in P be reciprocally as any power SP^{n+1} of the distance SP whose index is $n + 1$; it will be collected, as above, that the time in which the body describes any arc PQ, will be as $PQ \times PS^{\frac{1}{2}n}$; and the resistance in P as $\frac{Rr}{PQ^2 \times SP^n}$, or as $\frac{\overline{1 - \frac{1}{2}n} \times VQ}{PQ \times SP^n \times SQ}$, and therefore as $\frac{\overline{1 - \frac{1}{2}n} \times OS}{OP \times SP^{n+1}}$, that is, (because $\frac{\overline{1 - \frac{1}{2}n} \times OS}{OP}$ is a given quantity), reciprocally as SP^{n+1}. And therefore, since the velocity is reciprocally as $SP^{\frac{1}{2}n}$, the density in P will be reciprocally as SP.

Cor. 1. The resistance is to the centripetal force as $\overline{1 - \frac{1}{2}n} \times OS$ to OP.

Cor. 2. If the centripetal force be reciprocally as SP^3, $1 - \frac{1}{2}n$ will be $= 0$; and therefore the resistance and density of the medium will be nothing, as in prop. 9, book 1.

Cor. 3. If the centripetal force be reciprocally as any power of the radius SP, whose index is greater than the num-

ber 3, the affirmative reſiſtance will be changed into a negative.

SCHOLIUM.

This propoſition and the former, which relate to mediums of unequal denſity, are to be underſtood of the motion of bodies that are ſo ſmall, that the greater denſity of the medium on one ſide of the body above that on the other is not to be conſidered. I ſuppoſe alſo the reſiſtance, *cæteris paribus*, to be proportional to its denſity. Whence, in mediums whoſe force of reſiſtance is not as the denſity, the denſity muſt be ſo much augmented or diminiſhed, that either the exceſs of the reſiſtance may be taken away, or the defect ſupplied.

PROPOSITION XVII. PROBLEM IV.

To find the centripetal force and the reſiſting force of the medium, by which a body, the law of the velocity being given, ſhall revolve in a given ſpiral. (Pl. 4. Fig. 4.)

Let that ſpiral be PQR. From the velocity, with which the body goes over the very ſmall arc PQ, the time will be given; and from the altitude TQ, which is as the centripetal force, and the ſquare of the time, that force will be given. Then from the difference RSr of the areas PSQ and QSR deſcribed in equal particles of time, the retardation of the body will be given; and from the retardation will be found the reſiſting force and denſity of the medium.

PROPOSITION XVIII. PROBLEM V.

The law of centripetal force being given, to find the denſity of the medium in each of the places thereof, by which a body may deſcribe a given ſpiral.

From the centripetal force the velocity in each place muſt be found; then from the retardation of the velocity the denſity of the medium is found, as in the foregoing propoſition.

But I have explained the method of managing theſe problems in the tenth propoſition and ſecond lemma of this book; and will no longer detain the reader in theſe perplexed diſquiſitions. I ſhall now add ſome things relating to the forces of progreſſive bodies, and to the denſity and reſiſtance of thoſe mediums in which the motions hitherto treated of, and thoſe akin to them, are performed.

SECTION V.

Of the denſity and compreſſion of fluids; and of hydroſtatics.

THE DEFINITION OF A FLUID.

A fluid is any body whoſe parts yield to any force impreſſed on it, and, by yielding, are eaſily moved among themſelves.

PROPOSITION XIX. THEOREM XIV.

All the parts of a homogeneous and unmoved fluid included in any unmoved veſſel, and compreſſed on every ſide (ſetting aſide the conſideration of condenſation, gravity, and all centripetal forces), will be equally preſſed on every ſide, and remain in their places without any motion ariſing from that preſſure. (Pl. 4, Fig. 5.)

CASE 1. Let a fluid be included in the ſpherical veſſel ABC, and uniformly compreſſed on every ſide: I ſay, that no part of it will be moved by that preſſure. For if any part, as D, be moved, all ſuch parts at the ſame diſtance from the centre on every ſide muſt neceſſarily be moved at the ſame time by a like motion; becauſe the preſſure of them all is ſimilar and equal; and all other motion is excluded that does not ariſe from that preſſure. But if theſe parts come all of them nearer to the centre, the fluid muſt be condenſed towards the centre, contrary to the ſuppoſition. If they recede from it, the fluid muſt be condenſed towards the circumference; which is alſo contrary to the ſuppoſition. Neither can they move in any one direction retaining their diſtance from the centre, becauſe, for the ſame reaſon, they may move in a contrary direction; but the ſame part cannot be moved contrary ways at the ſame time. Therefore no part of the fluid will be moved from its place. Q.E.D.

CASE 2. I ſay now, that all the ſpherical parts of this fluid are equally preſſed on every ſide. For let EF be a ſpherical part of the fluid; if this be not preſſed equally on every ſide, augment the leſſer preſſure till it be preſſed equally on every ſide; and its parts (by caſe 1.) will remain in their places. But before the increaſe of the preſſure, they would remain in their places (by caſe 1); and by

the addition of a new preſſure they will be moved, by the definition of a fluid, from thoſe places. Now theſe two concluſions contradict each other. Therefore it was falſe to ſay that the ſphere EF was not preſſed equally on every ſide. Q.E.D.

Case 3. I ſay beſides, that different ſpherical parts have equal preſſures. For the contiguous ſpherical parts preſs each other mutually and equally in the point of contact (by law 3), But (by caſe 2) they are preſſed on every ſide with the ſame force. Therefore any two ſpherical parts not contiguous, ſince an intermediate ſpherical part can touch both, will be preſſed with the ſame force. Q.E.D.

Case 4. I ſay now, that all the parts of the fluid are every where preſſed equally. For any two parts may be touched by ſpherical parts in any points whatever; and there they will equally preſs thoſe ſpherical parts (by caſe 3), and are reciprocally equally preſſed by them (by law 3). Q.E.D.

Case 5. Since, therefore, any part GHI of the fluid is incloſed by the reſt of the fluid as in a veſſel, and is equally preſſed on every ſide; and alſo its parts equally preſs one another, and are at reſt among themſelves; it is manifeſt that all the parts of any fluid as GHI, which is preſſed equally on every ſide, do preſs each other mutually and equally, and are at reſt among themſelves. Q.E.D.

Case 6. Therefore if that fluid be included in a veſſel of a yielding ſubſtance, or that is not rigid, and be not equally preſſed on ever ſide, the ſame will give way to a ſtronger preſſure, by the definition of fluidity.

Case 7. And therefore, in an inflexible or rigid veſſel, a fluid will not ſuſtain a ſtronger preſſure on one ſide than on the other, but will give way to it, and that in a moment of time; becauſe the rigid ſide of the veſſel does not follow the yielding liquor. But the fluid, by thus yielding, will preſs againſt the oppoſite ſide, and ſo the preſſure will tend on every ſide to equality. And becauſe the fluid, as ſoon as it endeavours to recede from the part that is moſt preſſed, is withſtood by the reſiſtance of the veſſel on the oppoſite ſide, the preſſure will

on every ſide be reduced to equality, in a moment of time, without any local motion: and from thence the parts of the fluid (by caſe 5) will preſs each other mutually and equally, and be at reſt among themſelves. Q.E.D.

Cor. Whence neither will a motion of the parts of the fluid among themſelves be changed by a preſſure communicated to the external ſuperficies, except ſo far as either the figure of the ſuperficies may be ſomewhere altered, or that all the parts of the fluid, by preſſing one another more intenſely or remiſsly, may ſlide with more or leſs difficulty among themſelves.

PROPOSITION XX. THEOREM XV.

If all the parts of a ſpherical fluid, homogeneous at equal diſtances from the centre, lying on a ſpherical concentric bottom, gravitate towards the centre of the whole, the bottom will ſuſtain the weight of a cylinder, whoſe baſe is equal to the ſuperficies of the bottom, and whoſe altitude is the ſame with that of the incumbent fluid. (Pl. 4, Fig. 6.)

Let DHM be the ſuperficies of the bottom, and AEI the upper ſuperficies of the fluid. Let the fluid be diſtinguiſhed into concentric orbs of equal thickneſs, by the innumerable ſpherical ſuperficies BFK, CGL; and conceive the force of gravity to act only in the upper ſuperficies of every orb, and the actions to be equal on the equal parts of all the ſuperficies. Therefore the upper ſuperficies AE is preſſed by the ſingle force of its own gravity, by which all the parts of the upper orb, and the ſecond ſuperficies BFK, will (by prop. 19), according to its meaſure, be equally preſſed. The ſecond ſuperficies BFK is preſſed likewiſe by the force of its own gravity, which, added to the former force, makes the preſſure double. The third ſuperficies CGL is, according to its meaſure, acted on by this preſſure and the force of its own gravity beſides, which makes its preſſure triple. And in like manner the fourth ſuperficies receives a quadruple preſſure, the fifth ſuperficies a quintuple, and ſo on. Therefore the preſſure acting on every ſuperficies is not as the ſolid quantity of the incumbent fluid, but as the number of the orbs reaching to the upper ſurface of the fluid; and is equal to the

Fig 1.

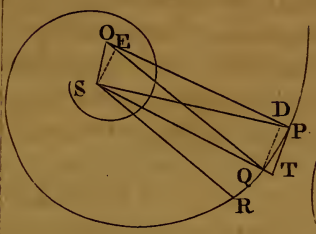


Fig 2.

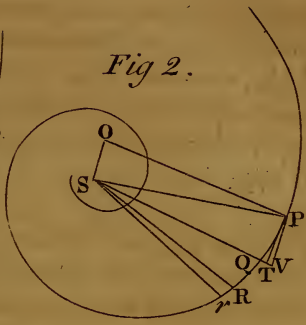


Fig 3.

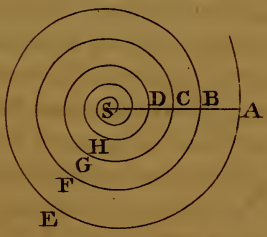


Fig 4.

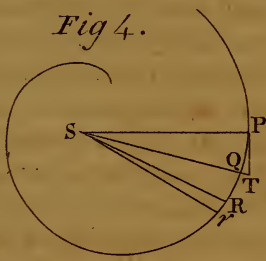


Fig 5.

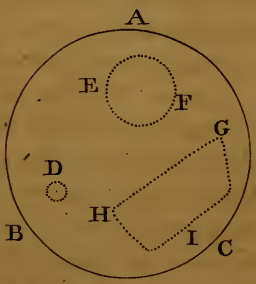
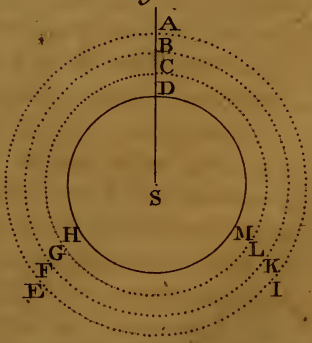


Fig 6.



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gravity of the loweſt orb multiplied by the number of orbs: that is, to the gravity of a ſolid whoſe ultimate ratio to the cylinder above-mentioned (when the number of the orbs is increaſed and their thickneſs diminiſhed, *ad infinitum*, ſo that the action of gravity from the loweſt ſuperficies to the uppermoſt may become continued) is the ratio of equality. Therefore the loweſt ſuperficies ſuſtains the weight of the cylinder above determined. Q.E.D. And by a like reaſoning the propoſition will be evident, where the gravity of the fluid decreaſes in any aſſigned ratio of the diſtance from the centre, and alſo where the fluid is more rare above and denſer below. Q.E.D.

Cor. 1. Therefore the bottom is not preſſed by the whole weight of the incumbent fluid, but only ſuſtains that part of it which is deſcribed in the propoſition; the reſt of the weight being ſuſtained archwiſe by the ſpherical figure of the fluid.

Cor. 2. The quantity of the preſſure is the ſame always at equal diſtances from the centre, whether the ſuperficies preſſed be parallel to the horizon, or perpendicular, or oblique; or whether the fluid, continued upwards from the compreſſed ſuperficies, riſes perpendicularly in a rectilinear direction, or creeps obliquely through crooked cavities and canals, whether thoſe paſſages be regular or irregular, wide or narrow. That the preſſure is not altered by any of theſe circumſtances, may be collected by applying the demonſtration of this theorem to the ſeveral caſes of fluids.

Cor. 3. From the ſame demonſtration it may alſo be collected (by prop. 19), that the parts of a heavy fluid acquire no motion among themſelves by the preſſure of the incumbent weight, except that motion which ariſes from condenſation.

Cor. 4. And therefore if another body of the ſame ſpecific gravity, incapable of condenſation, be immerſed in this fluid, it will acquire no motion by the preſſure of the incumbent weight: it will neither deſcend nor aſcend, nor change its figure. If it be ſpherical, it will remain ſo, notwithſtanding the preſſure; if it be ſquare, it will remain ſquare; and that, whether it be ſoft or fluid; whether it ſwims freely in the

fluid, or lies at the bottom. For any internal part of a fluid is in the ſame ſtate with the ſubmerſed body; and the caſe of all ſubmerſed bodies that have the ſame magnitude, figure, and ſpecific gravity, is alike. If a ſubmerſed body, retaining its weight, ſhould diſſolve and put on the form of a fluid, this body, if before it would have aſcended, deſcended, or from any preſſure aſſume a new figure, would now likewiſe aſcend, deſcend, or put on a new figure; and that, becauſe its gravity and the other cauſes of its motion remain. But (by caſe 5, prop. 19) it would now be at reſt, and retain its figure. Therefore alſo in the former caſe.

Cor. 5. Therefore a body that is ſpecifically heavier than a fluid contiguous to it will ſink; and that which is ſpecifically lighter will aſcend, and attain ſo much motion and change of figure as that exceſs or defect of gravity is able to produce. For that exceſs or defect is the ſame thing as an impulſe, by which a body, otherwiſe *in equilibrio* with the parts of the fluid, is acted on; and may be compared with the exceſs or defect of a weight in one of the ſcales of a balance.

Cor. 6. Therefore bodies placed in fluids have a twofold gravity; the one true and abſolute, the other apparent, vulgar, and comparative. Abſolute gravity is the whole force with which the body tends downwards; relative and vulgar gravity is the exceſs of gravity with which the body tends downwards more than the ambient fluid. By the firſt kind of gravity the parts of all fluids and bodies gravitate in their proper places; and therefore their weights taken together compoſe the weight of the whole. For the whole taken together is heavy, as may be experienced in veſſels full of liquor; and the weight of the whole is equal to the weights of all the parts, and is therefore compoſed of them. By the other kind of gravity bodies do not gravitate in their places; that is, compared with one another, they do not preponderate, but, hindering one another's endeavours to deſcend, remain in their proper places, as if they were not heavy. Thoſe things which are in the air, and do not preponderate, are commonly looked on as not heavy. Thoſe which do preponderate are commonly reckoned heavy, in as much as they are not ſuſtained

by the weight of the air. The common weights are nothing elſe but the exceſs of the true weights above the weight of the air. Hence alſo, vulgarly, thoſe things are called light which are leſs heavy, and, by yielding to the preponderating air, mount upwards. But theſe are only comparatively light, and not truly ſo, becauſe they deſcend *in vacuo*. Thus, in water, bodies which, by their greater or leſs gravity, deſcend or aſcend, are comparatively and apparently heavy or light; and their comparative and apparent gravity or levity is the exceſs or defect by which their true gravity either exceeds the gravity of the water or is exceeded by it. But thoſe things which neither by preponderating deſcend, nor, by yielding to the preponderating fluid, aſcend, although by their true weight they do increaſe the weight of the whole, yet comparatively, and in the ſenſe of the vulgar, they do not gravitate in the water. For theſe caſes are alike demonſtrated.

Cor. 7. Theſe things which have been demonſtrated concerning gravity take place in any other centripetal forces.

Cor. 8. Therefore if the medium in which any body moves be acted on either by its own gravity, or by any other centripetal force, and the body be urged more powerfully by the ſame force; the difference of the forces is that very motive force, which, in the foregoing propoſitions, I have conſidered as a centripetal force. But if the body be more lightly urged by that force, the difference of the forces becomes a centrifugal force, and is to be conſidered as ſuch.

Cor. 9. But ſince fluids by preſſing the included bodies do not change their external figures, it appears alſo (by cor. prop. 19) that they will not change the ſituation of their internal parts in relation to one another; and therefore if animals were immerſed therein, and that all ſenſation did ariſe from the motion of their parts, the fluid will neither hurt the immerſed bodies, nor excite any ſenſation, unleſs ſo far as thoſe bodies may be condenſed by the compreſſion. And the caſe is the ſame of any ſyſtem of bodies encompaſſed with a compreſſing fluid. All the parts of the ſyſtem will be agitated with the ſame motions as as if they were placed in a vacuum, and would only retain their comparative gravity;

unleſs ſo far as the fluid may ſomewhat reſiſt their motions, or be requiſite to conglutinate them by compreſſion.

PROPOSITION XXI. THEOREM XVI.

Let the denſity of any fluid be proportional to the compreſſion, and its parts be attracted downwards by a centripetal force reciprocally proportional to the diſtances from the centre: I ſay, that, if thoſe diſtances be taken continually proportional, the denſities of the fluid at the ſame diſtances will be alſo continually proportional. (Pl. 5, Fig. 1.)

Let ATV denote the ſpherical bottom of the fluid, S the centre, SA, SB, SC, SD, SE, SF, &c. diſtances continually proportional. Erect the perpendiculars AH, BI, CK, DL, EM, FN, &c. which ſhall be as the denſities of the medium in the places A, B, C, D, E, F; and the ſpecific gravities in thoſe places will be as $\frac{AH}{AS}$, $\frac{BI}{BS}$, $\frac{CK}{CS}$, &c. or, which is all one, as $\frac{AH}{AB}$, $\frac{BI}{BC}$, $\frac{CK}{CD}$, &c. Suppoſe, firſt, theſe gravities to be uniformly continued from A to B, from B to C, from C to D, &c. the decrements in the points B, C, D, &c. being taken by ſteps. And theſe gravities drawn into the altitudes AB, BC, CD, &c. will give the preſſures AH, BI, CK, &c. by which the bottom ATV is acted on (by theor. 15). Therefore the particle A ſuſtains all the preſſures AH, BI, CK, DL, &c. proceeding *in infinitum*; and the particle B ſuſtains the preſſures of all but the firſt AH; and the particle C all but the two firſt AH, BI; and ſo on: and therefore the denſity AH of the firſt particle A is to the denſity BI of the ſecond particle B as the ſum of all AH + BI + CK + DL, *in infinitum*, to the ſum of all BI + CK + DL, &c. And BI the denſity of the ſecond particle B is to CK the denſity of the third C, as the ſum of all BI + CK + DL, &c. to the ſum of all CK + DL, &c. Therefore theſe ſums are proportional to their differences AH, BI, CK, &c. and therefore continually proportional (by lem. 1 of this book); and therefore the differences AH, BI, CK, &c. proportional to the ſums, are alſo continually proportional. Wherefore ſince the denſities in the places A, B, C, &c. are as AH, BI, CK, &c.

they will alſo be continually proportional. Proceed interminiſſively, and, *ex æquo*, at the diſtances SA, SC, SE, continually proportional, the denſities AH, CK, EM will be continually proportional. And by the ſame reaſoning, at any diſtances SA, SD, SG, continually proportional, the denſities AH, DL, GO, will be continually proportional. Let now the points A, B, C, D, E, &c. coincide, ſo that the progreſſion of the ſpecific gravities from the bottom A to the top of the fluid may be made continual; and at any diſtances SA, SD, SG, continually proportional, the denſities AH, DL, GO, being all along continually proportional, will ſtill remain continually proportional. Q.E.D.

Cor. Hence if the denſity of the fluid in two places, as A and E, be given, its denſity in any other place Q may be collected. With the centre S, and the rectangular aſymptotes SQ, SX, deſcribe (Fig. 2) an hyperbola cutting the perpendiculars AH, EM, QT in a, e, and q, as alſo the perpendiculars HX, MY, TZ, let fall upon the aſymptote SX, in h, m, and t. Make the area YmtZ to the given area YmhX as the given area EeqQ to the given area EeaA; and the line Zt produced will cut off the line QT proportional to the denſity. For if the lines SA, SE, SQ are continually proportional, the areas EeqQ, EeaA will be equal, and thence the areas YmtZ, XhmY, proportional to them, will be alſo equal; and the lines SX, SY, SZ, that is, AH, EM, QT continually proportional, as they ought to be. And if the lines SA, SE, SQ, obtain any other order in the ſeries of continued proportionals, the lines AH, EM, QT, becauſe of the proportional hyperbolic areas, will obtain the ſame order in another ſeries of quantities continually proportional.

PROPOSITION XXII. THEOREM XVII.

Let the denſity of any fluid be proportional to the compreſſion, and its parts be attracted downwards by a gravitation reciprocally proportional to the ſquares of the diſtances from the centre: I ſay, that, if the diſtances be taken in harmonic progreſſion, the denſities of the fluid at thoſe diſtances will be in a geometrical progreſſion. (Pl. 5, Fig. 3.)

Let S denote the centre, and SA, SB, SC, SD, SE, the diſtances in geometrical progreſſion. Erect the perpendiculars AH,

BI, CK, &c. which shall be as the densities of the fluid in the places A, B, C, D, E, &c. and the specific gravities thereof in those places will be as $\frac{AH}{SA^2}$, $\frac{BI}{SB^2}$, $\frac{CK}{SC^2}$, &c. Suppose these gravities to be uniformly continued, the first from A to B, the second from B to C, the third from C to D, &c. And these drawn into the altitudes AB, BC, CD, DE, &c. or, which is the same thing, into the distances SA, SB, SC, &c. proportional to those altitudes, will give $\frac{AH}{SA}$, $\frac{BI}{SB}$, $\frac{CK}{SC}$, &c. the exponents of the pressures. Therefore since the densities are as the sums of those pressures, the differences AH — BI, BI — CK, &c. of the densities will be as the differences of those sums $\frac{AH}{SA}$, $\frac{BI}{SB}$, $\frac{CK}{SC}$, &c. With the centre S, and the asymptotes SA, Sx, describe any hyperbola, cutting the perpendiculars AH, BI, CK, &c. in a, b, c, &c. and the perpendiculars Ht, Iu, Kw, let fall upon the asymptote Sx, in h, i, k; and the differences of the densities tu, uw, &c. will be as $\frac{AH}{SA}$, $\frac{BI}{SB}$, &c. And the rectangles tu × th, uw × ui, &c. or tp, uq, &c. as $\frac{AH \times th}{SA}$, $\frac{BI \times ui}{SB}$, &c. that is, as Aa, Bb, &c. For, by the nature of the hyperbola, SA is to AH or St as th to Aa, and therefore $\frac{AH \times th}{SA}$ is equal to Aa. And, by a like reasoning, $\frac{BI \times ui}{SB}$ is equal to Bb, &c. But Aa, Bb, Cc, &c. are continually proportional, and therefore proportional to their differences Aa — Bb, Bb — Cc, &c., and therefore the rectangles tp, uq, &c. are proportional to those differences; as also the sums of the rectangles tp + uq, or tp + uq + wr to the sums of the differences Aa — Cc or Aa — Dd. Suppose several of these terms, and the sum of all the differences, as Aa — Ff, will be proportional to the sum of all the rectangles, as zthn. Increase the number of terms, and diminish the distances of the points A, B, C, &c. *in infinitum,*

and thoſe rectangles will become equal to the hyperbolic area zthn, and therefore the difference Aa — Ff is proportional to this area. Take now any diſtances, as SA, SD, SF, in harmonic progreſſion, and the differences Aa — Dd, Dd — Ff will be equal; and therefore the areas thlx, xlnz, proportional to thoſe differences will be equal among themſelves, and the denſities St, Sx, Sz, that is, AH, DL, FN, continually proportional. Q.E.D.

Cor. 2. Hence if any two denſities of the fluid, as AH and BI, be given, the area thiu, anſwering to their difference tu, will be given; and thence the denſity FN will be found at any height SF, by taking the area thnz to that given area thiu as the difference Aa — Ff to the difference Aa — Bb.

SCHOLIUM.

By a like reaſoning it may be proved, that if the gravity of the particles of a fluid be diminiſhed in a triplicate ratio of the diſtances from the centre; and the reciprocals of the ſquares of the diſtances SA, SB, SC, &c. (namely, $\frac{SA^3}{SA^2}, \frac{SA^3}{SB^2}, \frac{SA^3}{SC^2}$) be taken in an arithmetical progreſſion, the denſities AH, BI, CK, &c. will be in a geometrical progreſſion. And if the gravity be diminiſhed in a quadruplicate ratio of the diſtances, and the reciprocals of the cubes of the diſtances (as $\frac{SA^4}{SA^3}, \frac{SA^4}{SB^3}, \frac{SA^4}{SC^3}$, &c.) be taken in arithmetical progreſſion, the denſities AH, BI, CK, &c. will be in geometrical progreſſion. And ſo *in infinitum*. Again; if the gravity of the particles of the fluid be the ſame at all diſtances, and the diſtances be in arithmetical progreſſion, the denſities will be in a geometrical progreſſion, as Dr. *Halley* has found. If the gravity be as the diſtance, and the ſquares of the diſtances be in arithmetical progreſſion, the denſities will be in geometrical progreſſion. And ſo *in infinitum*. Theſe things will be ſo, when the denſity of the fluid condenſed by compreſſion is as the force of compreſſion; or, which is the ſame thing, when the ſpace poſſeſſed by the fluid is reciprocally as this force. Other laws of condenſation may be ſuppoſed, as that the cube of the compreſſing force may be as the biqua-

drate of the denſity; or the triplicate ratio of the force the ſame with the quadruplicate ratio of the denſity: in which caſe, if the gravity be reciprocally as the ſquare of the diſtance from the centre, the denſity will be reciprocally as the cube of the diſtance. Suppoſe that the cube of the compreſſing force be as the quadrato-cube of the denſity; and if the gravity be reciprocally as the ſquare of the diſtance, the denſity will be reciprocally in a ſeſquiplicate ratio of the diſtance. Suppoſe the compreſſing force to be in a duplicate ratio of the denſity, and the gravity reciprocally in a duplicate ratio of the diſtance, and the denſity will be reciprocally as the diſtance. To run over all the caſes that might be offered would be tedious. But as to our own air, this is certain from experiment, that its denſity is either accurately, or very nearly at leaſt, as the compreſſing force; and therefore the denſity of the air in the atmoſphere of the earth is as the weight of the whole incumbent air, that is, as the height of the mercury in the barometer.

PROPOSITION XXIII. THEOREM XVIII.

If a fluid be compoſed of particles mutually flying each other, and the denſity be as the compreſſion, the centrifugal forces of the particles will be reciprocally proportional to the diſtances of their centres. And, vice verſa, *particles flying each other with forces that are reciprocally proportional to the diſtances of their centres, compoſe an elaſtic fluid, whoſe denſity is as the compreſſion.* (Pl. 5, Fig. 4.)

Let the fluid be ſuppoſed to be included in a cubic ſpace ACE, and then to be reduced by compreſſion into a leſſer cubic ſpace ace; and the diſtances of the particles retaining a like ſituation with reſpect to each other in both the ſpaces, will be as the ſides AB, ab of the cubes; and the denſities of the mediums will be reciprocally as the containing ſpaces AB^3, ab^3. In the plane ſide of the greater cube ABCD take the ſquare DP equal to the plane ſide db of the leſſer cube: and, by the ſuppoſition, the preſſure with which the ſquare DP urges the incloſed fluid will be to the preſſure with which that ſquare db urges the incloſed fluid as the denſities of the mediums are to each other, that is, as ab^3 to AB^3. But the

preſſure with which the ſquare DB urges the included fluid is to the preſſure with which the ſquare DP urges the ſame fluid as the ſquare DB to the ſquare DP, that is, as AB^2 to ab^2. Therefore, *ex æquo*, the preſſure with which the ſquare DB urges the fluid is to the preſſure with which the ſquare db urges the fluid as ab to AB. Let the planes FGH, fgh, be drawn through the middles of the two cubes, and divide the fluid into two parts. Theſe parts will preſs each other mutually with the ſame forces with which they are themſelves preſſed by the planes AC, ac, that is, in the proportion of ab to AB: and therefore the centrifugal forces by which theſe preſſures are ſuſtained are in the ſame ratio. The number of the particles being equal, and the ſituation alike, in both cubes, the forces which all the particles exert, according to the planes FGH, fgh, upon all, are as the forces which each exerts on each. Therefore the forces which each exerts on each, according to the plane FGH in the greater cube, are to the forces which each exerts on each, according to the plane fgh in the leſſer cube, as ab to AB, that is, reciprocally as the diſtances of the particles from each other. Q.E.D.

And, *vice verſa*, if the forces of the ſingle particles are reciprocally as the diſtances, that is, reciprocally as the ſides of the cubes AB, ab; the ſums of the forces will be in the ſame ratio, and the preſſures of the ſides DB, db as the ſums of the forces; and the preſſure of the ſquare DP to the preſſure of the ſide DB as ab^2 to AB^2. And, *ex æquo*, the preſſure of the ſquare DP to the preſſure of the ſide db as ab^3 to AB^3; that is, the force of compreſſion in the one to the force of compreſſion in the other as the denſity in the former to the denſity in the latter. Q.E.D.

SCHOLIUM.

By a like reaſoning, if the centrifugal forces of the particles are reciprocally in the duplicate ratio of the diſtances between the centres, the cubes of the compreſſing forces will be as the biquadrates of the denſities. If the centrifugal forces be reciprocally in the triplicate or quadruplicate ratio of the diſtances, the cubes of the compreſſing forces will be as the quadrato-cubes, or cubo-cubes of the denſities. And univer-

fally, if D be put for the diſtance, and E for the denſity of the compreſſed fluid, and the centrifugal forces be reciprocally as any power D^n of the diſtance, whoſe index is the number n, the compreſſing forces will be as the cube roots of the power E^{n+2}, whoſe index is the number $n + 2$: and the contrary. All theſe things are to be underſtood of particles whoſe centrifugal forces terminate in thoſe particles that are next them, or are diffuſed not much further. We have an example of this in magnetical bodies. Their attractive virtue is terminated nearly in bodies of their own kind that are next them. The virtue of the magnet is contracted by the interpoſition of an iron plate, and is almoſt terminated at it; for bodies further off are not attracted by the magnet ſo much as by the iron plate. If in this manner particles repel others of their own kind that lie next them, but do not exert their virtue on the more remote, particles of this kind will compoſe ſuch fluids as are treated of in this propoſition. If the virtue of any particle diffuſe itſelf every way *in infinitum,* there will be required a greater force to produce an equal condenſation of a greater quantity of the fluid. But whether elaſtic fluids do really conſiſt of particles ſo repelling each other, is a phyſical queſtion. We have here demonſtrated mathematically the property of fluids conſiſting of particles of this kind, that hence philoſophers may take occaſion to diſcuſs that queſtion.

SECTION VI.

Of the motion and reſiſtance of funependulous bodies.

PROPOSITION XXIV. THEOREM XIX.

The quantities of matter in funependulous bodies, whoſe centres of oſcillation are equally diſtant from the centre of ſuſpenſion, are in a ratio compounded of the ratio of the weights and the duplicate ratio of the times of the oſcillations in vacuo.

For the velocity which a given force can generate in a given matter in a given time is as the force and the time directly, and the matter inverſely. The greater the force or the time is, or the leſs the matter, the greater velocity will be generated. This is manifeſt from the ſecond law of motion. Now if pendulums are of the ſame length, the motive forces

in places equally distant from the perpendicular are as the weights: and therefore if two bodies by oscillating describe equal arcs, and those arcs are divided into equal parts; since the times in which the bodies describe each of the correspondent parts of the arcs are as the times of the whole oscillations, the velocities in the correspondent parts of the oscillations will be to each other as the motive forces and the whole times of the oscillations directly, and the quantities of matter reciprocally: and therefore the quantities of matter are as the forces and the times of the oscillations directly and the velocities reciprocally. But the velocities reciprocally are as the times, and therefore the times directly and the velocities reciprocally are as the squares of the times; and therefore the quantities of matter are as the motive forces and the squares of the times, that is, as the weights and the squares of the times. Q.E.D.

Cor. 1. Therefore if the times are equal, the quantities of matter in each of the bodies are as the weights.

Cor. 2. If the weights are equal, the quantities of matter will be as the squares of the times.

Cor. 3. If the quantities of matter are equal, the weights will be reciprocally as the squares of the times.

Cor. 4. Whence since the squares of the times, *cæteris paribus*, are as the lengths of the pendulums, therefore if both the times and quantities of matter are equal, the weights will be as the lengths of the pendulums.

Cor. 5. And universally, the quantity of matter in the pendulous body is as the weight and the square of the time directly, and the length of the pendulum inversely.

Cor. 6. But in a non-resisting medium, the quantity of matter in the pendulous body is as the comparative weight and the square of the time directly, and the length of the pendulum inversely. For the comparative weight is the motive force of the body in any heavy medium, as was shewn above; and therefore does the same thing in such a non-resisting medium as the absolute weight does in a vacuum.

Cor. 7. And hence appears a method both of comparing bodies one among another, as to the quantity of matter in

each; and of comparing the weights of the ſame body in different places, to know the variation of its gravity. And by experiments made with the greateſt accuracy, I have always found the quantity of matter in bodies to be proportional to their weight.

PROPOSITION XXV. THEOREM XX.

Funependulous bodies that are, in any medium, reſiſted in the ratio of the moments of time, and funependulous bodies that move in a non-reſiſting medium of the ſame ſpecific gravity, perform their oſcillations in a cycloid in the ſame time, and deſcribe proportional parts of arcs together. (Pl. 5, Fig. 5.)

Let AB be an arc of a cycloid, which a body D, by vibrating in a non-reſiſting medium, ſhall deſcribe in any time. Biſect that arc in C, ſo that C may be the loweſt point thereof; and the accelerative force with which the body is urged in any place D, or d, or E, will be as the length of the arc CD, or Cd, or CE. Let that force be expreſſed by that ſame arc; and ſince the reſiſtance is as the moment of the time, and therefore given, let it be expreſſed by the given part CO of the cycloidal arc, and take the arc Od in the ſame ratio to the arc CD that the arc OB has to the arc CB: and the force with which the body in d is urged in a reſiſting medium, being the exceſs of the force Cd above the reſiſtance CO, will be expreſſed by the arc Od, and will therefore be to the force with which the body D is urged in a non-reſiſting medium in the place D, as the arc Od to the arc CD; and therefore alſo in the place B, as the arc OB to the arc CB. Therefore if two bodies D, d go from the place B, and are urged by theſe forces; ſince the forces at the beginning are as the arcs CB and OB, the firſt velocities and arcs firſt deſcribed will be in the ſame ratio. Let thoſe arcs be BD and Bd, and the remaining arcs CD, Od, will be in the ſame ratio. Therefore the forces, being proportional to thoſe arcs CD, Od, will remain in the ſame ratio as at the beginning, and therefore the bodies will continue deſcribing together arcs in the ſame ratio. Therefore the forces and velocities and the remaining arcs CD, Od, will be always as the whole arcs CB, OB, and therefore thoſe remaining arcs will be deſcribed

together. Therefore the two bodies D and d will arrive together at the places C and O; that which moves in the non-reſiſting medium, at the place C, and the other, in the reſiſting medium, at the place O. Now ſince the velocities in C and O are as the arcs CB, OB, the arcs which the bodies deſcribe when they go farther will be in the ſame ratio. Let thoſe arcs be CE and Oe. The force with which the body D in a non-reſiſting medium is retarded in E is as CE, and the force with which the body d in the reſiſting medium is retarded in e, is as the ſum of the force Ce and the reſiſtance CO, that is, as Oe; and therefore the forces with which the bodies are retarded are as the arcs CB, OB, proportional to the arcs CE, Oe; and therefore the velocities, retarded in that given ratio, remain in the ſame given ratio. Therefore the velocities and the arcs deſcribed with thoſe velocities are always to each other in that given ratio of the arcs CB and OB; and therefore if the entire arcs AB, aB are taken in the ſame ratio, the bodies D and d will deſcribe thoſe arcs together, and in the places A and a will loſe all their motion together. Therefore the whole oſcillations are iſochronal, or are performed in equal times; and any parts of the arcs, as BD, Bd, or BE, Be, that are deſcribed together, are proportional to the whole arcs BA, Ba. Q.E.D.

Cor. Therefore the ſwifteſt motion in a reſiſting medium does not fall upon the loweſt point C, but is found in that point O, in which the whole arc deſcribed Ba is biſected. And the body proceeding from thence to a, is retarded at the ſame rate with which it was accelerated before in its deſcent from B to O.

PROPOSITION XXVI. THEOREM XXI.

Funependulous bodies, that are reſiſted in the ratio of the velocity, have their oſcillations in a cycloid iſochronal.

For if two bodies, equally diſtant from their centres of ſuſpenſion, deſcribe, in oſcillating, unequal arcs, and the velocities in the correſpondent parts of the arcs be to each other as the whole arcs; the reſiſtances, proportional to the velocities, will be alſo to each other as the ſame arcs. Therefore if theſe reſiſtances be ſubducted from or added to the motive

forces arifing from gravity which are as the fame arcs, the differences or fums will be to each other in the fame ratio of the arcs; and fince the increments and decrements of the velocities are as thefe differences or fums, the velocities will be always as the whole arcs; therefore if the velocities are in any one cafe as the whole arcs, they will remain always in the fame ratio. But at the beginning of the motion, when the bodies begin to defcend and defcribe thofe arcs, the forces, which at that time are proportional to the arcs, will generate velocities proportional to the arcs. Therefore the velocities will be always as the whole arcs to be defcribed, and therefore thofe arcs will be defcribed in the fame time. Q.E.D.

PROPOSITION XXVII. THEOREM XXII.

If funependulous bodies are refifted in the duplicate ratio of their velocities, the differences between the times of the ofcillations in a refifting medium, and the times of the ofcillations in a non-refifting medium of the fame fpecific gravity, will be proportional to the arcs defcribed in ofcillating nearly.

For let equal pendulums in a refifting medium defcribe the unequal arcs A, B; and the refiftance of the body in the arc A will be to the refiftance of the body in the correfpondent part of the arc B in the duplicate ratio of the velocities, that is, as AA to BB nearly. If the refiftance in the arc B were to the refiftance in the arc A as AB to AA, the times in the arcs A and B would be equal (by the laft prop.) Therefore the refiftance AA in the arc A, or AB in the arc B, caufes the excefs of the time in the arc A above the time in a non-refifting medium; and the refiftance BB caufes the excefs of the time in the arc B above the time in a non-refifting medium. But thofe exceffes are as the efficient forces AB and BB nearly, that is, as the arcs A and B. Q.E.D.

Cor. 1. Hence from the times of the ofcillations in unequal arcs in a refifting medium, may be known the times of the ofcillations in a non-refifting medium of the fame fpecific gravity. For the difference of the times will be to the excefs of the time in the leffer arc above the time in a non-refifting medium as the difference of the arcs to the leffer arc.

Cor. 2. The ſhorter oſcillations are more iſochronal, and very ſhort ones are performed nearly in the ſame times as in a non-reſiſting medium. But the times of thoſe which are performed in greater arcs are a little greater, becauſe the reſiſtance in the deſcent of the body, by which the time is prolonged, is greater, in proportion to the length deſcribed in the deſcent than the reſiſtance in the ſubſequent aſcent, by which the time is contracted. But the time of the oſcillations, both ſhort and long, ſeems to be prolonged in ſome meaſure by the motion of the medium. For retarded bodies are reſiſted ſomewhat leſs in proportion to the velocity, and accelerated bodies ſomewhat more than thoſe that proceed uniformly forwards; becauſe the medium, by the motion it has received from the bodies, going forwards the ſame way with them, is more agitated in the former caſe, and leſs in the latter; and ſo conſpires more or leſs with the bodies moved. Therefore it reſiſts the pendulums in their deſcent more, and in their aſcent leſs, than in proportion to the velocity; and theſe two cauſes concurring prolong the time.

PROPOSITION XXVIII. THEOREM XXIII.

If a funependulous body, oſcillating in a cycloid, be reſiſted in the ratio of the moments of the time, its reſiſtance will be to the force of gravity as the exceſs of the arc deſcribed in the whole deſcent above the arc deſcribed in the ſubſequent aſcent to twice the length of the pendulum. (Pl. 5, Fig. 5.)

Let BC repreſent the arc deſcribed in the deſcent, Ca the arc deſcribed in the aſcent, and Aa the difference of the arcs: and things remaining as they were conſtructed and demonſtrated in prop. 25, the force with which the oſcillating body is urged in any place D will be to the force of reſiſtance as the arc CD to the arc CO, which is half of that difference Aa. Therefore the force with which the oſcillating body is urged at the beginning or the higheſt point of the cycloid, that is, the force of gravity, will be to the reſiſtance as the arc of the cycloid, between that higheſt point and loweſt point C, is to the arc CO; that is (doubling thoſe arcs), as the whole cycloidal arc, or twice the length of the pendulum, to the arc Aa. Q.E.D.

PROPOSITION XXIX. PROBLEM VI.

Suppoſing that a body oſcillating in a cycloid is reſiſted in a duplicate ratio of the velocity: to find the reſiſtance in each place. (Pl. 5, Fig. 6.)

Let Ba be an arc deſcribed in one entire oſcillation, C the loweſt point of the cycloid, and CZ half the whole cycloidal arc, equal to the length of the pendulum; and let it be required to find the reſiſtance of the body in any place D. Cut the indefinite right line OQ in the points O, S, P, Q, ſo that (erecting the perpendiculars OK, ST, PI, QE, and with the centre O, and the aſymptotes OK, OQ, deſcribing the hyperbola TIGE cutting the perpendiculars ST, PI, QE in T, I, and E, and through the point I drawing KF, parallel to the aſymptote OQ, meeting the aſymptote OK in K, and the perpendiculars ST and QE in L and F) the hyperbolic area PIEQ may be to the hyperbolic area PITS as the arc BC, deſcribed in the deſcent of the body, to the arc Ca deſcribed in the aſcent; and that the area IEF may be to the area ILT as OQ to OS. Then with the perpendicular MN cut off the hyperbolic area PINM, and let that area be to the hyperbolic area PIEQ as the arc CZ to the arc BC deſcribed in the deſcent. And if the perpendicular RG cut off the hyperbolic area PIGR, which ſhall be to the area PIEQ as any arc CD to the arc BC deſcribed in the whole deſcent, the reſiſtance in any place D will be to the force of gravity as the area $\frac{OR}{OQ}$ IEF — IGH to the area PINM.

For ſince the forces ariſing from gravity with which the body is urged in the places Z, B, D, a, are as the arcs CZ, CB, CD, Ca, and thoſe arcs are as the areas PINM, PIEQ, PIGR, PITS; let thoſe areas be the exponents both of the arcs and of the forces reſpectively. Let Dd be a very ſmall ſpace deſcribed by the body in its deſcent; and let it be expreſſed by the very ſmall area RGgr comprehended between the parallels RG, rg; and produce rg to h, ſo that GHhg and RGgr may be the contemporaneous decrements of the areas IGH, PIGR. And the increment GHhg — $\frac{Rr}{OQ}$ IEF,

or $Rr \times HG - \frac{Rr}{OQ}$ IEF, of the area $\frac{OR}{OQ}$ IEF — IGH will be to the decrement RGgr, or $Rr \times RG$, of the area PIGR, as $HG - \frac{IEF}{OQ}$ to RG; and therefore as $OR \times HG - \frac{OR}{OQ}$ IEF to $OR \times GR$ or $OP \times PI$, that is (becauſe of the equal quantities $OR \times HG$, $OR \times HR - OR \times GR$, ORHK — OPIK, PIHR and PIGR + IGH), as PIGR + IGH $- \frac{OR}{OQ}$ IEF to OPIK. Therefore if the area $\frac{OR}{OQ}$ IEF — IGH be called Y, and RGgr the decrement of the area PIGR be given, the increment of the area Y will be as PIGR — Y.

Then if V repreſent the force ariſing from the gravity, proportional to the arc CD to be deſcribed, by which the body is acted upon in D, and R be put for the reſiſtance, V — R will be the whole force with which the body is urged in D. Therefore the increment of the velocity is as V — R and the particle of time in which it is generated conjunctly. But the velocity itſelf is as the contemporaneous increment of the ſpace deſcribed directly and the ſame particle of time inverſely. Therefore, ſince the reſiſtance is, by the ſuppoſition, as the ſquare of the velocity, the increment of the reſiſtance will (by lem. 2) be as the velocity and the increment of the velocity conjunctly, that is, as the moment of the ſpace and V — R conjunctly; and, therefore, if the moment of the ſpace be given, as V — R; that is, if for the force V we put its exponent PIGR, and the reſiſtance R be expreſſed by any other area Z, as PIGR — Z.

Therefore the area PIGR uniformly decreaſing by the ſubduction of given moments, the area Y increaſes in proportion of PIGR — Y, and the area Z in proportion of PIGR — Z. And therefore if the areas Y and Z begin together, and at the beginning are equal, theſe, by the addition of equal moments, will continue to be equal; and in like manner decreaſing by equal moments, will vaniſh together. And, *vice verſa*, if they together begin and vaniſh, they will have equal mo-

ments and be always equal; and that, becauſe if the reſiſtance Z be augmented, the velocity together with the arc Ca, deſcribed in the aſcent of the body, will be diminiſhed; and the point in which all the motion together with the reſiſtance ceaſes coming nearer to the point C, the reſiſtance vaniſhes ſooner than the area Y. And the contrary will happen when the reſiſtance is diminiſhed.

Now the area Z begins and ends where the reſiſtance is nothing, that is, at the beginning of the motion where the arc CD is equal to the arc CB, and the right line RG falls upon the right line QE; and at the end of the motion where the arc CD is equal to the arc Ca, and RG falls upon the right line ST. And the area Y or $\frac{OR}{OQ}$ IEF —IGH begins and ends alſo where the reſiſtance is nothing, and therefore where $\frac{OR}{OQ}$ IEF and IGH are equal; that is (by the conſtruction), where the right line RG falls ſucceſſively upon the right lines QE and ST. Therefore thoſe areas begin and vaniſh together, and are therefore always equal. Therefore the area $\frac{OR}{OQ}$ IEF — IGH is equal to the area Z, by which the reſiſtance is expreſſed, and therefore is to the area PINM, by which the gravity is expreſſed, as the reſiſtance to the gravity. Q.E.D.

Cor. 1. Therefore the reſiſtance in the loweſt place C is to the force of gravity as the area $\frac{OP}{OQ}$ IEF to the area PINM.

Cor. 2. But it becomes greateſt where the area PIHR is to the area IEF as OR to OQ. For in that caſe its moment (that is, PIGR — Y) becomes nothing.

Cor. 3. Hence alſo may be known the velocity in each place, as being in the ſubduplicate ratio of the reſiſtance, and at the beginning of the motion equal to the velocity of the body oſcillating in the ſame cycloid without any reſiſtance.

However, by reaſon of the difficulty of the calculation by which the reſiſtance and the velocity are found by this pro-

Fig. 1.

Fig 2

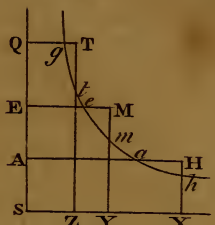


Fig. 3.

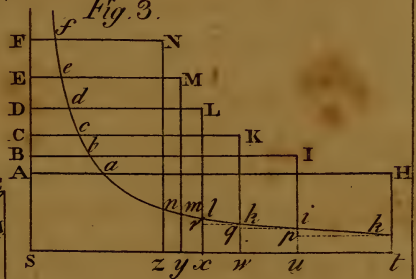
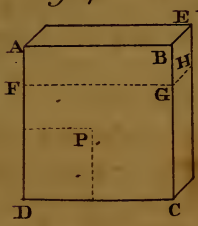


Fig. 4.



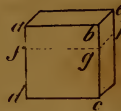


Fig. 5.

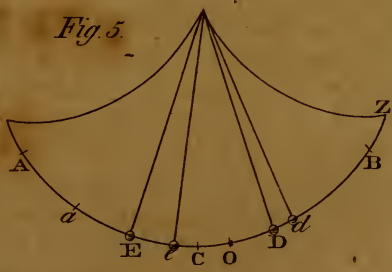
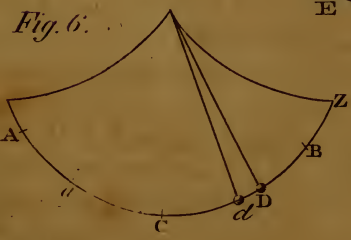
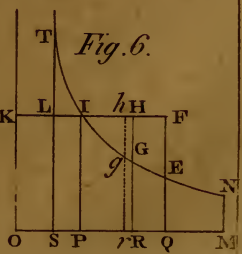


Fig. 6.





poſition, we have thought fit to ſubjoin the propoſition following.

PROPOSITION XXX. THEOREM XXIV.

If a right line aB (Pl. 6, Fig. 1) *be equal to the arc of a cycloid which an oſcillating body deſcribes, and at each of its points* D *the perpendiculars* DK *be erected, which ſhall be to the length of the pendulum as the reſiſtance of the body in the correſponding points of the arc to the force of gravity; I ſay, that the difference between the arc deſcribed in the whole deſcent and the arc deſcribed in the whole ſubſequent aſcent drawn into half the ſum of the ſame arcs will be equal to the area* BKa *which all thoſe perpendiculars take up.*

Let the arc of the cycloid, deſcribed in one entire oſcillation, be expreſſed by the right line aB, equal to it, and the arc which would have been deſcribed *in vacuo* by the length AB. Biſect AB in C, and the point C will repreſent the loweſt point of the cycloid, and CD will be as the force ariſing from gravity, with which the body in D is urged in the direction of the tangent of the cycloid, and will have the ſame ratio to the length of the pendulum as the force in D has to the force of gravity. Let that force, therefore, be expreſſed by that length CD, and the force of gravity by the length of the pendulum; and if in DE you take DK in the ſame ratio to the length of the pendulum as the reſiſtance has to the gravity, DK will be the exponent of the reſiſtance. From the centre C with the interval CA or CB deſcribe a ſemi-circle BEeA. Let the body deſcribe, in the leaſt time, the ſpace Dd; and, erecting the perpendiculars DE, de, meeting the circumference in E and e, they will be as the velocities which the body deſcending *in vacuo* from the point B would acquire in the places D and d. This appears by prop. 52, book 1. Let, therefore, theſe velocities be expreſſed by thoſe perpendiculars DE, de; and let DF be the velocity which it acquires in D by falling from B in the reſiſting medium. And if from the centre C with the interval CF we deſcribe the circle FfM meeting the right lines de and AB in f and M, then M will be the place to which it would thenceforward, without farther reſiſtance, aſcend, and df the velocity it would acquire in

d. Whence, alſo, if Fg repreſent the moment of the velocity which the body D, in deſcribing the leaſt ſpace Dd, loſes by the reſiſtance of the medium; and CN be taken equal to Cg; then will N be the place to which the body, if it met no farther reſiſtance, would thenceforward aſcend, and MN will be the decrement of the aſcent ariſing from the loſs of that velocity. Draw Fm perpendicular to df, and the decrement Fg of the velocity DF generated by the reſiſtance DK will be to the increment fm of the ſame velocity, generated by the force CD, as the generating force DK to the generating force CD. But becauſe of the ſimilar triangles Fmf, Fhg, FDC, fm is to Fm or Dd as CD to DF; and, *ex æquo*, Fg to Dd as DK to DF. Alſo Fh is to Fg as DF to CF; and, *ex æquo perturbatè*, Fh or MN to Dd as DK to CF or CM; and therefore the ſum of all the MN × CM will be equal to the ſum of all the Dd × DK. At the moveable point M ſuppoſe always a rectangular ordinate erected equal to the indeterminate CM, which by a continual motion is drawn into the whole length Aa; and the trapezium deſcribed by that motion, or its equal, the rectangle Aa × ½aB, will be equal to the ſum of all the MN × CM, and therefore to the ſum of all the Dd × DK, that is, to the area BKVTa. Q.E.D.

Cor. Hence from the law of reſiſtance, and the difference Aa of the arcs Ca, CB, may be collected the proportion of the reſiſtance to the gravity nearly.

For if the reſiſtance DK be uniform, the figure BKTa will be a rectangle under Ba and DK; and thence the rectangle under ½Ba and Aa will be equal to the rectangle under Ba and DK, and DK will be equal to ½Aa. Wherefore ſince DK is the exponent of the reſiſtance, and the length of the pendulum the exponent of the gravity, the reſiſtance will be to the gravity as ½Aa to the length of the pendulum; altogether as in prop. 28 is demonſtrated.

If the reſiſtance be as the velocity, the figure BKTa will be nearly an ellipſis. For if a body, in a non-reſiſting medium, by one entire oſcillation, ſhould deſcribe the length BA, the velocity in any place D would be as the ordinate

DE of the circle described on the diameter AB. Therefore since Ba in the resisting medium, and BA in the non-resisting one, are described nearly in the same times; and therefore the velocities in each of the points of Ba are to the velocities in the correspondent points of the length BA nearly as Ba is to BA; the velocity in the point D in the resisting medium will be as the ordinate of the circle or ellipsis described upon the diameter Ba; and therefore the figure BKVTa will be nearly an ellipsis. Since the resistance is supposed proportional to the velocity, let OV be the exponent of the resistance in the middle point O; and an ellipsis BRVSa described with the centre O, and the semi-axes OB, OV, will be nearly equal to the figure BKVTa, and to its equal the rectangle Aa × BO. Therefore Aa × BO is to OV × BO as the area of this ellipsis to OV × BO; that is, Aa is to OV as the area of the semi-circle to the square of the radius, or as 11 to 7 nearly; and, therefore, $\frac{7}{11}$Aa is to the length of the pendulum as the resistance of the oscillating body in O to its gravity.

Now if the resistance DK be in the duplicate ratio of the velocity, the figure BKVTa will be almost a parabola having V for its vertex and OV for its axis, and therefore will be nearly equal to the rectangle under $\frac{2}{3}$Ba and OV. Therefore the rectangle under $\frac{1}{2}$Ba and Aa is equal to the rectangle $\frac{2}{3}$Ba × OV, and therefore OV is equal to $\frac{3}{4}$Aa; and therefore the resistance in O made to the oscillating body is to its gravity as $\frac{3}{4}$Aa to the length of the pendulum.

And I take these conclusions to be accurate enough for practical uses. For since an ellipsis or parabola BRVSa falls in with the figure BKVTa in the middle point V, that figure, if greater towards the part BRV or VSa than the other, is less towards the contrary part, and is therefore nearly equal to it.

PROPOSITION XXXI. THEOREM XXV.

If the resistance made to an oscillating body in each of the proportional parts of the arcs described be augmented or diminished in a given ratio, the difference between the arc described in the descent and the arc described in the subsequent ascent will be augmented or diminished in the same ratio.

For that difference arises from the retardation of the pendulum by the resistance of the medium, and therefore is as the whole retardation and the retarding resistance proportional thereto. In the foregoing proposition the rectangle under the right line $\frac{1}{2}$aB and the difference Aa of the arcs CB, Ca, was equal to the area BKTa. And that area, if the length aB remains, is augmented or diminished in the ratio of the ordinates DK; that is, in the ratio of the resistance, and is therefore as the length aB and the resistance conjunctly. And therefore the rectangle under Aa and $\frac{1}{2}$aB is as aB and the resistance conjunctly, and therefore Aa is as the resistance. Q.E.D.

Cor. 1. Hence if the resistance be as the velocity, the difference of the arcs in the same medium will be as the whole arc described: and the contrary.

Cor. 2. If the resistance be in the duplicate ratio of the velocity, that difference will be in the duplicate ratio of the whole arc: and the contrary.

Cor. 3. And universally, if the resistance be in the triplicate or any other ratio of the velocity, the difference will be in the same ratio of the whole arc: and the contrary.

Cor. 4. If the resistance be partly in the simple ratio of the velocity, and partly in the duplicate ratio of the same, the difference will be partly in the ratio of the whole arc, and partly in the duplicate ratio of it: and the contrary. So that the law and ratio of the resistance will be the same for the velocity as the law and ratio of that difference for the length of the arc.

Cor. 5. And therefore if a pendulum describe successively unequal arcs, and we can find the ratio of the increment or decrement of this difference for the length of the arc described, there will be had also the ratio of the increment or decrement of the resistance for a greater or less velocity.

GENERAL SCHOLIUM.

From these propositions we may find the resistance of mediums by pendulums oscillating therein. I found the resistance of the air by the following experiments. I suspended a wooden globe or ball weighing $57\frac{7}{22}$ ounces troy, its diameter $6\frac{7}{8}$ *London* inches, by a fine thread on a firm hook, so that

the diſtance between the hook and the centre of oſcillation of the globe was $10\frac{1}{2}$ feet. I marked on the thread a point 10 feet and 1 inch diſtant from the centre of ſuſpenſion; and even with that point I placed a ruler divided into inches, by the help whereof I obſerved the lengths of the arcs deſcribed by the pendulum. Then I numbered the oſcillations in which the globe would loſe $\frac{1}{8}$ part of its motion. If the pendulum was drawn aſide from the perpendicular to the diſtance of 2 inches, and thence let go, ſo that in its whole deſcent it deſcribed an arc of 2 inches, and in the firſt whole oſcillation, compounded of the deſcent and ſubſequent aſcent, an arc of almoſt 4 inches, the ſame in 164 oſcillations loſt $\frac{1}{8}$ part of its motion, ſo as in its laſt aſcent to deſcribe an arc of $1\frac{1}{4}$ inches. If in the firſt deſcent it deſcribed an arc of 4 inches, it loſt $\frac{1}{8}$ part of its motion in 121 oſcillations, ſo as in its laſt aſcent to deſcribe an arc of $3\frac{1}{2}$ inches. If in the firſt deſcent it deſcribed an arc of 8, 16, 32, or 64 inches, it loſt $\frac{1}{8}$ part of its motion in 69, $35\frac{1}{2}$, $18\frac{1}{2}$, $9\frac{2}{3}$ oſcillations, reſpectively. Therefore the difference between the arcs deſcribed in the firſt deſcent and the laſt aſcent was in the 1ſt, 2d, 3d, 4th, 5th, 6th caſes, $\frac{1}{4}$, $\frac{1}{2}$, 1, 2, 4, 8 inches, reſpectively. Divide thoſe differences by the number of oſcillations in each caſe, and in one mean oſcillation, wherein an arc of $3\frac{3}{4}$, $7\frac{1}{2}$, 15, 30, 60, 120 inches was deſcribed, the difference of the arcs deſcribed in the deſcent and ſubſequent aſcent will be $\frac{1}{656}$, $\frac{1}{2442}$, $\frac{1}{699}$, $\frac{4}{711}$, $\frac{8}{377}$, $\frac{24}{299}$ parts of an inch, reſpectively. But theſe differences in the greater oſcillations are in the duplicate ratio of the arcs deſcribed nearly, but in leſſer oſcillations ſomething greater than in that ratio; and therefore (by cor. 2, prop. 31 of this book) the reſiſtance of the globe, when it moves very ſwift, is in the duplicate ratio of the velocity, nearly; and when it moves ſlowly, ſomewhat greater than in that ratio.

Now let V repreſent the greateſt velocity in any oſcillation, and let A, B, and C be given quantities, and let us ſuppoſe the difference of the arcs to be $AV + BV^{\frac{3}{2}} + CV^2$. Since the greateſt velocities are in the cycloid as $\frac{1}{2}$ the arcs deſcribed in oſcillating, and in the circle as $\frac{1}{2}$ the chords of thoſe arcs; and

therefore in equal arcs are greater in the cycloid than in the circle in the ratio of $\frac{1}{2}$ the arcs to their chords; but the times in the circle are greater than in the cycloid, in a reciprocal ratio of the velocity; it is plain that the differences of the arcs (which are as the reſiſtance and the ſquare of the time conjunctly) are nearly the ſame in both curves: for in the cycloid thoſe differences muſt be on the one hand augmented, with the reſiſtance, in about the duplicate ratio of the arc to the chord, becauſe of the velocity augmented in the ſimple ratio of the ſame; and on the other hand diminiſhed, with the ſquare of the time, in the ſame duplicate ratio. Therefore to reduce theſe obſervations to the cycloid, we muſt take the ſame differences of the arcs as were obſerved in the circle, and ſuppoſe the greateſt velocities analogous to the half, or the whole arcs, that is, to the numbers $\frac{1}{2}$, 1, 2, 4, 8, 16. Therefore in the 2d, 4th, and 6th caſes, put 1, 4, and 16 for V; and the difference of the arcs in the 2d caſe will become $\frac{\frac{1}{2}}{121} = A + B + C$; in the 4th caſe, $\frac{2}{35\frac{1}{2}} = 4A + 8B + 16C$; in the 6th caſe, $\frac{8}{9\frac{2}{3}} = 16A + 64B + 256C$. Theſe equations reduced give $A = 0{,}0000916$, $B = 0{,}0010847$, and $C = 0{,}0029558$. Therefore the difference of the arcs is as $0{,}0000916V + 0{,}0010847V^{\frac{3}{2}} + 0{,}0029558V^2$: and therefore ſince (by cor. prop. 30, applied to this caſe) the reſiſtance of the globe in the middle of the arc deſcribed in oſcillating, where the velocity is V, is to its weight as $\frac{1}{11}AV + \frac{7}{10}BV^{\frac{3}{2}} + \frac{3}{4}CV^2$ to the length of the pendulum, if for A, B, and C you put the numbers found, the reſiſtance of the globe will be to its weight as $0{,}0000583V + 0{,}0007593V^{\frac{3}{2}} + 0{,}0022169V^2$ to the length of the pendulum between the centre of ſuſpenſion and the ruler, that is, to 121 inches. Therefore ſince V in the 2d caſe repreſents 1, in the 4th caſe 4, and in the 6th caſe 16, the reſiſtance will be to the weight of the globe, in the 2d caſe, as 0,0030345 to 121; in the 4th, as 0,041748 to 121; in the 6th, as 0,61705 to 121.

The arc which the point marked in the thread described in the 6th case, was of $120 - \frac{8}{9\frac{2}{3}}$, or $119\frac{5}{29}$ inches. And therefore since the radius was 121 inches, and the length of the pendulum between the point of suspension and the centre of the globe was 126 inches, the arc which the centre of the globe described was $124\frac{3}{31}$ inches. Because the greatest velocity of the oscillating body, by reason of the resistance of the air, does not fall on the lowest point of the arc described, but near the middle place of the whole arc, this velocity will be nearly the same as if the globe in its whole descent in a non-resisting medium should describe $62\frac{3}{62}$ inches, the half of that arc, and that in a cycloid, to which we have above reduced the motion of the pendulum; and therefore that velocity will be equal to that which the globe would acquire by falling perpendicularly from a height equal to the versed sine of that arc. But that versed sine in the cycloid is to that arc $62\frac{3}{62}$ as the same arc to twice the length of the pendulum 252, and therefore equal to 15,278 inches. Therefore the velocity of the pendulum is the same which a body would acquire by falling, and in its fall describing a space of 15,278 inches. Therefore with such a velocity the globe meets with a resistance which is to its weight as 0,61705 to 121, or (if we take that part only of the resistance which is in the duplicate ratio of the velocity) as 0,56752 to 121.

I found, by an hydrostatical experiment, that the weight of this wooden globe was to the weight of a globe of water of the same magnitude as 55 to 97: and therefore since 121 is to 213,4 in the same ratio, the resistance made to this globe of water, moving forwards with the above-mentioned velocity, will be to its weight as 0,56752 to 213,4, that is, as 1 to $376\frac{4}{5}$. Whence since the weight of a globe of water, in the time in which the globe with a velocity uniformly continued describes a length of 30,556 inches, will generate all that velocity in the falling globe, it is manifest that the force of resistance uniformly continued in the same time will take away a velocity, which will be less than the other in the ratio of 1 to $376\frac{4}{5}$.

that is, the $\frac{1}{376\frac{1}{50}}$ part of the whole velocity. And therefore in the time that the globe, with the same velocity uniformly continued, would describe the length of its semi-diameter, or $3\frac{7}{16}$ inches, it would lose the $\frac{1}{3342}$ part of its motion.

I also counted the oscillations in which the pendulum lost $\frac{1}{4}$ part of its motion. In the following table the upper numbers denote the length of the arc described in the first descent, expressed in inches and parts of an inch; the middle numbers denote the length of the arc described in the last ascent; and in the lowest place are the numbers of the oscillations. I give an account of this experiment, as being more accurate than that in which only $\frac{1}{8}$ part of the motion was lost. I leave the calculation to such as are disposed to make it.

First descent	2	4	8	16	32	64
Last ascent	$1\frac{1}{2}$	3	6	12	24	48
Numb. of oscill...374	272	$162\frac{1}{2}$	$83\frac{1}{3}$	$41\frac{2}{3}$	$22\frac{2}{3}$	

I afterwards suspended a leaden globe of 2 inches in diameter, weighing $26\frac{1}{4}$ ounces troy by the same thread, so that between the centre of the globe and the point of suspension there was an interval of $10\frac{1}{2}$ feet, and I counted the oscillations in which a given part of the motion was lost. The first of the following tables exhibits the number of oscillations in which $\frac{1}{8}$ part of the whole motion was lost; the second the number of oscillations in which there was lost $\frac{1}{4}$ part of the same.

First descent	1	2	4	8	16	32	64
Last ascent	$\frac{7}{8}$	$\frac{7}{4}$	$3\frac{1}{2}$	7	14	28	56
Numb. of oscill...226	228	193	140	$90\frac{1}{2}$	53	30	
First descent	1	2	4	8	16	32	64
Last ascent	$\frac{3}{4}$	$1\frac{1}{2}$	3	6	12	24	48
Numb. of oscill...510	518	420	318	204	121	70	

Selecting in the first table the 3d, 5th, and 7th observations, and expressing the greatest velocities in these observations particularly by the numbers 1, 4, 16 respectively, and generally by the quantity V as above, there will come out in the 3d observation $\frac{\frac{1}{2}}{193} = A + B + C$, in the 5th observation $\frac{2}{90\frac{1}{2}} =$

$4A + 8B + 16C$, in the 7th obſervation $\frac{8}{30} = 16A + 64B + 256C$. Theſe equations reduced give $A = 0{,}001414$, $B = 0{,}000297$, $C = 0{,}000879$. And thence the reſiſtance of the globe moving with the velocity V will be to its weight $26\frac{1}{4}$ ounces in the ſame ratio as $0{,}0009V + 0{,}000208V^{\frac{3}{2}} + 0{,}000659V^2$ to 121 inches, the length of the pendulum. And if we regard that part only of the reſiſtance which is in the duplicate ratio of the velocity, it will be to the weight of the globe as $0{,}000659V^2$ to 121 inches. But this part of the reſiſtance in the 1ſt experiment was to the weight of the wooden globe of $57\frac{7}{22}$ ounces as $0{,}002217V^2$ to 121; and thence the reſiſtance of the wooden globe is to the reſiſtance of the leaden one (their velocities being equal) as $57\frac{7}{22}$ into 0,002217 to $26\frac{1}{4}$ into 0,000659, that is, as $7\frac{1}{3}$ to 1. The diameters of the two globes were $6\frac{7}{8}$ and 2 inches, and the ſquares of theſe are to each other as $47\frac{1}{4}$ and 4, or $11\frac{13}{16}$ and 1, nearly. Therefore the reſiſtances of theſe equally ſwift globes were in leſs than a duplicate ratio of the diameters. But we have not yet conſidered the reſiſtance of the thread, which was certainly very conſiderable, and ought to be ſubducted from the reſiſtance of the pendulums here found. I could not determine this accurately, but I found it greater than a third part of the whole reſiſtance of the leſſer pendulum; and thence I gathered that the reſiſtances of the globes, when the reſiſtance of the thread is ſubducted, are nearly in the duplicate ratio of their diameters. For the ratio of $7\frac{1}{3} - \frac{1}{3}$ to $1 - \frac{1}{3}$, or $10\frac{1}{2}$ to 1, is not very different from the duplicate ratio of the diameters $11\frac{13}{16}$ to 1.

Since the reſiſtance of the thread is of leſs moment in greater globes, I tried the experiment alſo with a globe whoſe diameter was $18\frac{3}{4}$ inches. The length of the pendulum between the point of ſuſpenſion and the centre of oſcillation was $122\frac{1}{2}$ inches, and between the point of ſuſpenſion and the knot in the thread $109\frac{1}{2}$ inches. The arc deſcribed by the knot at the firſt deſcent of the pendulum was 32 inches. The arc deſcribed by the ſame knot in the laſt aſcent after five oſ-

cillations was 28 inches. The ſum of the arcs, or the whole arc deſcribed in one mean oſcillation, was 60 inches. The difference of the arcs 4 inches. The $\frac{1}{10}$ part of this, or the difference between the deſcent and aſcent in one mean oſcillation, is $\frac{2}{5}$ of an inch. Then as the radius $109\frac{1}{2}$ to the radius $122\frac{1}{2}$, ſo is the whole arc of 60 inches deſcribed by the knot in one mean oſcillation to the whole arc of $67\frac{1}{8}$ inches deſcribed by the centre of the globe in one mean oſcillation; and ſo is the difference $\frac{2}{5}$ to a new difference 0,4475. If the length of the arc deſcribed were to remain, and the length of the pendulum ſhould be augmented in the ratio of 126 to $122\frac{1}{2}$, the time of the oſcillation would be augmented, and the velocity of the pendulum would be diminiſhed in the ſubduplicate of that ratio; ſo that the difference 0,4475 of the arcs deſcribed in the deſcent and ſubſequent aſcent would remain. Then if the arc deſcribed be augmented in the ratio of $124\frac{3}{31}$ to $67\frac{1}{8}$, that difference 0,4475 would be augmented in the duplicate of that ratio, and ſo would become 1,5295. Theſe things would be ſo upon the ſuppoſition that the reſiſtance of the pendulum were in the duplicate ratio of the velocity. Therefore if the pendulum deſcribe the whole arc of $124\frac{3}{31}$ inches, and its length between the point of ſuſpenſion and the centre of oſcillation be 126 inches, the difference of the arcs deſcribed in the deſcent and ſubſequent aſcent would be 1,5295 inches. And this difference multiplied into the weight of the pendulous globe, which was 208 ounces, produces 318,136. Again; in the pendulum above-mentioned, made of a wooden globe, when its centre of oſcillation, being 126 inches from the point of ſuſpenſion, deſcribed the whole arc of $124\frac{3}{31}$ inches, the difference of the arcs deſcribed in the deſcent and aſcent was $\frac{126}{121}$ into $\frac{8}{9\frac{2}{3}}$. This multiplied into the weight of the globe, which was $57\frac{7}{22}$ ounces, produces 49,396. But I multiply theſe differences into the weights of the globes, in order to find their reſiſtances. For the differences ariſe from the reſiſtances, and are as the reſiſtances directly and the weights inverſely. Therefore the reſiſtances are as the numbers 318,136 and 49,396. But that part of the reſiſtance of the leſſer globe,

which is in the duplicate ratio of the velocity, was to the whole reſiſtance as 0,56752 to 0,61675, that is, as 45,453 to 49,396; whereas that part of the reſiſtance of the greater globe is almoſt equal to its whole reſiſtance; and ſo thoſe parts are nearly as 318,136 and 45,453, that is, as 7 and 1. But the diameters of the globes are $18\frac{3}{4}$ and $6\frac{7}{8}$; and their ſquares $351\frac{9}{16}$ and $47\frac{17}{64}$ are as 7,438 and 1, that is, as the reſiſtances of the globes 7 and 1, nearly. The difference of theſe ratios is ſcarce greater than may ariſe from the reſiſtance of the thread. Therefore thoſe parts of the reſiſtances which are, when the globes are equal, as the ſquares of the velocities, are alſo, when the velocities are equal, as the ſquares of the diameters of the globes.

But the greateſt of the globes I uſed in theſe experiments was not perfectly ſpherical, and therefore in this calculation I have, for brevity's ſake, neglected ſome little niceties; being not very ſolicitous for an accurate calculus in an experiment that was not very accurate. So that I could wiſh that theſe experiments were tried again with other globes, of a larger ſize, more in number, and more accurately formed; ſince the demonſtration of a vacuum depends thereon. If the globes be taken in a geometrical proportion, as ſuppoſe whoſe diameters are 4, 8, 16, 32 inches; one may collect from the progreſſion obſerved in the experiments what would happen if the globes were ſtill larger.

In order to compare the reſiſtances of different fluids with each other, I made the following trials. I procured a wooden veſſel 4 feet long, 1 foot broad, and 1 foot high. This veſſel, being uncovered, I filled with ſpring water, and, having immerſed pendulums therein, I made them oſcillate in the water. And I found that a leaden globe weighing $166\frac{1}{6}$ ounces, and in diameter $3\frac{5}{8}$ inches, moved therein as it is ſet down in the following table; the length of the pendulum from the point of ſuſpenſion to a certain point marked in the thread being 126 inches, and to the centre of oſcillation $134\frac{3}{8}$ inches.

The arc deſcribed in the firſt deſcent by a point marked in the thread, was inches	64 . 32 . 16 . 8 . 4 . 2 . 1 . $\frac{1}{2}$. $\frac{1}{4}$
The arc deſcribed in the laſt aſcent was inches	48 . 24 . 12 . 6 . 3 . $1\frac{1}{2}$. $\frac{3}{4}$. $\frac{3}{8}$. $\frac{3}{16}$
The difference of the arcs proportional to the motion loſt, was inches	16 . 8 . 4 . 2 . 1 . $\frac{1}{2}$. $\frac{1}{4}$. $\frac{1}{8}$. $\frac{1}{16}$
The number of the oſcillations in water	$\frac{29}{60}$. $1\frac{1}{5}$. 3 . 7 . $11\frac{1}{4}$. $12\frac{2}{3}$. $13\frac{1}{3}$
The number of the oſcillations in air	$85\frac{1}{2}$. 287 . 535

In the experiments of the 4th column there were equal motions loſt in 535 oſcillations made in the air, and $1\frac{1}{5}$ in water. The oſcillations in the air were indeed a little ſwifter than thoſe in the water. But if the oſcillations in the water were accelerated in ſuch a ratio that the motions of the pendulums might be equally ſwift in both mediums, there would be ſtill the ſame number $1\frac{1}{5}$ of oſcillations in the water, and by theſe the ſame quantity of motion would be loſt as before; becauſe the reſiſtance is increaſed, and the ſquare of the time diminiſhed in the ſame duplicate ratio. The pendulums, therefore, being of equal velocities, there were equal motions loſt in 535 oſcillations in the air, and $1\frac{1}{5}$ in the water; and therefore the reſiſtance of the pendulum in the water is to its reſiſtance in the air as 535 to $1\frac{1}{5}$. This is the proportion of the whole reſiſtances in the caſe of the 4th column.

Now let $AV + CV^2$ repreſent the difference of the arcs deſcribed in the deſcent and ſubſequent aſcent by the globe moving in air with the greateſt velocity V; and ſince the greateſt velocity is in the caſe of the 4th column to the greateſt velocity in the caſe of the 1ſt column as 1 to 8; and that difference of the arcs in the caſe of the 4th column to the dif-

ference in the case of the 1st column as $\frac{2}{535}$ to $\frac{16}{85\frac{1}{2}}$, or as $85\frac{1}{2}$ to 4280; put in these cases 1 and 8 for the velocities, and $85\frac{1}{2}$ and 4280 for the differences of the arcs, and A + C will be $= 85\frac{1}{2}$, and 8A + 64C = 4280 or A + 8C = 535; and then, by reducing these equations, there will come out 7C $= 449\frac{1}{2}$ and C $= 64\frac{3}{14}$ and A $= 21\frac{2}{7}$; and therefore the resistance, which is as $\frac{7}{11}$AV + $\frac{3}{4}$CV², will become as $13\frac{6}{11}$V + $48\frac{9}{56}$V². Therefore in the case of the 4th column, where the velocity was 1, the whole resistance is to its part proportional to the square of the velocity as $13\frac{6}{11}$ + $48\frac{9}{56}$ or $61\frac{12}{17}$ to $48\frac{9}{56}$; and therefore the resistance of the pendulum in water is to that part of the resistance in air, which is proportional to the square of the velocity, and which in swift motions is the only part that deserves consideration, as $61\frac{12}{17}$ to $48\frac{9}{56}$ and 535 to $1\frac{1}{5}$ conjunctly, that is, as 571 to 1. If the whole thread of the pendulum oscillating in the water had been immersed, its resistance would have been still greater; so that the resistance of the pendulum oscillating in the water, that is, that part which is proportional to the square of the velocity, and which only needs to be considered in swift bodies, is to the resistance of the same whole pendulum, oscillating in air with the same velocity, as about 850 to 1, that is, as the density of water to the density of air, nearly.

In this calculation we ought also to have taken in that part of the resistance of the pendulum in the water which was as the square of the velocity; but I found (which will perhaps seem strange) that the resistance in the water was augmented in more than a duplicate ratio of the velocity. In searching after the cause, I thought upon this, that the vessel was too narrow for the magnitude of the pendulous globe, and by its narrowness obstructed the motion of the water as it yielded to the oscillating globe. For when I immersed a pendulous globe, whose diameter was one inch only, the resistance was augmented nearly in a duplicate ratio of the velocity. I tried this by making a pendulum of two globes, of which the lesser and lower oscillated in the water, and the greater and higher was fastened to the thread just above the

water, and, by oſcillating in the air, aſſiſted the motion of the pendulum, and continued it longer. The experiments made by this contrivance proved according to the following table.

Arc deſcr. in firſt deſcent ..	16	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$
Arc deſcr. in laſt aſcent ..	12	6	3	$1\frac{1}{2}$	$\frac{3}{4}$	$\frac{3}{8}$	$\frac{3}{16}$
Diff. of arcs, proport. to motion loſt	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$
Number of oſcillations ...	$3\frac{3}{8}$	$6\frac{1}{2}$	$12\frac{1}{12}$	$21\frac{1}{5}$	34	53	$62\frac{1}{5}$

In comparing the reſiſtances of the mediums with each other, I alſo cauſed iron pendulums to oſcillate in quickſilver. The length of the iron wire was about 3 feet, and the diameter of the pendulous globe about $\frac{1}{3}$ of an inch. To the wire, juſt above the quickſilver, there was fixed another leaden globe of a bigneſs ſufficient to continue the motion of the pendulum for ſome time. Then a veſſel, that would hold about 3 pounds of quickſilver, was filled by turns with quickſilver and common water, that, by making the pendulum oſcillate ſucceſſively in theſe two different fluids, I might find the proportion of their reſiſtances; and the reſiſtance of the quickſilver proved to be to the reſiſtance of water as about 13 or 14 to 1; that is, as the denſity of quickſilver to the denſity of water. When I made uſe of a pendulous globe ſomething bigger, as of one whoſe diameter was about $\frac{1}{2}$ or $\frac{2}{3}$ of an inch, the reſiſtance of the quickſilver proved to be to the reſiſtance of the water as about 12 or 10 to 1. But the former experiment is more to be relied on, becauſe in the latter the veſſel was too narrow in proportion to the magnitude of the immerſed globe; for the veſſel ought to have been enlarged together with the globe. I intended to have repeated theſe experiments with larger veſſels, and in melted metals, and other liquors both cold and hot; but I had not leiſure to try all; and beſides, from what is already deſcribed, it appears ſufficiently that the reſiſtance of bodies moving ſwiftly is nearly proportional to the denſities of the fluids in which they move. I do not ſay accurately; for more tenacious fluids, of equal denſity, will undoubtedly reſiſt more than thoſe that are more liquid; as cold oil more than warm, warm oil more than rain-water, and water more than ſpirit of wine. But in liquors,

which are ſenſibly fluid enough, as in air, in ſalt and freſh water, in ſpirit of wine, of turpentine, and ſalts, in oil cleared of its fæces by diſtillation and warmed, in oil of vitriol, and in mercury, and melted metals, and any other ſuch like, that are fluid enough to retain for ſome time the motion impreſſed upon them by the agitation of the veſſel, and which being poured out are eaſily reſolved into drops, I doubt not but the rule already laid down may be accurate enough, eſpecially if the experiments be made with larger pendulous bodies, and more ſwiftly moved.

Laſtly, ſince it is the opinion of ſome that there is a certain æthereal medium extremely rare and ſubtile, which freely pervades the pores of all bodies; and from ſuch a medium, ſo pervading the pores of bodies, ſome reſiſtance muſt needs ariſe; in order to try whether the reſiſtance, which we experience in bodies in motion, be made upon their outward ſuperficies only, or whether their internal parts meet with any conſiderable reſiſtance upon their ſuperficies, I thought of the following experiment. I ſuſpended a round deal box by a thread 11 feet long, on a ſteel hook, by means of a ring of the ſame metal, ſo as to make a pendulum of the aforeſaid length. The hook had a ſharp hollow edge on its upper part, ſo that the upper arc of the ring preſſing on the edge might move the more freely; and the thread was faſtened to the lower arc of the ring. The pendulum being thus prepared, I drew it aſide from the perpendicular to the diſtance of about 6 feet, and that in a plane perpendicular to the edge of the hook, left the ring, while the pendulum oſcillated, ſhould ſlide to and fro on the edge of the hook: for the point of ſuſpenſion, in which the ring touches the hook, ought to remain immovable. I therefore accurately noted the place to which the pendulum was brought, and, letting it go, I marked three other places, to which it returned at the end of the 1ſt, 2d, and 3d oſcillation. This I often repeated, that I might find thoſe places as accurately as poſſible. Then I filled the box with lead and other heavy metals that were near at hand. But, firſt, I weighed the box when empty, and that part of the thread that went round it, and half the remaining part, extended

between the hook and the ſuſpended box; for the thread ſo extended always acts upon the pendulum, when drawn aſide from the perpendicular, with half its weight. To this weight I added the weight of the air contained in the box. And this whole weight was about $\frac{1}{768}$ of the weight of the box when filled with the metals. Then becauſe the box when full of the metals, by extending the thread with its weight, increaſed the length of the pendulum, I ſhortened the thread ſo as to make the length of the pendulum, when oſcillating, the ſame as before. Then drawing aſide the pendulum to the place firſt marked, and letting it go, I reckoned about 77 oſcillations before the box returned to the ſecond mark, and as many afterwards before it came to the third mark, and as many after that before it came to the fourth mark. From whence I conclude that the whole reſiſtance of the box, when full, had not a greater proportion to the reſiſtance of the box, when empty, than 78 to 77. For if their reſiſtances were equal, the box, when full, by reaſon of its *vis inſita*, which was 78 times greater than the *vis inſita* of the ſame when empty, ought to have continued its oſcillating motion ſo much the longer, and therefore to have returned to thoſe marks at the end of 78 oſcillations. But it returned to them at the end of 77 oſcillations.

Let, therefore, A repreſent the reſiſtance of the box upon its external ſuperficies, and B the reſiſtance of the empty box on its internal ſuperficies; and if the reſiſtances to the internal parts of bodies equally ſwift be as the matter, or the number of particles that are reſiſted, then 78B will be the reſiſtance made to the internal parts of the box, when full; and therefore the whole reſiſtance A + B of the empty box will be to the whole reſiſtance A + 78B of the full box as 77 to 78, and, by diviſion, A + B to 77B as 77 to 1; and thence A + B to B as 77 × 77 to 1, and, by diviſion again, A to B as 5928 to 1. Therefore the reſiſtance of the empty box in its internal parts will be above 5000 times leſs than the reſiſtance on its external ſuperficies. This reaſoning depends upon the ſuppoſition that the greater reſiſtance of the full box ariſes

not from any other latent cauſe, but only from the action of ſome ſubtile fluid upon the included metal.

This experiment is related by memory, the paper being loſt in which I had deſcribed it; ſo that I have been obliged to omit ſome fractional parts, which are ſlipt out of my memory; and I have no leiſure to try it again. The firſt time I made it, the hook being weak, the full box was retarded ſooner. The cauſe I found to be, that the hook was not ſtrong enough to bear the weight of the box; ſo that, as it oſcillated to and fro, the hook was bent ſometimes this and ſometimes that way. I therefore procured a hook of ſufficient ſtrength, ſo that the point of ſuſpenſion might remain unmoved, and then all things happened as is above deſcribed.

SECTION VII.

Of the motion of fluids, and the reſiſtance made to projected bodies.

PROPOSITION XXXII. THEOREM XXVI.

Suppoſe two ſimilar ſyſtems of bodies conſiſting of an equal number of particles, and let the correſpondent particles be ſimilar and proportional, each in one ſyſtem to each in the other, and have a like ſituation among themſelves, and the ſame given ratio of denſity to each other; and let them begin to move among themſelves in proportional times, and with like motions (that is, thoſe in one ſyſtem among one another, and thoſe in the other among one another). And if the particles that are in the ſame ſyſtem do not touch one another, except in the moments of reflexion; nor attract, nor repel each other, except with accelerative forces that are as the diameters of the correſpondent particles inverſely, and the ſquares of the velocities directly; I ſay, that the particles of thoſe ſyſtems will continue to move among themſelves with like motions and in proportional times.

Like bodies in like ſituations are ſaid to be moved among themſelves with like motions and in proportional times, when their ſituations at the end of thoſe times are always found alike in reſpect of each other: as ſuppoſe we compare the particles in one ſyſtem with the correſpondent particles in the other. Hence the times will be proportional, in which ſimi-

lar and proportional parts of ſimilar figures will be deſcribed by correſpondent particles. Therefore if we ſuppoſe two ſyſtems of this kind, the correſpondent particles, by reaſon of the ſimilitude of the motions at their beginning, will continue to be moved with like motions, ſo long as they move without meeting one another; for if they are acted on by no forces, they will go on uniformly in right lines, by the 1ſt law. But if they do agitate one another with ſome certain forces, and thoſe forces are as the diameters of the correſpondent particles inverſely and the ſquares of the velocities directly, then, becauſe the particles are in like ſituations, and their forces are proportional, the whole forces with which correſpondent particles are agitated, and which are compounded of each of the agitating forces (by corol. 2 of the laws), will have like directions, and have the ſame effect as if they reſpected centres placed alike among the particles; and thoſe whole forces will be to each other as the ſeveral forces which compoſe them, that is, as the diameters of the correſpondent particles inverſely, and the ſquares of the velocities directly: and therefore will cauſe correſpondent particles to continue to deſcribe like figures. Theſe things will be ſo (by cor. 1 and 8, prop. 4, book 1), if thoſe centres are at reſt; but if they are moved, yet, by reaſon of the ſimilitude of the tranſlations, their ſituations among the particles of the ſyſtem will remain ſimilar; ſo that the changes introduced into the figures deſcribed by the particles will ſtill be ſimilar. So that the motions of correſpondent and ſimilar particles will continue ſimilar till their firſt meeting with each other; and thence will ariſe ſimilar colliſions, and ſimilar reflexions; which will again beget ſimilar motions of the particles among themſelves (by what was juſt now ſhewn), till they mutually fall upon one another again, and ſo on *ad infinitum*.

Cor. 1. Hence if any two bodies, which are ſimilar and in like ſituations to the correſpondent particles of the ſyſtems, begin to move amongſt them in like manner and in proportional times, and their magnitudes and denſities be to each other as the magnitudes and denſities of the correſponding particles, theſe bodies will continue to be moved in like man-

ner and in proportional times; for the caſe of the greater parts of both ſyſtems and of the particles is the very ſame.

Cor. 2. And if all the ſimilar and ſimilarly ſituated parts of both ſyſtems be at reſt among themſelves; and two of them, which are greater than the reſt, and mutually correſpondent in both ſyſtems, begin to move in lines alike poſited, with any ſimilar motion whatſoever, they will excite ſimilar motions in the reſt of the parts of the ſyſtems, and will continue to move among thoſe parts in like manner and in proportional times; and will therefore deſcribe ſpaces proportional to their diameters.

PROPOSITION XXXIII. THEOREM XXVII.

The ſame things being ſuppoſed, I ſay, that the greater parts of the ſyſtems are reſiſted in a ratio compounded of the duplicate ratio of their velocities, and the duplicate ratio of their diameters, and the ſimple ratio of the denſity of the parts of the ſyſtems.

For the reſiſtance ariſes partly from the centripetal or centrifugal forces with which the particles of the ſyſtem mutually act on each other, partly from the colliſions and reflexions of the particles and the greater parts. The reſiſtances of the firſt kind are to each other as the whole motive forces from which they ariſe, that is, as the whole accelerative forces and the quantities of matter in correſponding parts; that is (by the ſuppoſition), as the ſquares of the velocities directly, and the diſtances of the correſponding particles inverſely, and the quantities of matter in the correſpondent parts directly: and therefore ſince the diſtances of the particles in one ſyſtem are to the correſpondent diſtances of the particles of the other as the diameter of one particle or part in the former ſyſtem to the diameter of the correſpondent particle or part in the other, and ſince the quantities of matter are as the denſities of the parts and the cubes of the diameters; the reſiſtances are to each other as the ſquares of the velocities and the ſquares of the diameters and the denſities of the parts of the ſyſtems. Q.E.D. The reſiſtances of the latter ſort are as the number of correſpondent reflexions and the forces of thoſe reflexions conjunctly; but the number

of the reflexions are to each other as the velocities of the corresponding parts directly and the spaces between their reflexions inversely. And the forces of the reflexions are as the velocities and the magnitudes and the densities of the corresponding parts conjunctly; that is, as the velocities and the cubes of the diameters and the densities of the parts. And, joining all these ratios, the resistances of the corresponding parts are to each other as the squares of the velocities and the squares of the diameters and the densities of the parts conjunctly. Q.E.D.

Cor. 1. Therefore if those systems are two elastic fluids, like our air, and their parts are at rest among themselves; and two similar bodies proportional in magnitude and density to the parts of the fluids, and similarly situated among those parts, be any how projected in the direction of lines similarly posited; and the accelerative forces with which the particles of the fluids mutually act upon each other are as the diameters of the bodies projected inversely and the squares of their velocities directly; those bodies will excite similar motions in the fluids in proportional times, and will describe similar spaces and proportional to their diameters.

Cor. 2. Therefore in the same fluid a projected body that moves swiftly meets with a resistance that is in the duplicate ratio of its velocity, nearly. For if the forces, with which distant particles act mutually upon one another should be augmented in the duplicate ratio of the velocity, the projected body would be resisted in the same duplicate ratio accurately; and therefore in a medium, whose parts when at a distance do not act mutually with any force on one another, the resistance is in the duplicate ratio of the velocity accurately. Let there be, therefore, three mediums A, B, C, consisting of similar and equal parts regularly disposed at equal distances. Let the parts of the mediums A and B recede from each other with forces that are among themselves as T and V; and let the parts of the medium C be entirely destitute of any such forces. And if four equal bodies D, E, F, G, move in these mediums, the two first D and E in the two first A and B, and the other two F and G in the third C; and if

the velocity of the body D be to the velocity of the body E, and the velocity of the body F to the velocity of the body G, in the ſubduplicate ratio of the force T to the force V; the reſiſtance of the body D to the reſiſtance of the body E, and the reſiſtance of the body F to the reſiſtance of the body G, will be in the duplicate ratio of the velocities; and therefore the reſiſtance of the body D will be to the reſiſtance of the body F as the reſiſtance of the body E to the reſiſtance of the body G. Let the bodies D and F be equally ſwift, as alſo the bodies E and G; and, augmenting the velocities of the bodies D and F in any ratio, and diminiſhing the forces of the particles of the medium B in the duplicate of the ſame ratio, the medium B will approach to the form and condition of the medium C at pleaſure; and therefore the reſiſtances of the equal and equally ſwift bodies E and G in theſe mediums will perpetually approach to equality, ſo that their difference will at laſt become leſs than any given. Therefore, ſince the reſiſtances of the bodies D and F are to each other as the reſiſtances of the bodies E and G, thoſe will alſo in like manner approach to the ratio of equality. Therefore the bodies D and F, when they move with very great ſwiftneſs, meet with reſiſtances very nearly equal; and therefore ſince the reſiſtance of the body F is in a duplicate ratio of the velocity, the reſiſtance of the body D will be nearly in the ſame ratio.

Cor. 3. The reſiſtance of a body moving very ſwift in an elaſtic fluid is almoſt the ſame as if the parts of the fluid were deſtitute of their centrifugal forces, and did not fly from each other; if ſo be that the elaſticity of the fluid ariſe from the centrifugal forces of the particles, and the velocity be ſo great as not to allow the particles time enough to act.

Cor. 4. Therefore, ſince the reſiſtances of ſimilar and equally ſwift bodies, in a medium whoſe diſtant parts do not fly from each other, are as the ſquares of the diameters, the reſiſtances made to bodies moving with very great and equal velocities in an elaſtic fluid will be as the ſquares of the diameters, nearly.

Cor. 5. And ſince ſimilar, equal, and equally ſwift bodies, moving through mediums of the ſame denſity, whoſe parti-

cles do not fly from each other mutually, will ſtrike againſt an equal quantity of matter in equal times, whether the particles of which the medium conſiſts be more and ſmaller, or fewer and greater, and therefore impreſs on that matter an equal quantity of motion, and in return (by the 3d law of motion) ſuffer an equal re-action from the ſame, that is, are equally reſiſted; it is manifeſt, alſo, that in elaſtic fluids of the ſame denſity, when the bodies move with extreme ſwiftneſs, their reſiſtances are nearly equal, whether the fluids conſiſt of groſs parts, or of parts ever ſo ſubtile. For the reſiſtance of projectiles moving with exceedingly great celerities is not much diminiſhed by the ſubtilty of the medium.

Cor. 6. All theſe things are ſo in fluids whoſe elaſtic force takes its riſe from the centrifugal forces of the particles. But if that force ariſe from ſome other cauſe, as from the expanſion of the particles after the manner of wool, or the boughs of trees, or any other cauſe, by which the particles are hindered from moving freely among themſelves, the reſiſtance, by reaſon of the leſſer fluidity of the medium, will be greater than in the corollaries above.

PROPOSITION XXXIV. THEOREM XXVIII.

If in a rare medium, conſiſting of equal particles freely diſpoſed at equal diſtances from each other, a globe and a cylinder deſcribed on equal diameters move with equal velocities in the direction of the axis of the cylinder, the reſiſtance of the globe will be but half ſo great as that of the cylinder.

For ſince the action of the medium upon the body is the ſame (by cor. 5 of the laws) whether the body move in a quieſcent medium, or whether the particles of the medium impinge with the ſame velocity upon the quieſcent body, let us conſider the body as if it were quieſcent, and ſee with what force it would be impelled by the moving medium. Let, therefore, ABKI (Pl. 6, Fig. 2) repreſent a ſpherical body deſcribed from the centre C with the ſemi-diameter CA, and let the particles of the medium impinge with a given velocity upon that ſpherical body in the directions of right lines parallel to AC; and let FB be one of thoſe right lines. In FB take LB equal to the ſemi-diameter CB, and draw BD touch-

ing the ſphere in B. Upon KC and BD let fall the perpendiculars BE, LD; and the force with which a particle of the medium, impinging on the globe obliquely in the direction FB, would ſtrike the globe in B, will be to the force with which the ſame particle, meeting the cylinder ONGQ, deſcribed about the globe with the axis AC1, would ſtrike it perpendicularly in b, as LD to LB, or BE to BC. Again; the efficacy of this force to move the globe according to the direction of its incidence FB or AC, is to the efficacy of the ſame to move the globe according to the direction of its determination, that is, in the direction of the right line BC in which it impels the globe directly, as BE to BC. And, joining theſe ratios, the efficacy of a particle, falling upon the globe obliquely in the direction of the right line FB, to move the globe in the direction of its incidence, is to the efficacy of the ſame particle falling in the ſame line perpendicularly on the cylinder, to move it in the ſame direction, as BE^2 to BC^2. Therefore if in bE, which is perpendicular to the circular baſe of the cylinder NAO, and equal to the radius AC, we take bH equal to $\frac{BE^2}{CB}$; then bH will be to bE as the effect of the particle upon the globe to the effect of the particle upon the cylinder. And therefore the ſolid which is formed by all the right lines bH will be to the ſolid formed by all the right lines bE as the effect of all the particles upon the globe to the effect of all the particles upon the cylinder. But the former of theſe ſolids is a paraboloid whoſe vertex is C, its axis CA, and latus rectum CA, and the latter ſolid is a cylinder circumſcribing the paraboloid; and it is known that a paraboloid is half its circumſcribed cylinder. Therefore the whole force of the medium upon the globe is half of the entire force of the ſame upon the cylinder. And therefore if the particles of the medium are at reſt, and the cylinder and globe move with equal velocities, the reſiſtance of the globe will be half the reſiſtance of the cylinder. Q.E.D.

SCHOLIUM.

By the ſame method other figures may be compared together as to their reſiſtance; and thoſe may be found which

are moſt apt to continue their motions in reſiſting mediums. As if upon the circular baſe CEBH (Pl. 6, Fig. 3) from the centre O, with the radius OC, and the altitude OD, one would conſtruct a fruſtum CBGF of a cone, which ſhould meet with leſs reſiſtance than any other fruſtum conſtructed with the ſame baſe and altitude, and going forwards towards D in the direction of its axis: biſect the altitude OD in Q, and produce OQ to S, ſo that QS may be equal to QC, and S will be the vertex of the cone whoſe fruſtum is ſought.

Whence, by the bye, ſince the angle CSB is always acute, it follows, that, if the ſolid ADBE (Pl. 6, Fig. 4) be generated by the convolution of an elliptical or oval figure ADBE about its axis AB, and the generating figure be touched by three right lines FG, GH, HI, in the points F, B, and I, ſo that GH ſhall be perpendicular to the axis in the point of contact B, and FG, HI may be inclined to GH in the angles FGB, BHI of 135 degrees; the ſolid ariſing from the convolution of the figure ADFGHIE about the ſame axis AB will be leſs reſiſted than the former ſolid; if ſo be that both move forward in the direction of their axis AB, and that the extremity B of each go foremoſt. Which propoſition I conceive may be of uſe in the building of ſhips.

If the figure DNFG be ſuch a curve, that if, from any point thereof, as N, the perpendicular NM be let fall on the axis AB, and from the given point G there be drawn the right line GR parallel to a right line touching the figure in N, and cutting the axis produced in R, MN becomes to GR as GR^3 to $4BR \times GB^2$, the ſolid deſcribed by the revolution of this figure about its axis AB, moving in the before-mentioned rare medium from A towards B, will be leſs reſiſted than any other circular ſolid whatſoever, deſcribed of the ſame length and breadth.

☞ *The demonſtration of theſe curious Theorems being omitted by the author, the analyſis thereof, communicated by a friend, is added at the end of this volume.*

PROPOSITION XXXV. PROBLEM VII.

If a rare medium conſiſt of very ſmall quieſcent particles of equal magnitudes, and freely diſpoſed at equal diſtances from

Fig: 1.

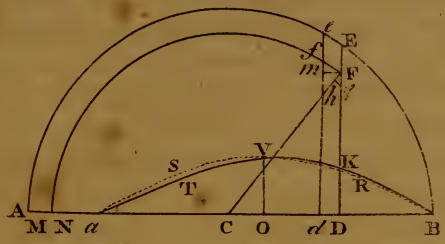


Fig: 2.

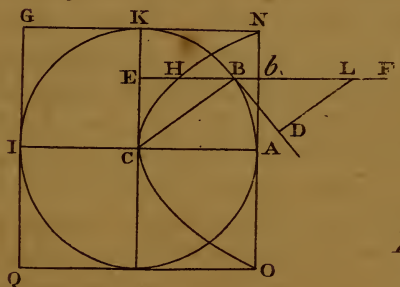


Fig: 3.

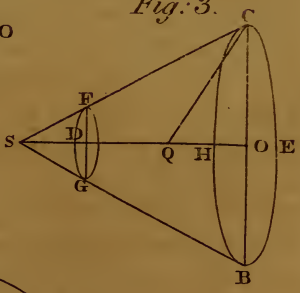
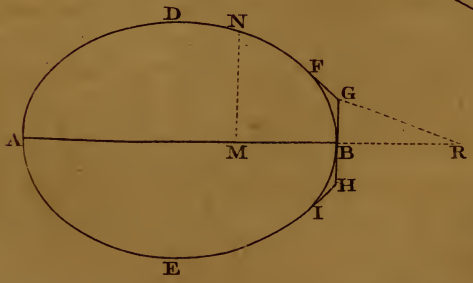


Fig: 4.



one another: to find the reſiſtance of a globe moving uniformly forwards in this medium.

CASE 1. Let a cylinder deſcribed with the ſame diameter and altitude be conceived to go forward with the ſame velocity in the direction of its axis through the ſame medium; and let us ſuppoſe that the particles of the medium, on which the globe or cylinder falls, fly back with as great a force of reflexion as poſſible. Then ſince the reſiſtance of the globe (by the laſt propoſition) is but half the reſiſtance of the cylinder, and ſince the globe is to the cylinder as 2 to 3, and ſince the cylinder by falling perpendicularly on the particles, and reflecting them with the utmoſt force, communicates to them a velocity double to its own; it follows that the cylinder, in moving forward uniformly half the length of its axis, will communicate a motion to the particles which is to the whole motion of the cylinder as the denſity of the medium to the denſity of the cylinder; and that the globe, in the time it deſcribes one length of its diameter in moving uniformly forwards, will communicate the ſame motion to the particles; and in the time that it deſcribes two thirds of its diameter, will communicate a motion to the particles which is to the whole motion of the globe as the denſity of the medium to the denſity of the globe. And therefore the globe meets with a reſiſtance, which is to the force by which its whole motion may be either taken away or generated in the time in which it deſcribes two thirds of its diameter moving uniformly forwards, as the denſity of the medium to the denſity of the globe.

CASE 2. Let us ſuppoſe that the particles of the medium incident on the globe or cylinder are not reflected; and then the cylinder falling perpendicularly on the particles will communicate its own ſimple velocity to them, and therefore meets a reſiſtance but half ſo great as in the former caſe, and the globe alſo meets with a reſiſtance but half ſo great.

CASE 3. Let us ſuppoſe the particles of the medium to fly back from the globe with a force which is neither the greateſt, nor yet none at all, but with a certain mean force; then the reſiſtance of the globe will be in the ſame mean ratio between

the reſiſtance in the firſt caſe and the reſiſtance in the ſecond. Q.E.I.

Cor. 1. Hence if the globe and the particles are infinitely hard, and deſtitute of all elaſtic force, and therefore of all force of reflexion; the reſiſtance of the globe will be to the force by which its whole motion may be deſtroyed or generated, in the time that the globe deſcribes four third parts of its diameter, as the denſity of the medium to the denſity of the globe.

Cor. 2. The reſiſtance of the globe, *cæteris paribus*, is in the duplicate ratio of the velocity.

Cor. 3. The reſiſtance of the globe, *cæteris paribus*, is in the duplicate ratio of the diameter.

Cor. 4. The reſiſtance of the globe is, *cæteris paribus*, as the denſity of the medium.

Cor. 5. The reſiſtance of the globe is in a ratio compounded of the duplicate ratio of the velocity, and the duplicate ratio of the diameter, and the ratio of the denſity of the medium.

Cor. 6. The motion of the globe and its reſiſtance may be thus expounded. Let AB (Pl. 7, Fig. 1) be the time in which the globe may, by its reſiſtance uniformly continued, loſe its whole motion. Erect AD, BC perpendicular to AB. Let BC be that whole motion, and through the point C, the aſymptotes being AD, AB, deſcribe the hyperbola CF. Produce AB to any point E. Erect the perpendicular EF meeting the hyperbola in F. Complete the parallelogram CBEG, and draw AF meeting BC in H. Then if the globe in any time BE, with its firſt motion BC uniformly continued, deſcribes in a non-reſiſting medium the ſpace CBEG expounded by the area of the parallelogram, the ſame in a reſiſting medium will deſcribe the ſpace CBEF expounded by the area of the hyperbola; and its motion at the end of that time will be expounded by EF, the ordinate of the hyperbola, there being loſt of its motion the part FG. And its reſiſtance at the end of the ſame time will be expounded by the length BH, there being loſt of its reſiſtance the part CH. All theſe things appear by cor. 1 and 3, prop. 5, book 2.

Cor. 7. Hence if the globe in the time T by the reſiſtance R uniformly continued loſe its whole motion M, the ſame globe in the time t in a reſiſting medium, wherein the reſiſtance R decreaſes in a duplicate ratio of the velocity, will loſe out of its motion M the part $\frac{tM}{T \times t}$, the part $\frac{TM}{T \times t}$ remaining; and will deſcribe a ſpace which is to the ſpace deſcribed in the ſame time t, with the uniform motion M, as the logarithm of the number $\frac{T + t}{T}$ multiplied by the number 2,302585092994 is to the number $\frac{t}{T}$, becauſe the hyperbolic area BCFE is to the rectangle BCGE in that proportion.

SCHOLIUM.

I have exhibited in this propoſition the reſiſtance and retardation of ſpherical projectiles in mediums that are not continued, and ſhewn that this reſiſtance is to the force by which the whole motion of the globe may be deſtroyed or produced in the time in which the globe can deſcribe two thirds of its diameter, with a velocity uniformly continued, as the denſity of the medium to the denſity of the globe, if ſo be the globe and the particles of the medium be perfectly elaſtic, and are endued with the utmoſt force of reflexion; and that this force, where the globe and particles of the medium are infinitely hard and void of any reflecting force, is diminiſhed one half. But in continued mediums, as water, hot oil, and quickſilver, the globe as it paſſes through them does not immediately ſtrike againſt all the particles of the fluid that generate the reſiſtance made to it, but preſſes only the particles that lie next to it, which preſs the particles beyond, which preſs other particles, and ſo on; and in theſe mediums the reſiſtance is diminiſhed one other half. A globe in theſe extremely fluid mediums meets with a reſiſtance that is to the force by which its whole motion may be deſtroyed or generated in the time wherein it can deſcribe, with that motion uniformly continued, eight third parts of its diameter, as the denſity of the medium to the denſity of the globe. This I ſhall endeavour to ſhew in what follows.

PROPOSITION XXXVI. PROBLEM VIII.

To define the motion of water running out of a cylindrical veſſel through a hole made at the bottom.

Let ACDB (Pl. 7, Fig. 2) be a cylindrical veſſel, AB the mouth of it, CD the bottom parallel to the horizon, EF a circular hole in the middle of the bottom, G the centre of the hole, and GH the axis of the cylinder perpendicular to the horizon. And ſuppoſe a cylinder of ice APQB to be of the ſame breadth with the cavity of the veſſel, and to have the ſame axis, and to deſcend perpetually with an uniform motion, and that its parts, as ſoon as they touch the ſuperficies AB, diſſolve into water, and flow down by their weight into the veſſel, and in their fall compoſe the cataract or column of water ABNFEM, paſſing through the hole EF, and filling up the ſame exactly. Let the uniform velocity of the deſcending ice and of the contiguous water in the circle AB be that which the water would acquire by falling through the ſpace IH; and let IH and HG lie in the ſame right line; and through the point I let there be drawn the right line KL parallel to the horizon, and meeting the ice on both the ſides thereof in K and L. Then the velocity of the water running out at the hole EF will be the ſame that it would acquire by falling from I through the ſpace IG. Therefore, by *Galileo*'s theorems, IG will be to IH in the duplicate ratio of the velocity of the water that runs out at the hole to the velocity of the water in the circle AB, that is, in the duplicate ratio of the circle AB to the circle EF; thoſe circles being reciprocally as the velocities of the water which in the ſame time and in equal quantities paſſes ſeverally through each of them, and completely fills them both. We are now conſidering the velocity with which the water tends to the plane of the horizon. But the motion parallel to the ſame, by which the parts of the falling water approach to each other, is not here taken notice of; ſince it is neither produced by gravity, nor at all changes the motion perpendicular to the horizon which the gravity produces. We ſuppoſe, indeed, that the parts of the water cohere a little, that by their coheſion they may in falling approach to each other with motions parallel to the horizon, in

order to form one ſingle cataract, and to prevent their being divided into ſeveral: but the motion parallel to the horizon ariſing from this coheſion does not come under our preſent conſideration.

CASE 1. Conceive now the whole cavity in the veſſel, which encompaſſes the falling water ABNFEM, to be full of ice, ſo that the water may paſs through the ice as through a funnel. Then if the water paſs very near to the ice only, without touching it; or, which is the ſame thing, if, by reaſon of the perfect ſmoothneſs of the ſurface of the ice, the water, through touching it, glides over it with the utmoſt freedom, and without the leaſt reſiſtance; the water will run through the hole EF with the ſame velocity as before, and the whole weight of the column of water ABNFEM will be all taken up as before in forcing out the water, and the bottom of the veſſel will ſuſtain the weight of the ice encompaſſing that column.

Let now the ice in the veſſel diſſolve into water; yet will the efflux of the water remain, as to its velocity, the ſame as before. It will not be leſs, becauſe the ice now diſſolved will endeavour to deſcend; it will not be greater, becauſe the ice, now become water, cannot deſcend without hindering the deſcent of other water equal to its own deſcent. The ſame force ought always to generate the ſame velocity in the effluent water.

But the hole at the bottom of the veſſel, by reaſon of the oblique motions of the particles of the effluent water, muſt be a little greater than before. For now the particles of the water do not all of them paſs through the hole perpendicularly, but, flowing down on all parts from the ſides of the veſſel, and converging towards the hole, paſs through it with oblique motions; and in tending downwards meet in a ſtream whoſe diameter is a little ſmaller below the hole than at the hole itſelf; its diameter being to the diameter of the hole as 5 to 6, or as $5\frac{1}{2}$ to $6\frac{1}{2}$, very nearly, if I took the meaſures of thoſe diameters right. I procured a very thin flat plate, having a hole pierced in the middle, the diameter of the circular hole being $\frac{5}{8}$ parts of an inch. And that the ſtream of running waters

might not be accelerated in falling, and by that acceleration become narrower, I fixed this plate not to the bottom, but to the ſide of the veſſel, ſo as to make the water go out in the direction of a line parallel to the horizon. Then, when the veſſel was full of water, I opened the hole to let it run out; and the diameter of the ſtream, meaſured with great accuracy at the diſtance of about half an inch from the hole, was $\frac{21}{25}$ of an inch. Therefore the diameter of this circular hole was to the diameter of the ſtream very nearly as 25 to 21. So that the water in paſſing through the hole converges on all ſides, and, after it has run out of the veſſel, becomes ſmaller by converging in that manner, and by becoming ſmaller is accelerated till it comes to the diſtance of half an inch from the hole, and at that diſtance flows in a ſmaller ſtream and with greater celerity than in the hole itſelf, and this in the ratio of 25 × 25 to 21 × 21, or 17 to 12, very nearly; that is, in about the ſubduplicate ratio of 2 to 1. Now it is certain from experiments, that the quantity of water running out in a given time through a circular hole made in the bottom of a veſſel is equal to the quantity, which, flowing with the aforeſaid velocity, would run out in the ſame time through another circular hole, whoſe diameter is to the diameter of the former as 21 to 25. And therefore that running water in paſſing through the hole itſelf has a velocity downwards equal to that which a heavy body would acquire in falling through half the height of the ſtagnant water in the veſſel, nearly. But, then, after it has run out, it is ſtill accelerated by converging, till it arrives at a diſtance from the hole that is nearly equal to its diameter, and acquires a velocity greater than the other in about the ſubduplicate ratio of 2 to 1; which velocity a heavy body would nearly acquire by falling through the whole height of the ſtagnant water in the veſſel.

Therefore in what follows let the diameter of the ſtream be repreſented by that leſſer hole which we called EF. And imagine another plane VW above the hole EF (Pl. 7, Fig. 3), and parallel to the plane thereof, to be placed at a diſtance equal to the diameter of the ſame hole, and to be pierced through with a greater hole ST, of ſuch a magnitude that

a ſtream which will exactly fill the lower hole EF may paſs through it; the diameter of which hole will therefore be to the diameter of the lower hole as 25 to 21, nearly. By this means the water will run perpendicularly out at the lower hole; and the quantity of the water running out will be, according to the magnitude of this laſt hole, the ſame, very nearly, which the ſolution of the problem requires. The ſpace included between the two planes and the falling ſtream may be conſidered as the bottom of the veſſel. But, to make the ſolution more ſimple and mathematical, it is better to take the lower plane alone for the bottom of the veſſel, and to ſuppoſe that the water which flowed through the ice as through a funnel, and ran out of the veſſel through the hole EF made in the lower plane, preſerves its motion continually, and that the ice continues at reſt. Therefore in what follows let ST be the diameter of a circular hole deſcribed from the centre Z, and let the ſtream run out of the veſſel through that hole, when the water in the veſſel is all fluid. And let EF be the diameter of the hole, which the ſtream, in falling through, exactly fills up, whether the water runs out of the veſſel by that upper hole ST, or flows through the middle of the ice in the veſſel, as through a funnel. And let the diameter of the upper hole ST be to the diameter of the lower EF as about 25 to 21, and let the perpendicular diſtance between the planes of the holes be equal to the diameter of the leſſer hole EF. Then the velocity of the water downwards, in running out of the veſſel through the hole ST, will be in that hole the ſame that a body may acquire by falling from half the height IZ; and the velocity of both the falling ſtreams will be in the hole EF, the ſame which a body would acquire by falling from the whole height IG.

Case 2. If the hole EF be not in the middle of the bottom of the veſſel, but in ſome other part thereof, the water will ſtill run out with the ſame velocity as before, if the magnitude of the hole be the ſame. For though an heavy body takes a longer time in deſcending to the ſame depth, by an oblique line, than by a perpendicular line, yet in both caſes it acquires in its deſcent the ſame velocity; as *Galileo* has demonſtrated.

Case 3. The velocity of the water is the ſame when it runs out through a hole in the ſide of the veſſel. For if the hole be ſmall, ſo that the interval between the ſuperficies AB and KL may vaniſh as to ſenſe, and the ſtream of water horizontally iſſuing out may form a parabolic figure: from the latus rectum of this parabola may be collected, that the velocity of the effluent water is that which a body may acquire by falling the height IG or HG of the ſtagnant water in the veſſel. For, by making an experiment, I found that if the height of the ſtagnant water above the hole were 20 inches, and the height of the hole above a plane parallel to the horizon were alſo 20 inches, a ſtream of water ſpringing out from thence would fall upon the plane, at the diſtance of 37 inches, very nearly, from a perpendicular let fall upon that plane from the hole. For without reſiſtance the ſtream would have fallen upon the plane at the diſtance of 40 inches, the latus rectum of the parabolic ſtream being 80 inches.

Case 4. If the effluent water tend upwards, it will ſtill iſſue forth with the ſame velocity. For the ſmall ſtream of water ſpringing upwards, aſcends with a perpendicular motion to GH or GI, the height of the ſtagnant water in the veſſel; excepting in ſo far as its aſcent is hindered a little by the reſiſtance of the air; and therefore it ſprings out with the ſame velocity that it would acquire in falling from that height. Every particle of the ſtagnant water is equally preſſed on all ſides (by prop. 19, book 2), and, yielding to the preſſure, tends all ways with an equal force, whether it deſcends through the hole in the bottom of the veſſel, or guſhes out in an horizontal direction through an hole in the ſide, or paſſes into a canal, and ſprings up from thence through a little hole made in the upper part of the canal. And it may not only be collected from reaſoning, but is manifeſt alſo from the well-known experiments juſt mentioned, that the velocity with which the water runs out is the very ſame that is aſſigned in this propoſition.

Case 5. The velocity of the effluent water is the ſame, whether the figure of the hole be circular, or ſquare, or triangular, or any other figure equal to the circular; for the ve-

locity of the effluent water does not depend upon the figure of the hole, but arifes from its depth below the plane KL.

Case 6. If the lower part of the veffel ABDC be immerfed into ftagnant water, and the height of the ftagnant water above the bottom of the veffel be GR, the velocity with which the water that is in the veffel will run out at the hole EF into the ftagnant water will be the fame which the water would acquire by falling from the height IR; for the weight of all the water in the veffel that is below the fuperficies of the ftagnant water will be fuftained in equilibrio by the weight of the ftagnant water, and therefore does not at all accelerate the motion of the defcending water in the veffel. This cafe will alfo appear by experiments, meafuring the times in which the water will run out.

Cor. 1. Hence if CA the depth of the water be produced to K, fo that AK may be to CK in the duplicate ratio of the area of a hole made in any part of the bottom to the area of the circle AB, the velocity of the effluent water will be equal to the velocity which the water would acquire by falling from the height KC.

Cor. 2. And the force with which the whole motion of the effluent water may be generated is equal to the weight of a cylindric column of water, whofe bafe is the hole EF, and its altitude 2GI or 2CK. For the effluent water, in the time it becomes equal to this column, may acquire, by falling by its own weight from the height GI, a velocity equal to that with which it runs out.

Cor. 3. The weight of all the water in the veffel ABDC is to that part of the weight which is employed in forcing out the water as the fum of the circles AB and EF to twice the circle EF. For let IO be a mean proportional between IH and IG, and the water running out at the hole EF will, in the time that a drop falling from I would defcribe the altitude IG, become equal to a cylinder whofe bafe is the circle EF and its altitude 2IG, that is, to a cylinder whofe bafe is the circle AB, and whofe altitude is 2IO. For the circle EF is to the circle AB in the fubduplicate ratio of the altitude IH to the altitude IG; that is, in the fimple ratio of the

mean proportional IO to the altitude IG. Moreover, in the time that a drop falling from I can deſcribe the altitude IH, the water that runs out will have become equal to a cylinder whoſe baſe is the circle AB, and its altitude 2IH; and in the time that a drop falling from I through H to G deſcribes HG, the difference of the altitudes, the effluent water, that is, the water contained within the ſolid ABNFEM, will be equal to the difference of the cylinders, that is, to a cylinder whoſe baſe is AB, and its altitude 2HO. And therefore all the water contained in the veſſel ABDC is to the whole falling water contained in the ſaid ſolid ABNFEM as HG to 2HO, that is, as HO + OG to 2HO, or IH + IO to 2IH. But the weight of all the water in the ſolid ABNFEM is employed in forcing out the water; and therefore the weight of all the water in the veſſel is to that part of the weight that is employed in forcing out the water as IH + IO to 2IH, and therefore as the ſum of the circles EF and AB to twice the circle EF.

Cor. 4. And hence the weight of all the water in the veſſel ABDC is to the other part of the weight which is ſuſtained by the bottom of the veſſel as the ſum of the circles AB and EF to the difference of the ſame circles.

Cor. 5. And that part of the weight which the bottom of the veſſel ſuſtains is to the other part of the weight employed in forcing out the water as the difference of the circles AB and EF to twice the leſſer circle EF, or as the area of the bottom to twice the hole.

Cor. 6. That part of the weight which preſſes upon the bottom is to the whole weight of the water perpendicularly incumbent thereon as the circle AB to the ſum of the circles AB and EF, or as the circle AB to the exceſs of twice the circle AB above the area of the bottom. For that part of the weight which preſſes upon the bottom is to the weight of the whole water in the veſſel as the difference of the circles AB and EF to the ſum of the ſame circles (by cor. 4); and the weight of the whole water in the veſſel is to the weight of the whole water perpendicularly incumbent on the bottom as the circle AB to the difference of the circles AB and EF. There-

fore, *ex æquo perturbatè*, that part of the weight which preſſes upon the bottom is to the weight of the whole water perpendicularly incumbent thereon as the circle AB to the ſums of the circles AB and EF, or the exceſs of twice the circle AB above the bottom.

Cor. 7. If in the middle of the hole EF there be placed the little circle PQ deſcribed about the centre G, and parallel to the horizon, the weight of water which that little circle ſuſtains is greater than the weight of a third part of a cylinder of water whoſe baſe is that little circle and its height GH. For let ABNFEM (Pl. 7, Fig. 4) be the cataract or column of falling water whoſe axis is GH, as above, and let all the water, whoſe fluidity is not requiſite for the ready and quick deſcent of the water, be ſuppoſed to be congealed, as well round about the cataract, as above the little circle. And let PHQ be the column of water congealed above the little circle, whoſe vertex is H, and its altitude GH. And ſuppoſe this cataract to fall with its whole weight downwards, and not in the leaſt to lie againſt or to preſs PHQ, but to glide freely by it without any friction, unleſs, perhaps, juſt at the very vertex of the ice, where the cataract at the beginning of its fall may tend to a concave figure. And as the congealed water AMEC, BNFD, lying round the cataract, is convex in its internal ſuperficies AME, BNF, towards the falling cataract, ſo this column PHQ will be convex towards the cataract alſo, and will therefore be greater than a cone whoſe baſe is that little circle PQ and its altitude GH; that is, greater than a third part of a cylinder deſcribed with the ſame baſe and altitude. Now that little circle ſuſtains the weight of this column, that is, a weight greater than the weight of the cone, or a third part of the cylinder.

Cor. 8. The weight of water which the circle PQ, when very ſmall, ſuſtains, ſeems to be leſs than the weight of two thirds of a cylinder of water whoſe baſe is that little circle, and its altitude HG. For, things ſtanding as above ſuppoſed, imagine the half of a ſpheroid deſcribed whoſe baſe is that little circle, and its ſemi-axis or altitude HG. This figure will be equal to two thirds of that cylinder, and will compre-

hend within it the column of congealed water PHQ, the weight of which is ſuſtained by that little circle. For though the motion of the water tends directly downwards, the external ſuperficies of that column muſt yet meet the baſe PQ in an angle ſomewhat acute, becauſe the water in its fall is perpetually accelerated, and by reaſon of that acceleration become narrower. Therefore, ſince that angle is leſs than a right one, this column in the lower parts thereof will lie within the hemi-ſpheroid. In the upper parts alſo it will be acute or pointed; becauſe, to make it otherwiſe, the horizontal motion of the water muſt be at the vertex infinitely more ſwift than its motion towards the horizon. And the leſs this circle PQ is, the more acute will the vertex of this column be; and the circle being diminiſhed *in infinitum*, the angle PHQ will be diminiſhed *in infinitum*, and therefore the column will lie within the hemi-ſpheroid. Therefore that column is leſs than that hemi-ſpheroid, or than two third parts of the cylinder whoſe baſe is that little circle, and its altitude GH. Now the little circle ſuſtains a force of water equal to the weight of this column, the weight of the ambient water being employed in cauſing its efflux out at the hole.

Cor. 9. The weight of water which the little circle PQ ſuſtains, when it is very ſmall, is very nearly equal to the weight of a cylinder of water whoſe baſe is that little circle, and its altitude $\frac{1}{2}$GH; for this weight is an arithmetical mean between the weights of the cone and the hemi-ſpheroid above-mentioned. But if that little circle be not very ſmall, but on the contrary increaſed till it be equal to the hole EF, it will ſuſtain the weight of all the water lying perpendicularly above it, that is, the weight of a cylinder of water whoſe baſe is that little circle, and its altitude GH.

Cor. 10. And (as far as I can judge) the weight which this little circle ſuſtains is always to the weight of a cylinder of water whoſe baſe is that little circle, and its altitude $\frac{1}{2}$GH, as EF^2 to $EF^2 - \frac{1}{2}PQ^2$, or as the circle EF to the exceſs of this circle above half the little circle PQ, very nearly.

LEMMA IV.

If a cylinder move uniformly forwards in the direction of its length, the reſiſtance made thereto is not at all changed by

augmenting or diminiſhing that length; and is therefore the ſame with the reſiſtance of a circle, deſcribed with the ſame diameter, and moving forwards with the ſame velocity in the direction of a right line perpendicular to its plane.

For the ſides are not at all oppoſed to the motion; and a cylinder becomes a circle when its length is diminiſhed *in infinitum.*

PROPOSITION XXXVII. THEOREM XXIX.

If a cylinder move uniformly forwards in a compreſſed, infinite, and non-elaſtic fluid, in the direction of its length, the reſiſtance ariſing from the magnitude of its tranſverſe ſection is to the force by which its whole motion may be deſtroyed or generated, in the time that it moves four times its length, as the denſity of the medium to the denſity of the cylinder, nearly.

For let the veſſel ABDC (Pl. 7, Fig. 5) touch the ſurface of ſtagnant water with its bottom CD, and let the water run out of this veſſel into the ſtagnant water through the cylindric canal EFTS perpendicular to the horizon; and let the little circle PQ be placed parallel to the horizon any where in the middle of the canal; and produce CA to K, ſo that AK may be to CK in the duplicate of the ratio, which the exceſs of the orifice of the canal EF above the little circle PQ bears to the circle AB. Then it is manifeſt (by caſe 5, caſe 6, and cor. 1, prop. 36) that the velocity of the water paſſing through the annular ſpace between the little circle and the ſides of the veſſel will be the very ſame which the water would acquire by falling, and in its fall deſcribing the altitude KC or IG.

And (by cor. 10, prop. 36) if the breadth of the veſſel be infinite, ſo that the lineolæ HI may vaniſh, and the altitudes IG, HG become equal; the force of the water that flows down and preſſes upon the circle will be to the weight of a cylinder whoſe baſe is that little circle, and the altitude $\frac{1}{2}$IG, as EF^2 to $EF^2 - \frac{1}{2}PQ^2$, very nearly. For the force of the water flowing downwards uniformly through the whole canal will be the ſame upon the little circle PQ in whatſoever part of the canal it be placed.

Let now the orifices of the canal EF, ST be clofed, and let the little circle afcend in the fluid compreffed on every fide, and by its afcent let it oblige the water that lies above it to defcend through the annular fpace between the little circle and the fides of the canal. Then will the velocity of the afcending little circle be to the velocity of the defcending water as the difference of the circles EF and PQ is to the circle PQ; and the velocity of the afcending little circle will be to the fum of the velocities, that is, to the relative velocity of the defcending water with which it paffes by the little circle in its afcent, as the difference of the circles EF and PQ to the circle EF, or as $EF^2 - PQ^2$ to EF^2. Let that relative velocity be equal to the velocity with which it was fhewn above that the water would pafs through the annular fpace, if the circle were to remain unmoved, that is, to the velocity which the water would acquire by falling, and in its fall defcribing the altitude IG; and the force of the water upon the afcending circle will be the fame as before (by cor. 5, of the laws of motion); that is, the refiftance of the afcending little circle will be to the weight of a cylinder of water whofe bafe is that little circle, and its altitude $\frac{1}{2}$IG, as EF^2 to $EF^2 - \frac{1}{2}PQ^2$, nearly. But the velocity of the little circle will be to the velocity which the water acquires by falling, and in its fall defcribing the altitude IG, as $EF^2 - PQ^2$ to EF^2.

Let the breadth of the canal be increafed *in infinitum;* and the ratios between $EF^2 - PQ^2$ and EF^2, and between EF^2 and $EF^2 - \frac{1}{2}PQ^2$, will become at laft ratios of equality. And therefore the velocity of the little circle will now be the fame which the water would acquire in falling, and in its fall defcribing the altitude IG; and the refiftance will become equal to the weight of a cylinder whofe bafe is that little circle, and its altitude half the altitude IG, from which the cylinder muft fall to acquire the velocity of the afcending circle; and with this velocity the cylinder in the time of its fall will defcribe four times its length. But the refiftance of the cylinder moving forwards with this velocity in the direction of its length is the fame with the refiftance of the little circle (by lem. 4), and is therefore nearly equal to the force by which its

motion may be generated while it deſcribes four times its length.

If the length of the cylinder be augmented or diminiſhed, its motion, and the time in which it deſcribes four times its length, will be augmented or diminiſhed in the ſame ratio and therefore the force by which the motion, ſo increaſed or diminiſhed, may be deſtroyed or generated, will continue the ſame; becauſe the time is increaſed or diminiſhed in the ſame proportion; and therefore that force remains ſtill equal to the reſiſtance of the cylinder, becauſe (by lem. 4) that reſiſtance will alſo remain the ſame.

If the denſity of the cylinder be augmented or diminiſhed, its motion, and the force by which its motion may be generated or deſtroyed in the ſame time, will be augmented or diminiſhed in the ſame ratio. Therefore the reſiſtance of any cylinder whatſoever will be to the force by which its whole motion may be generated or deſtroyed, in the time during which it moves four times its length, as the denſity of the medium to the denſity of the cylinder, nearly. Q.E.D.

A fluid muſt be compreſſed to become continued; it muſt be continued and non-elaſtic, that all the preſſure ariſing from its compreſſion may be propagated in an inſtant; and ſo, acting equally upon all parts of the body moved, may produce no change of the reſiſtance. The preſſure ariſing from the motion of the body is ſpent in generating a motion in the parts of the fluid, and this creates the reſiſtance. But the preſſure ariſing from the compreſſion of the fluid, be it ever ſo forcible, if it be propagated in an inſtant, generates no motion in the parts of a continued fluid, produces no change at all of motion therein; and therefore neither augments nor leſſens the reſiſtance. This is certain, that the action of the fluid ariſing from the compreſſion cannot be ſtronger on the hinder parts of the body moved than on its fore parts, and therefore cannot leſſen the reſiſtance deſcribed in this propoſition. And if its propagation be infinitely ſwifter than the motion of the body preſſed, it will not be ſtronger on the fore parts than on the hinder parts. But that action will be infinitely ſwifter, and propagated in an inſtant, if the fluid be continued and non-elaſtic.

Cor. 1. The resistances made to cylinders going uniformly forwards in the direction of their lengths through continued infinite mediums, are in a ratio compounded of the duplicate ratio of the velocities and the duplicate ratio of the diameters, and the ratio of the density of the mediums.

Cor. 2. If the breadth of the canal be not infinitely increased, but the cylinder go forwards in the direction of its length through an included quiescent medium, its axis all the while coinciding with the axis of the canal, its resistance will be to the force by which its whole motion, in the time in which it describes four times its length, may be generated or destroyed, in a ratio compounded of the ratio of EF^2 to $EF^2 - \frac{1}{2}PQ^2$ once, and the ratio of EF^2 to $EF^2 - PQ^2$ twice, and the ratio of the density of the medium to the density of the cylinder.

Cor. 3. The same things supposed, and that a length L is to the quadruple of the length of the cylinder in a ratio compounded of the ratio $EF^2 - \frac{1}{2}PQ^2$ to EF^2 once, and the ratio of $EF^2 - PQ^2$ to EF^2 twice; the resistance of the cylinder will be to the force by which its whole motion, in the time during which it describes the length L, may be destroyed or generated, as the density of the medium to the density of the cylinder.

SCHOLIUM.

In this proposition we have investigated that resistance alone which arises from the magnitude of the transverse section of the cylinder, neglecting that part of the same which may arise from the obliquity of the motions. For as, in case 1, of prop. 36, the obliquity of the motions with which the parts of the water in the vessel converged on every side to the hole EF hindered the efflux of the water through the hole, so, in this proposition, the obliquity of the motions, with which the parts of the water, pressed by the antecedent extremity of the cylinder, yield to the pressure, and diverge on all sides, retards their passage through the places that lie round that antecedent extremity, towards the hinder parts of the cylinder, and causes the fluid to be moved to a greater distance; which increases the resistance, and that in the same ratio almost in

which it diminiſhed the efflux of the water out of the veſſel, that is, in the duplicate ratio of 25 to 21, nearly. And as, in caſe 1, of that propoſition, we made the parts of the water paſs through the hole EF perpendicularly and in the greateſt plenty, by ſuppoſing all the water in the veſſel lying round the cataract to be frozen, and that part of the water whoſe motion was oblique and uſeleſs to remain without motion, ſo in this propoſition, that the obliquity of the motions may be taken away, and the parts of the water may give the freeſt paſſage to the cylinder, by yielding to it with the moſt direct and quick motion poſſible, ſo that only ſo much reſiſtance may remain as ariſes from the magnitude of the tranſverſe ſection, and which is incapable of diminution, unleſs by diminiſhing the diameter of the cylinder; we muſt conceive thoſe parts of the fluid whoſe motions are oblique and uſeleſs, and produce reſiſtance, to be at reſt among themſelves at both extremities of the cylinder, and there to cohere, and be joined to the cylinder. Let ABCD (Pl. 7, Fig. 6) be a rectangle, and let AE and BE be two parabolic arcs, deſcribed with the axis AB, and with a latus rectum that is to the ſpace HG, which muſt be deſcribed by the cylinder in falling, in order to acquire the velocity with which it moves, as HG to $\frac{1}{2}$AB. Let CF and DF be two other parabolic arcs deſcribed with the axis CD, and a latus rectum quadruple of the former; and by the convolution of the figure about the axis EF let there be generated a ſolid, whoſe middle part ABDC is the cylinder we are here ſpeaking of, and whoſe extreme parts ABE and CDF contain the parts of the fluid at reſt among themſelves, and concreted into two hard bodies, adhering to the cylinder at each end like a head and tail. Then if this ſolid EACFDB move in the direction of the length of its axis FE towards the parts beyond E, the reſiſtance will be the ſame which we have here determined in this propoſition, nearly; that is, it will have the ſame ratio to the force with which the whole motion of the cylinder may be deſtroyed or generated, in the time that it is deſcribing the length 4AC with that motion uniformly continued, as the denſity of the

fluid has to the denſity of the cylinder, nearly. And (by cor. 7, prop. 36) the reſiſtance muſt be to this force in the ratio of 2 to 3, at the leaſt.

LEMMA V.

If a cylinder, a ſphere, and a ſpheroid, of equal breadths be placed ſucceſſively in the middle of a cylindric canal, ſo that their axes may coincide with the axis of the canal, theſe bodies will equally hinder the paſſage of the water through the canal.

For the ſpaces lying between the ſides of the canal, and the cylinder, ſphere, and ſpheroid, through which the water paſſes, are equal; and the water will paſs equally through equal ſpaces.

This is true, upon the ſuppoſition that all the water above the cylinder, ſphere, or ſpheroid, whoſe fluidity is not neceſſary to make the paſſage of the water the quickeſt poſſible, is congealed, as was explained above in cor. 7, prop. 36.

LEMMA VI.

The ſame ſuppoſition remaining, the forementioned bodies are equally acted on by the water flowing through the canal.

This appears by lem. 5, and the third law. For the water and the bodies act upon each other mutually and equally.

LEMMA VII.

If the water be at reſt in the canal, and theſe bodies move with equal velocity and the contrary way through the canal, their reſiſtances will be equal among themſelves.

This appears from the laſt lemma, for the relative motions remain the ſame among themſelves.

SCHOLIUM.

The caſe is the ſame of all convex and round bodies, whoſe axes coincide with the axis of the canal. Some difference may ariſe from a greater or leſs friction; but in theſe *lemmata* we ſuppoſe the bodies to be perfectly ſmooth, and the medium to be void of all tenacity and friction; and that thoſe parts of the fluid which by their oblique and ſuperfluous motions may diſturb, hinder, and retard the flux of the water through the canal, are at reſt amongſt themſelves; being fixed like water by froſt, and adhering to the fore and hinder

Fig. 1.

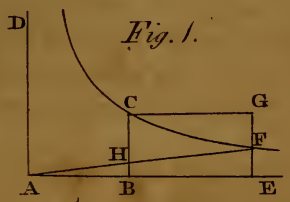


Fig. 2.

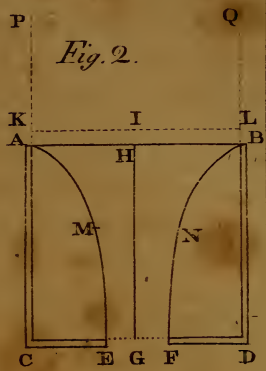


Fig 3

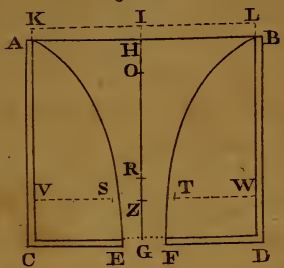


Fig. 4.

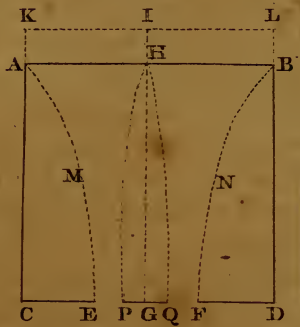


Fig. 5.

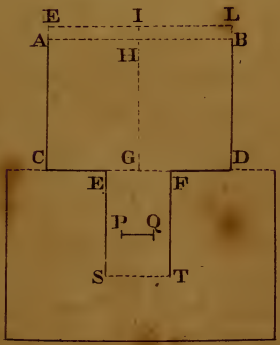
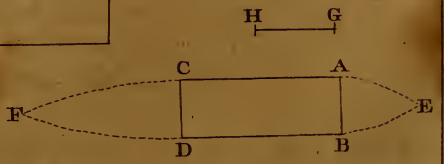


Fig. 6.



parts of the bodies in the manner explained in the ſcholium of the laſt propoſition; for in what follows we conſider the very leaſt reſiſtance that round bodies deſcribed with the greateſt given tranſverſe ſections can poſſibly meet with.

Bodies ſwimming upon fluids, when they move ſtraight forwards, cauſe the fluid to aſcend at their fore parts and ſubſide at their hinder parts, eſpecially if they are of an obtuſe figure; and thence they meet with a little more reſiſtance than if they were acute at the head and tail. And bodies moving in elaſtic fluids, if they are obtuſe behind and before, condenſe the fluid a little more at their fore parts, and relax the ſame at their hinder parts; and therefore meet alſo with a little more reſiſtance than if they were acute at the head and tail. But in theſe lemmas and propoſitions we are not treating of elaſtic but non-elaſtic fluids; not of bodies floating on the ſurface of the fluid, but deeply immerſed therein. And when the reſiſtance of bodies in non-elaſtic fluids is once known, we may then augment this reſiſtance a little in elaſtic fluids, as our air; and in the ſurfaces of ſtagnating fluids, as lakes and ſeas.

PROPOSITION XXXVIII. THEOREM XXX.

If a globe move uniformly forward in a compreſſed, infinite, and non-elaſtic fluid, its reſiſtance is to the force by which its whole motion may be deſtroyed or generated, in the time that it deſcribes eight third parts of its diameter, as the denſity of the fluid to the denſity of the globe, very nearly.

For the globe is to its circumſcribed cylinder as two to three; and therefore the force which can deſtroy all the motion of the cylinder while the ſame cylinder is deſcribing the length of four of its diameters, will deſtroy all the motion of the globe while the globe is deſcribing two thirds of this length, that is, eight third parts of its own diameter. Now the reſiſtance of the cylinder is to this force very nearly as the denſity of the fluid to the denſity of the cylinder or globe (by prop. 37), and the reſiſtance of the globe is equal to the reſiſtance of the cylinder (by lem. 5, 6, and 7). Q.E.D.

Cor. 1. The reſiſtances of globes in infinite compreſſed mediums are in a ratio compounded of the duplicate ratio of the

velocity, and the duplicate ratio of the diameter, and the ratio of the denſity of the mediums.

Cor. 2. The greateſt velocity with which a globe can deſcend by its comparative weight through a reſiſting fluid, is the ſame which it may acquire by falling with the ſame weight, and without any reſiſtance, and in its fall deſcribing a ſpace that is to four third parts of its diameter as the denſity of the globe to the denſity of the fluid. For the globe in the time of its fall, moving with the velocity acquired in falling, will deſcribe a ſpace that will be to eight third parts of its diameter as the denſity of the globe to the denſity of the fluid; and the force of its weight which generates this motion will be to the force that can generate the ſame motion, in the time that the globe deſcribes eight third parts of its diameter, with the ſame velocity as the denſity of the fluid to the denſity of the globe; and therefore (by this propoſition) the force of weight will be equal to the force of reſiſtance, and therefore cannot accelerate the globe.

Cor. 3. If there be given both the denſity of the globe and its velocity at the beginning of the motion, and the denſity of the compreſſed quieſcent fluid in which the globe moves, there is given at any time both the velocity of the globe and its reſiſtance, and the ſpace deſcribed by it (by cor. 7, prop. 35).

Cor. 4. A globe moving in a compreſſed quieſcent fluid of the ſame denſity with itſelf will loſe half its motion before it can deſcribe the length of two of its diameters (by the ſame cor. 7).

PROPOSITION XXXIX. THEOREM XXXI.

If a globe move uniformly forward through a fluid incloſed and compreſſed in a cylindric canal, its reſiſtance is to the force by which its whole motion may be generated or deſtroyed, in the time in which it deſcribes eight third parts of its diameter, in a ratio compounded of the ratio of the orifice of the canal to the exceſs of that orifice above half the greateſt circle of the globe; and the duplicate ratio of the orifice of the canal to the exceſs of that orifice above the

greateſt circle of the globe; and the ratio of the denſity of the fluid to the denſity of the globe, nearly.

This appears by cor. 2, prop. 37, and the demonſtration proceeds in the ſame manner as in the foregoing propoſition.

SCHOLIUM.

In the two laſt propoſitions we ſuppoſe (as was done before in lem. 5) that all the water which precedes the globe, and whoſe fluidity increaſes the reſiſtance of the ſame, is congealed. Now if that water becomes fluid, it will ſomewhat increaſe the reſiſtance. But in theſe propoſitions that increaſe is ſo ſmall, that it may be neglected, becauſe the convex ſuperficies of the globe produces the very ſame effect almoſt as the congelation of the water.

PROPOSITION XL. PROBLEM IX.

To find by phænomena the reſiſtance of a globe moving through a perfectly fluid compreſſed medium.

Let A be the weight of the globe *in vacuo*, B its weight in the reſiſting medium, D the diameter of the globe, F a ſpace which is to $\frac{4}{3}$D as the denſity of the globe to the denſity of the medium, that is, as A to A — B, G the time in which the globe falling with the weight B without reſiſtance deſcribes the ſpace F, and H the velocity which the body acquires by that fall. Then H will be the greateſt velocity with which the globe can poſſibly deſcend with the weight B in the reſiſting medium, by cor. 2, prop. 38; and the reſiſtance which the globe meets with, when deſcending with that velocity, will be equal to its weight B; and the reſiſtance it meets with in any other velocity will be to the weight B in the duplicate ratio of that velocity to the greateſt velocity H, by cor. 1, prop. 38.

This is the reſiſtance that ariſes from the inactivity of the matter of the fluid. That reſiſtance which ariſes from the elaſticity, tenacity, and friction of its parts, may be thus inveſtigated.

Let the globe be let fall ſo that it may deſcend in the fluid by the weight B; and let P be the time of falling, and let that time be expreſſed in ſeconds, if the time G be given in ſeconds. Find the abſolute number N agreeing to the lo-

garithm 0,4342944819 $\frac{2P}{G}$, and let L be the logarithm of the number $\frac{N+1}{N}$; and the velocity acquired in falling will be $\frac{N-1}{N+1}$H, and the height defcribed will be $\frac{2PF}{G}$ — 1,3862943611F + 4,605170186LF. If the fluid be of a fufficient depth, we may neglect the term 4,605170186LF; and $\frac{2PF}{G}$ — 1,3862943611F will be the altitude defcribed, nearly. Thefe things appear by prop. 9, book 2, and its corollaries, and are true upon this fuppofition, that the globe meets with no other refiftance but that which arifes from the inactivity of matter. Now if it really meet with any refiftance of another kind, the defcent will be flower, and from the quantity of that retardation will be known the quantity of this new refiftance.

That the velocity and defcent of a body falling in a fluid might more eafily be known, I have compofed the following table; the firft column of which denotes the times of defcent; the fecond fhews the velocities acquired in falling, the greateft velocity being 100000000; the third exhibits the fpaces defcribed by falling in thofe times, 2F being the fpace which the body defcribes in the time G with the greateft velocity; and the fourth gives the fpaces defcribed with the greateft velocity in the fame times. The numbers in the fourth column are $\frac{2P}{G}$, and by fubducting the number 1,3862944—4,6051702L, are found the numbers in the third column; and thefe numbers muft be multiplied by the fpace F to obtain the fpaces defcribed in falling. A fifth column is added to all thefe, containing the fpaces defcribed in the fame times by a body falling *in vacuo* with the force of B its comparative weight.

The Times P.	Velocities of the body falling in the fluid.	The ſpaces deſcribed in falling in the fluid.	The ſpaces deſcribed with the greateſt motion.	The ſpaces deſcribed by falling *in vacuo*.
0,001G	99999$\frac{2}{3}$9	0,000001F	0,002F	0,000001F
0,01G	999967	0,0001F	0,02F	0,0001F
0,1G	9966799	0,0099834F	0,2F	0,01F
0,2G	19737532	0,0397361F	0,4F	0,04F
0,3G	29131261	0,0886815F	0,6F	0,09F
0,4G	37994896	0,1559070F	0,8F	0,16F
0,5G	46211716	0,2402290F	1,0F	0,25F
0,6G	53704957	0,3402706F	1,2F	0,36F
0,7G	60436778	0,4545405F	1,4F	0,49F
0,8G	66403677	0,5815071F	1,6F	0,64F
0,9G	71629787	0,7196609F	1,8F	0,81F
1G	76159416	0,8675617F	2F	1F
2G	96402758	2,6500055F	4F	4F
3G	99505475	4,6186570F	6F	9F
4G	99932930	6,6143765F	8F	16F
5G	99990920	8,6137964F	10F	25F
6G	99998771	10,6137179F	12F	36F
7G	99999834	12,6137073F	14F	49F
8G	99999980	14,6137059F	16F	64F
9G	99999997	16,6137057F	18F	81F
10G	99999999$\frac{3}{5}$	18,6137056F	20F	100F

SCHOLIUM.

In order to inveſtigate the reſiſtances of fluids from experiments, I procured a ſquare wooden veſſel, whoſe length and breadth on the inſide was 9 inches *Engliſh* meaſure, and its depth 9 feet $\frac{1}{2}$; this I filled with rain-water: and having provided globes made up of wax, and lead included therein, I noted the times of the deſcents of theſe globes, the height through which they deſcended being 112 inches. A ſolid cubic foot of *Engliſh* meaſure contains 76 pounds *troy* weight of rain-water; and a ſolid inch contains $\frac{175}{576}$ ounces *troy* weight, or 253$\frac{1}{3}$ grains; and a globe of water of one inch in diameter contains 132,645 grains in air, or 132,8 grains *in vacuo*; and any other globe will be as the exceſs of its weight *in vacuo* above its weight in water.

Exper. 1. A globe whoſe weight was 156$\frac{1}{4}$ grains in air, and 77 grains in water, deſcribed the whole height of 112

inches in 4 ſeconds. And, upon repeating the experiment, the globe ſpent again the very ſame time of 4 ſeconds in falling.

The weight of this globe *in vacuo* is $156\frac{1}{3}\frac{3}{8}$ grains; and exceſs of this weight above the weight of the globe in water is $79\frac{1}{3}\frac{3}{8}$ grains. Hence the diameter of the globe appears to be 0,84224 parts of an inch. Then it will be, as that exceſs to the weight of the globe *in vacuo*, ſo is the denſity of the water to the denſity of the globe; and ſo is $\frac{8}{3}$ parts of the diameter of the globe (viz. 2,24597 inches) to the ſpace 2F, which will be therefore 4,4256 inches. Now a globe falling *in vacuo* with its whole weight of $156\frac{1}{3}\frac{3}{8}$ grains in one ſecond of time will deſcribe $193\frac{1}{3}$ inches; and falling in water in the ſame time with the weight of 77 grains without reſiſtance, will deſcribe 95,219 inches; and in the time G, which is to one ſecond of time in the ſubduplicate ratio of the ſpace F, or of 2,2128 inches to 95,219 inches, will deſcribe 2,2128 inches, and will acquire the greateſt velocity H with which it is capable of deſcending in water. Therefore the time G is $0''$,15244. And in this time G, with that greateſt velocity H, the globe will deſcribe the ſpace 2F, which is 4,4256 inches; and therefore in 4 ſeconds will deſcribe a ſpace of 116,1245 inches. Subduct the ſpace 1,3862944F, or 3,0676 inches, and there will remain a ſpace of 113,0569 inches, which the globe falling through water in a very wide veſſel will deſcribe in 4 ſeconds. But this ſpace, by reaſon of the narrowneſs of the wooden veſſel before-mentioned, ought to be diminiſhed in a ratio compounded of the ſubduplicate ratio of the orifice of the veſſel to the exceſs of this orifice above half a great circle of the globe, and of the ſimple ratio of the ſame orifice to its exceſs above a great circle of the globe, that is, in a ratio of 1 to 0,9914. This done, we have a ſpace of 112,08 inches, which a globe falling through the water in this wooden veſſel in 4 ſeconds of time ought nearly to deſcribe by this theory; but it deſcribed 112 inches by the experiment.

Exper. 2. Three equal globes, whoſe weights were ſeverally $76\frac{1}{3}$ grains in air, and $5\frac{1}{16}$ grains in water, were let fall

ſucceſſively; and every one fell through the water in 15 ſeconds of time, deſcribing in its fall a height of 112 inches.

By computation, the weight of each globe *in vacuo* is $76\frac{5}{12}$ grains; the exceſs of this weight above the weight in water is 71 grains $\frac{17}{48}$; the diameter of the globe 0,81296 of an inch; $\frac{8}{3}$ parts of this diameter 2,16789 inches; the ſpace 2F is 2,3217 inches; the ſpace which a globe of $5\frac{1}{16}$ grains in weight would deſcribe in one ſecond without reſiſtance, 12,808 inches, and the time G0″,301056. Therefore the globe, with the greateſt velocity it is capable of receiving from a weight of $5\frac{1}{16}$ grains in its deſcent through water, will deſcribe in the time 0″,301056 the ſpace of 2,3217 inches; and in 15 ſeconds the ſpace 115,678 inches. Subduct the ſpace 1,3862944F, or 1,609 inches, and there remains the ſpace 114,069 inches; which therefore the falling globe ought to deſcribe in the ſame time, if the veſſel were very wide. But becauſe our veſſel was narrow, the ſpace ought to be diminiſhed by about 0,895 of an inch. And ſo the ſpace will remain 113,174 inches, which a globe falling in this veſſel ought nearly to deſcribe in 15 ſeconds, by the theory. But by the experiment it deſcribed 112 inches. The difference is not ſenſible.

Exper. 3. Three equal globes, whoſe weights were ſeverally 121 grains in air, and 1 grain in water, were ſucceſſively let fall; and they fell through the water in the times 46″, 47″, and 50″, deſcribing a height of 112 inches.

By the theory, theſe globes ought to have fallen in about 40″. Now whether their falling more ſlowly were occaſioned from hence, that in ſlow motions the reſiſtance ariſing from the force of inactivity does really bear a leſs proportion to the reſiſtance ariſing from other cauſes; or whether it is to be attributed to little bubbles that might chance to ſtick to the globes, or to the rarefaction of the wax by the warmth of the weather, or of the hand that let them fall; or, laſtly, whether it proceeded from ſome inſenſible errors in weighing the globes in the water, I am not certain. Therefore the weight of the globe in water ſhould be of ſeveral grains, that the experiment may be certain, and to be depended on.

EXPER. 4. I began the foregoing experiments to inveſtigate the reſiſtances of fluids, before I was acquainted with the theory laid down in the propoſitions immediately preceding. Afterwards, in order to examine the theory after it was diſcovered, I procured a wooden veſſel, whoſe breadth on the inſide was $8\frac{2}{3}$ inches, and its depth 15 feet and $\frac{1}{3}$. Then I made four globes of wax, with lead included, each of which weighed $139\frac{1}{4}$ grains in air, and $7\frac{1}{8}$ grains in water. Theſe I let fall, meaſuring the times of their falling in the water with a pendulum oſcillating to half ſeconds. The globes were cold, and had remained ſo ſome time, both when they were weighed and when they were let fall; becauſe warmth rarefies the wax, and by rarefying it diminiſhes the weight of the globe in the water; and wax, when rarefied, is not inſtantly reduced by cold to its former denſity. Before they were let fall, they were totally immerſed under water, leſt, by the weight of any part of them that might chance to be above the water, their deſcent ſhould be accelerated in its beginning. Then, when after their immerſion they were perfectly at reſt, they were let go with the greateſt care, that they might not receive any impulſe from the hand that let them down. And they fell ſucceſſively in the times of $47\frac{1}{2}$, $48\frac{1}{2}$, 50, and 51 oſcillations, deſcribing a height of 15 feet and 2 inches. But the weather was now a little colder than when the globes were weighed, and therefore I repeated the experiment another day; and then the globes fell in the times of 49, $49\frac{1}{2}$, 50, and 53; and at a third trial in the times of $49\frac{1}{2}$, 50, 51, and 53 oſcillations. And by making the experiment ſeveral times over, I found that the globes fell moſtly in the times of $49\frac{1}{2}$ and 50 oſcillations. When they fell ſlower, I ſuſpect them to have been retarded by ſtriking againſt the ſides of the veſſel.

Now, computing from the theory, the weight of the globe *in vacuo* is $139\frac{2}{5}$ grains; the exceſs of this weight above the weight of the globe in water $132\frac{11}{40}$ grains; the diameter of the globe 0,99868 of an inch; $\frac{8}{3}$ parts of the diameter 2,66315 inches; the ſpace 2F 2,8066 inches; the ſpace which a globe weighing $7\frac{1}{8}$ grains falling without reſiſtance deſcribes in a ſe-

cond of time 9,88164 inches; and the time G0″,376843. Therefore the globe with the greateſt velocity with which it is capable of deſcending through the water by the force of a weight of $7\frac{1}{8}$ grains, will in the time 0″,376843 deſcribe a ſpace of 2,8066 inches, and in one ſecond of time a ſpace of 7,44766 inches, and in the time 25″, or in 50 oſcillations, the ſpace 186,1915 inches. Subduct the ſpace 1,386294F, or 1,9454 inches, and there will remain the ſpace 184,2461 inches which the globe will deſcribe in that time in a very wide veſſel. Becauſe our veſſel was narrow, let this ſpace be diminiſhed in a ratio compounded of the ſubduplicate ratio of the orifice of the veſſel to the exceſs of this orifice above half a great circle of the globe, and of the ſimple ratio of the ſame orifice to its exceſs above a great circle of the globe; and we ſhall have the ſpace of 181,86 inches, which the globe ought by the theory to deſcribe in this veſſel in the time of 50 oſcillations, nearly. But it deſcribed the ſpace of 182 inches, by experiment, in $49\frac{1}{2}$ or 50 oſcillations.

Exper. 5. Four globes weighing $154\frac{3}{8}$ grains in air, and $21\frac{1}{2}$ grains in water, being let fall ſeveral times, fell in the times of $28\frac{1}{2}$, 29, $29\frac{1}{2}$, and 30, and ſometimes of 31, 32, and 33 oſcillations, deſcribing a height of 15 feet and 2 inches.

They ought by the theory to have fallen in the time of 29 oſcillations, nearly.

Exper. 6. Five globes, weighing $212\frac{3}{8}$ grains in air, and $79\frac{1}{2}$ in water, being ſeveral times let fall, fell in the times of 15, $15\frac{1}{2}$, 16, 17, and 18 oſcillations, deſcribing a height of 15 feet and 2 inches.

By the theory they ought to have fallen in the time of 15 oſcillations, nearly.

Exper. 7. Four globes, weighing $293\frac{3}{8}$ grains in air, and $35\frac{7}{8}$ grains in water, being let fall ſeveral times, fell in the times of $29\frac{1}{2}$, 30, $30\frac{1}{2}$, 31, 32, and 33 oſcillations, deſcribing a height of 15 feet and 1 inch and $\frac{1}{2}$.

By the theory they ought to have fallen in the time of 28 oſcillations, nearly.

In ſearching for the cauſe that occaſioned theſe globes of the ſame weight and magnitude to fall, ſome ſwifter and

ſome ſlower, I hit upon this; that the globes, when they were firſt let go and began to fall, oſcillated about their centres; that ſide which chanced to be the heavier deſcending firſt, and producing an oſcillating motion. Now by oſcillating thus, the globe communicates a greater motion to the water than if it deſcended without any oſcillations; and by this communication loſes part of its own motion with which it ſhould deſcend; and therefore as this oſcillation is greater or leſs, it will be more or leſs retarded. Beſides, the globe always recedes from that ſide of itſelf which is deſcending in the oſcillation, and by ſo receding comes nearer to the ſides of the veſſel, ſo as even to ſtrike againſt them ſometimes. And the heavier the globes are, the ſtronger this oſcillation is; and the greater they are, the more is the water agitated by it. Therefore to diminiſh this oſcillation of the globes, I made new ones of lead and wax, ſticking the lead in one ſide of the globe very near its ſurface; and I let fall the globe in ſuch a manner, that, as near as poſſible, the heavier ſide might be loweſt at the beginning of the deſcent. By this means the oſcillations became much leſs than before, and the times in which the globes fell were not ſo unequal: as in the following experiments.

EXPER. 8. Four globes weighing 139 grains in air, and $6\frac{1}{2}$ in water, were let fall ſeveral times, and fell moſtly in the time of 51 oſcillations, never in more than 52, or in fewer than 50, deſcribing a height of 182 inches.

By the theory they ought to fall in about the time of 52 oſcillations.

EXPER. 9. Four globes weighing $273\frac{1}{4}$ grains in air, and $140\frac{3}{4}$ in water, being ſeveral times let fall, fell in never fewer than 12, and never more than 13 oſcillations, deſcribing a height of 182 inches.

Theſe globes by the theory ought to have fallen in the time of $11\frac{1}{3}$ oſcillations, nearly.

EXPER. 10. Four globes, weighing 384 grains in air, and $119\frac{1}{2}$ in water, being let fall ſeveral times, fell in the times of $17\frac{3}{4}$, 18, $18\frac{1}{2}$, and 19 oſcillations, deſcribing a height of $181\frac{1}{2}$ inches. And when they fell in the time of 19 oſcillations, I

ſometimes heard them hit againſt the ſides of the veſſel before they reached the bottom.

By the theory they ought to have fallen in the time of $15\frac{5}{9}$ oſcillations, nearly.

EXPER. 11. Three equal globes, weighing 48 grains in the air, and $3\frac{29}{32}$ in water, being ſeveral times let fall, fell in the times of $43\frac{1}{2}$, 44, $44\frac{1}{2}$, 45, and 46 oſcillations, and moſtly in 44 and 55, deſcribing a height of $182\frac{1}{2}$ inches, nearly.

By the theory they ought to have fallen in the time of 46 oſcillations and $\frac{5}{9}$, nearly.

EXPER. 12. Three equal globes, weighing 141 grains in air, and $4\frac{3}{8}$ in water, being let fall ſeveral times, fell in the times of 61, 62, 63, 64, and 65 oſcillations, deſcribing a ſpace of 182 inches.

And by the theory they ought to have fallen in $64\frac{1}{2}$ oſcillations, nearly.

From theſe experiments it is manifeſt, that when the globes fell ſlowly, as in the ſecond, fourth, fifth, eighth, eleventh, and twelfth experiments, the times of falling are rightly exhibited by the theory; but when the globes fell more ſwiftly, as in the ſixth, ninth, and tenth experiments, the reſiſtance was ſomewhat greater than in the duplicate ratio of the velocity. For the globes in falling oſcillate a little; and this oſcillation, in thoſe globes that are light and fall ſlowly, ſoon ceaſes by the weakneſs of the motion; but in greater and heavier globes, the motion being ſtrong, it continues longer, and is not to be checked by the ambient water till after ſeveral oſcillations. Beſides, the more ſwiftly the globes move, the leſs are they preſſed by the fluid at their hinder parts; and if the velocity be perpetually increaſed, they will at laſt leave an empty ſpace behind them, unleſs the compreſſion of the fluid be increaſed at the ſame time. For the compreſſion of the fluid ought to be increaſed (by prop. 32 and 33) in the duplicate ratio of the velocity, in order to preſerve the reſiſtance in the ſame duplicate ratio. But becauſe this is not done, the globes that move ſwiftly are not ſo much preſſed at their hinder parts as the others; and by the defect of

this preſſure it comes to paſs that their reſiſtance is a little greater than in a duplicate ratio of their velocity.

So that the theory agrees with the phænomena of bodies falling in water. It remains that we examine the phænomena of bodies falling in air.

EXPER. 13. From the top of St. *Paul*'s Church in *London*, in *June* 1710, there were let fall together two glaſs globes, one full of quickſilver, the other of air; and in their fall they deſcribed a height of 220 *Engliſh* feet. A wooden table was ſuſpended upon iron hinges on one ſide, and the other ſide of the ſame was ſupported by a wooden pin. The two globes lying upon this table were let fall together by pulling out the pin by means of an iron wire reaching from thence quite down to the ground; ſo that, the pin being removed, the table, which had then no ſupport but the iron hinges, fell downwards, and, turning round upon the hinges, gave leave to the globes to drop off from it. At the ſame inſtant, with the ſame pull of the iron wire that took out the pin, a pendulum oſcillating to ſeconds was let go, and began to oſcillate. The diameters and weights of the globes, and their times of falling, are exhibited in the following table.

The globes filled with mercury.			*The globes full of air.*		
Weights.	Diameters.	Times in falling.	Weights.	Diameters.	Times in falling.
908 grains	0,8 of an inch	4″	510 grains	5,1 inches	8″½
983	0,8	4 —	642	5,2	8
866	0,8	4	599	5,1	8
747	0,75	4 +	515	5,0	8¼
808	0,75	4	483	5,0	8½
784	0,75	4 +	641	5,2	8

But the times obſerved muſt be corrected; for the globes of mercury (by *Galileo*'s theory), in 4 ſeconds of time, will deſcribe 257 *Engliſh* feet, and 220 feet in only 3″42‴. So that the wooden table, when the pin was taken out, did not turn upon its hinges ſo quickly as it ought to have done; and the ſlowneſs of that revolution hindered the deſcent of the globes at the beginning. For the globes lay about the middle of the table, and indeed were rather nearer to the axis upon which it turned than to the pin. And hence the times of falling

were prolonged about 18′′′; and therefore ought to be corrected by ſubducting that exceſs, eſpecially in the larger globes, which, by reaſon of the largeneſs of their diameters, lay longer upon the revolving table than the others. This being done, the times in which the ſix larger globes fell will come forth 8′ 12″, 7′ 42′′′, 7″ 42′′′, 7″ 57′′′, 8′ 12″, and 7′ 42″.

Therefore the fifth in order among the globes that were full of air being 5 inches in diameter, and 483 grains in weight, fell in 8′ 12″, deſcribing a ſpace of 220 feet. The weight of a bulk of water equal to this globe is 16600 grains; and the weight of an equal bulk of air is $\frac{16600}{8600}$ grains, or $19\frac{3}{10}$ grains; and therefore the weight of the globe *in vacuo* is $502\frac{3}{10}$ grains; and this weight is to the weight of a bulk of air equal to the globe as $502\frac{3}{10}$ to $19\frac{3}{10}$; and ſo is 2F to $\frac{8}{3}$ of the diameter of the globe, that is, to $13\frac{1}{3}$ inches. Whence 2F becomes 28 feet 11 inches. A globe falling *in vacuo* with its whole weight of $502\frac{3}{10}$ grains, will in one ſecond of time deſcribe $193\frac{1}{3}$ inches as above; and with the weight of 483 grains will deſcribe 185,905 inches; and with that weight 483 grains *in vacuo* will deſcribe the ſpace F, or 14 feet $5\frac{1}{2}$ inches, in the time of 57″ 58′′′, and acquire the greateſt velocity it is capable of deſcending with in the air. With this velocity the globe in 8″ 12′′′ of time will deſcribe 245 feet and $5\frac{1}{3}$ inches. Subduct 1,3863F, or 20 feet and $\frac{1}{2}$ an inch, and there remain 225 feet 5 inches. This ſpace, therefore, the falling globe ought by the theory to deſcribe in 8″ 12′′′. But by the experiment it deſcribed a ſpace of 220 feet. The difference is inſenſible.

By like calculations applied to the other globes full of air, I compoſed the following table.

The weights of the globes.	The diameters.	The times of falling from a height of 220 feet.	The ſpaces which they would deſcribe by the theory.	The exceſſes.
510 grains	5,1 inches	8′ 12′′′	226 feet 1 inch.	6 feet 1 inch.
642	5,2	7 42	230 9	10 9
599	5,1	7 42	227 10	7 10
515	5	7 57	224 5	4 5
483	5	8 12	225 5	5 5
641	5,2	7 42	230 7	10 7

EXPER. 14. *Anno* 1719, in the month of *July*, Dr. *Deſaguliers* made ſome experiments of this kind again, by forming hogs' bladders into ſpherical orbs; which was done by means of a concave wooden ſphere, which the bladders, being wetted well firſt, were put into. After that, being blown full of air, they were obliged to fill up the ſpherical cavity that contained them; and then, when dry, were taken out. Theſe were let fall from the lantern on the top of the cupola of the ſame church, namely, from a height of 272 feet; and at the ſame moment of time there was let fall a leaden globe, whoſe weight was about 2 pounds *troy* weight. And in the mean time ſome perſons ſtanding in the upper part of the church where the globes were let fall obſerved the whole times of falling; and others ſtanding on the ground obſerved the differences of the times between the fall of the leaden weight and the fall of the bladder. The times were meaſured by pendulums oſcillating to half ſeconds. And one of thoſe that ſtood upon the ground had a machine vibrating four times in one ſecond; and another had another machine accurately made with a pendulum vibrating four times in a ſecond alſo. One of thoſe alſo who ſtood at the top of the church had a like machine; and theſe inſtruments were ſo contrived, that their motions could be ſtopped or renewed at pleaſure. Now the leaden globe fell in about four ſeconds and $\frac{1}{4}$ of time; and from the addition of this time to the difference of time above ſpoken of, was collected the whole time in which the bladder was falling. The times which the five bladders ſpent in falling, after the leaden globe had reached the ground, were, the firſt time, $14\frac{3}{4}''$, $12\frac{3}{4}''$, $14\frac{5}{8}''$, $17\frac{3}{4}''$, and $16\frac{7}{8}''$; and the ſecond time, $14\frac{1}{2}''$, $14\frac{1}{4}''$, $14''$, $19''$, and $16\frac{3}{4}''$. Add to theſe $4\frac{1}{4}''$, the time in which the leaden globe was falling, and the whole times in which the five bladders fell were, the firſt time, $19''$, $17''$, $18\frac{7}{8}''$, $22''$, and $21\frac{1}{8}''$; and the ſecond time, $18\frac{3}{4}''$, $18\frac{1}{2}''$, $18\frac{1}{4}''$, $23\frac{1}{4}''$, and $21''$. The times obſerved at the top of the church were, the firſt time, $19\frac{3}{8}''$, $17\frac{1}{4}''$, $18\frac{3}{4}''$, $22\frac{1}{8}''$, and $21\frac{5}{8}''$; and the ſecond time, $19''$, $18\frac{5}{8}''$, $18\frac{3}{8}''$, $24''$, and $21\frac{1}{4}''$. But the bladders did not always fall directly down, but ſometimes fluttered a little in the air, and waved to and fro as they were deſcending. And by theſe motions the times of their falling were prolonged,

and increaſed by half a ſecond ſometimes, and ſometimes by a whole ſecond. The ſecond and fourth bladder fell moſt directly the firſt time, and the firſt and third the ſecond time. The fifth bladder was wrinkled, and by its wrinkles was a little retarded. I found their diameters by their circumferences meaſured with a very fine thread wound about them twice. In the following table I have compared the experiments with the theory; making the denſity of air to be to the denſity of rain-water as 1 to 860, and computing the ſpaces which by the theory the globes ought to deſcribe in falling.

The weights of the bladders.	The diameters.	The times of falling from a height of 272 feet.	The ſpaces which by the theory ought to have been deſcribed in thoſe times.	The difference between the theory and the experiments.
128 grains	5,28 inches	19″	271 feet 11 in.	— 0 ft. 1 in.
156	5,19	17	272 0½	+ 0 0½
137½	5,3	18	272 7	+ 0 7
97½	5,26	22	277 4	+ 5 4
99⅓	5	21⅛	282 0	+ 10 0

Our theory, therefore, exhibits rightly, within a very little, all the reſiſtance that globes moving either in air or in water meet with; which appears to be proportional to the denſities of the fluids in globes of equal velocities and magnitudes.

In the ſcholium ſubjoined to the ſixth ſection, we ſhewed, by experiments of pendulums, that the reſiſtances of equal and equally ſwift globes moving in air, water, and quickſilver, are as the denſities of the fluids. We here prove the ſame more accurately by experiments of bodies falling in air and water. For pendulums at each oſcillation excite a motion in the fluid always contrary to the motion of the pendulum in its return; and the reſiſtance ariſing from this motion, as alſo the reſiſtance of the thread by which the pendulum is ſuſpended, makes the whole reſiſtance of a pendulum greater than the reſiſtance deduced from the experiments of falling bodies. For by the experiments of pendulums deſcribed in that ſcholium, a globe of the ſame denſity as water in deſcribing the length of its ſemi-diameter in air would loſe the $\frac{1}{3342}$ part of its motion. But by the theory delivered in this

ſeventh ſection, and confirmed by experiments of falling bodies, the ſame globe in deſcribing the ſame length would loſe only a part of its motion equal to $\frac{1}{4586}$, ſuppoſing the denſity of water to be to the denſity of air as 860 to 1. Therefore the reſiſtances were found greater by the experiments of pendulums (for the reaſons juſt mentioned) than by the experiments of falling globes; and that in the ratio of about 4 to 3. But yet ſince the reſiſtances of pendulums oſcillating in air, water, and quickſilver, are alike increaſed by like cauſes, the proportion of the reſiſtances in theſe mediums will be rightly enough exhibited by the experiments of pendulums, as well as by the experiments of falling bodies. And from all this it may be concluded, that the reſiſtances of bodies, moving in any fluids whatſoever, though of the moſt extreme fluidity, are, *cæteris paribus*, as the denſities of the fluids.

Theſe things being thus eſtabliſhed, we may now determine what part of its motion any globe projected in any fluid whatſoever would nearly loſe in a given time. Let D be the diameter of the globe, and V its velocity at the beginning of its motion, and T the time in which a globe with the velocity V can deſcribe *in vacuo* a ſpace that is to the ſpace $\frac{2}{3}$D as the denſity of the globe to the denſity of the fluid; and the globe projected in that fluid will, in any other time t, loſe the part $\frac{tV}{T+t}$, the part $\frac{TV}{T+t}$ remaining; and will deſcribe a ſpace, which will be to that deſcribed in the ſame time *in vacuo* with the uniform velocity V, as the logarithm of the number $\frac{T+t}{T}$ multiplied by the number 2,302585093 is to the number $\frac{t}{T}$, by cor. 7, prop. 35. In ſlow motions the reſiſtance may be a little leſs, becauſe the figure of a globe is more adapted to motion than the figure of a cylinder deſcribed with the ſame diameter. In ſwift motions the reſiſtance may be a little greater, becauſe the elaſticity and compreſſion of the fluid do not increaſe in the duplicate ratio of the velocity. But theſe little niceties I take no notice of.

And though air, water, quickſilver, and the like fluids, by the diviſion of their parts *in infinitum*, ſhould be ſubtilized, and become mediums infinitely fluid, nevertheleſs, the reſiſtance they would make to projected globes would be the ſame. For the reſiſtance conſidered in the preceding propoſitions ariſes from the inactivity of the matter; and the inactivity of matter is eſſential to bodies, and always proportional to the quantity of matter. By the diviſion of the parts of the fluid the reſiſtance ariſing from the tenacity and friction of the parts may be indeed diminiſhed; but the quantity of matter will not be at all diminiſhed by this diviſion; and if the quantity of matter be the ſame, its force of inactivity will be the ſame; and therefore the reſiſtance here ſpoken of will be the ſame, as being always proportional to that force. To diminiſh this reſiſtance, the quantity of matter in the ſpaces through which the bodies move muſt be diminiſhed; and therefore the celeſtial ſpaces, through which the globes of the planets and comets are perpetually paſſing towards all parts, with the utmoſt freedom, and without the leaſt ſenſible diminution of their motion, muſt be utterly void of any corporeal fluid, excepting, perhaps, ſome extremely rare vapours and the rays of light.

Projectiles excite a motion in fluids as they paſs through them; and this motion ariſes from the exceſs of the preſſure of the fluid at the fore-parts of the projectile above the preſſure of the ſame at the hinder parts; and cannot be leſs in mediums infinitely fluid than it is in air, water, and quickſilver, in proportion to the denſity of matter in each. Now this exceſs of preſſure does, in proportion to its quantity, not only excite a motion in the fluid, but alſo acts upon the projectile ſo as to retard its motion; and therefore the reſiſtance in every fluid is as the motion excited by the projectile in the fluid; and cannot be leſs in the moſt ſubtile æther in proportion to the denſity of that æther, than it is in air, water, and quickſilver, in proportion to the denſities of thoſe fluids.

SECTION VIII.

Of motion propagated through fluids.

PROPOSITION XLI. THEOREM XXXII.

A preſſure is not propagated through a fluid in rectilinear directions, unleſs where the particles of the fluid lie in a right line. (Pl. 8, Fig. 1.)

If the particles a, b, c, d, e, lie in a right line, the preſſure may be indeed directly propagated from a to e; but then the particle e will urge the obliquely poſited particles f and g obliquely, and thoſe particles f and g will not ſuſtain this preſſure, unleſs they be ſupported by the particles h and k lying beyond them; but the particles that ſupport them are alſo preſſed by them; and thoſe particles cannot ſuſtain that preſſure, without being ſupported by, and preſſing upon, thoſe particles that lie ſtill farther, as l and m, and ſo on *in infinitum*. Therefore the preſſure, as ſoon as it is propagated to particles that lie out of right lines, begins to deflect towards one hand and the other, and will be propagated obliquely *in infinitum;* and after it has begun to be propagated obliquely, if it reaches more diſtant particles lying out of the right line, it will deflect again on each hand; and this it will do as often as it lights on particles that do not lie exactly in a right line. Q.E.D.

Cor. If any part of a preſſure, propagated through a fluid from a given point, be intercepted by any obſtacle, the remaining part, which is not intercepted, will deflect into the ſpaces behind the obſtacle. This may be demonſtrated alſo after the following manner. Let a preſſure be propagated from the point A (Pl. 8, Fig. 2) towards any part, and, if it be poſſible, in rectilinear directions; and the obſtacle NBCK being perforated in BC, let all the preſſure be intercepted but the coniform part APQ paſſing through the circular hole BC. Let the cone APQ be divided into fruſtums by the tranſverſe planes de, fg, hi. Then while the cone ABC, propagating the preſſure, urges the conic fruſtum degf beyond it on the ſuperficies de, and this fruſtum urges the next fruſtum fgih on the ſuperficies fg, and that fruſtum urges a third fruſtum, and ſo *in infinitum;* it is manifeſt (by the third law) that the

firſt fruſtum defg is, by the re-action of the ſecond fruſtum fghi, as much urged and preſſed on the ſuperficies fg, as it urges and preſſes that ſecond fruſtum. Therefore the fruſtum degf is compreſſed on both ſides, that is, between the cone Ade and the fruſtum fhig; and therefore (by caſe 6, prop. 19) cannot preſerve its figure, unleſs it be compreſſed with the ſame force on all ſides. Therefore with the ſame force with which it is preſſed on the ſuperficies de, fg, it will endeavour to break forth at the ſides df, eg; and there (being not in the leaſt tenacious or hard, but perfectly fluid) it will run out, expanding itſelf, unleſs there be an ambient fluid oppoſing that endeavour. Therefore, by the effort it makes to run out, it will preſs the ambient fluid, at its ſides df, eg, with the ſame force that it does the fruſtum fghi; and therefore, the preſſure will be propagated as much from the ſides df, eg, into the ſpaces NO, KL this way and that way, as it is propagated from the ſuperficies fg towards PQ. Q.E.D.

PROPOSITION XLII. THEOREM XXXIII.

All motion propagated through a fluid diverges from a rectilinear progreſs into the unmoved ſpaces. (Pl. 8, Fig. 3.)

CASE 1. Let a motion be propagated from the point A through the hole BC, and, if it be poſſible, let it proceed in the conic ſpace BCQP according to right lines diverging from the point A. And let us firſt ſuppoſe this motion to be that of waves in the ſurface of ſtanding water; and let de, fg, hi, kl, &c. be the tops of the ſeveral waves, divided from each other by as many intermediate valleys or hollows. Then, becauſe the water in the ridges of the waves is higher than in the unmoved parts of the fluid KL, NO, it will run down from off the tops of thoſe ridges e, g, i, l, &c. d, f, h, k, &c. this way and that way towards KL and NO; and becauſe the water is more depreſſed in the hollows of the waves than in the unmoved parts of the fluid KL, NO, it will run down into thoſe hollows out of thoſe unmoved parts. By the firſt deflux the ridges of the waves will dilate themſelves this way and that way, and be propagated towards KL and NO. And becauſe the motion of the waves from A towards PQ is carried on by a continual deflux from the ridges of the waves into

the hollows next to them, and therefore cannot be ſwifter than in proportion to the celerity of the deſcent; and the deſcent of the water on each ſide towards KL and NO muſt be performed with the ſame velocity; it follows, that the dilatation of the waves on each ſide towards KL and NO will be propagated with the ſame velocity as the waves themſelves go forward with directly from A to PQ. And therefore the whole ſpace this way and that way towards KL and NO will be filled by the dilated waves rfgr, shis, tklt, vmnv, &c. Q.E.D. That theſe things are ſo, any one may find by making the experiment in ſtill water.

CASE 2. Let us ſuppoſe that de, fg, hi, kl, mn, repreſent pulſes ſucceſſively propagated from the point A through an elaſtic medium. Conceive the pulſes to be propagated by ſucceſſive condenſations and rarefactions of the medium, ſo that the denſeſt part of every pulſe may occupy a ſpherical ſuperficies deſcribed about the centre A, and that equal intervals intervene between the ſucceſſive pulſes. Let the lines de, fg, hi, kl, &c. repreſent the denſeſt parts of the pulſes, propagated through the hole BC; and becauſe the medium is denſer there than in the ſpaces on either ſide towards KL and NO, it will dilate itſelf as well towards thoſe ſpaces KL, NO, on each hand, as towards the rare intervals between the pulſes; and thence the medium, becoming always more rare next the intervals, and more denſe next the pulſes, will partake of their motion. And becauſe the progreſſive motion of the pulſes ariſes from the perpetual relaxation of the denſer parts towards the antecedent rare intervals; and ſince the pulſes will relax themſelves on each hand towards the quieſcent parts of the medium KL, NO, with very near the ſame celerity; therefore the pulſes will dilate themſelves on all ſides into the unmoved parts KL, NO, with almoſt the ſame celerity with which they are propagated directly from the centre A; and therefore will fill up the whole ſpace KLON. Q.E.D. And we find the ſame by experience alſo in ſounds which are heard though a mountain interpoſe; and, if they come into a chamber through the window, dilate themſelves into all the parts of the room, and are heard in every corner; and

not as reflected from the oppoſite walls, but directly propagated from the window, as far as our ſenſe can judge.

CASE 3. Let us ſuppoſe, laſtly, that a motion of any kind is propagated from A through the hole BC. Then ſince the cauſe of this propagation is that the parts of the medium that are near the centre A diſturb and agitate thoſe which lie farther from it; and ſince the parts which are urged are fluid, and therefore recede every way towards thoſe ſpaces where they are leſs preſſed, they will by conſequence recede towards all the parts of the quieſcent medium; as well to the parts on each hand, as KL and NO, as to thoſe right before, as PQ: and by this means all the motion, as ſoon as it has paſſed through the hole BC, will begin to dilate itſelf, and from thence, as from its principle and centre, will be propagated directly every way. Q.E.D.

PROPOSITION XLIII. THEOREM XXXIV.

Every tremulous body in an elaſtic medium propagates the motion of the pulſes on every ſide right forward; but in a non-elaſtic medium excites a circular motion.

CASE 1. The parts of the tremulous body alternately going and returning, do in going urge and drive before them thoſe parts of the medium that lie neareſt, and by that impulſe compreſs and condenſe them; and in returning ſuffer thoſe compreſſed parts to recede again, and expand themſelves. Therefore the parts of the medium that lie neareſt to the tremulous body move to and fro by turns, in like manner as the parts of the tremulous body itſelf do; and for the ſame cauſe that the parts of this body agitate theſe parts of the medium, theſe parts, being agitated by like tremors, will in their turn agitate others next to themſelves; and theſe others, agitated in like manner, will agitate thoſe that lie beyond them, and ſo on *in infinitum*. And in the ſame manner as the firſt parts of the medium were condenſed in going, and relaxed in returning, ſo will the other parts be condenſed every time they go, and expand themſelves every time they return. And therefore they will not be all going and all returning at the ſame inſtant (for in that caſe they would always preſerve determined

diſtances from each other, and there could be no alternate condenſation and rarefaction); but ſince, in the places where they are condenſed, they approach to, and, in the places where they are rarefied, recede from each other, therefore ſome of them will be going while others are returning; and ſo on *in infinitum*. The parts ſo going, and in their going condenſed, are pulſes, by reaſon of the progreſſive motion with which they ſtrike obſtacles in their way; and therefore the ſucceſſive pulſes produced by a tremulous body will be propagated in rectilinear directions; and that at nearly equal diſtances from each other, becauſe of the equal intervals of time in which the body, by its ſeveral tremors, produces the ſeveral pulſes. And though the parts of the tremulous body go and return in ſome certain and determinate direction, yet the pulſes propagated from thence through the medium will dilate themſelves towards the ſides, by the foregoing propoſition; and will be propagated on all ſides from that tremulous body, as from a common centre, in ſuperficies nearly ſpherical and concentrical. An example of this we have in waves excited by ſhaking a finger in water, which proceed not only forwards and backwards agreeably to the motion of the finger, but ſpread themſelves in the manner of concentrical circles all round the finger, and are propagated on every ſide. For the gravity of the water ſupplies the place of elaſtic force.

Case 2. If the medium be not elaſtic, then, becauſe its parts cannot be condenſed by the preſſure ariſing from the vibrating parts of the tremulous body, the motion will be propagated in an inſtant towards the parts where the medium yields moſt eaſily, that is, to the parts which the tremulous body would otherwiſe leave vacuous behind it. The caſe is the ſame with that of a body projected in any medium whatever. A medium yielding to projectiles does not recede *in infinitum*, but with a circular motion comes round to the ſpaces which the body leaves behind it. Therefore as often as a tremulous body tends to any part, the medium yielding to it comes round in a circle to the parts which the body leaves; and as often as the body returns to the firſt place, the me-

dium will be driven from the place it came round to, and return to its original place. And though the tremulous body be not firm and hard, but every way flexible, yet if it continue of a given magnitude, since it cannot impel the medium by its tremors any where without yielding to it somewhere else, the medium receding from the parts of the body where it is pressed will always come round in a circle to the parts that yield to it. Q.E.D.

Cor. It is a mistake, therefore, to think, as some have done, that the agitation of the parts of flame conduces to the propagation of a pressure in rectilinear directions through an ambient medium. A pressure of that kind must be derived not from the agitation only of the parts of flame, but from the dilatation of the whole.

PROPOSITION XLIV. THEOREM XXXV.

If water ascend and descend alternately in the erected legs KL, MN, *of a canal or pipe; and a pendulum be constructed whose length between the point of suspension and the centre of oscillation is equal to half the length of the water in the canal; I say, that the water will ascend and descend in the same times in which the pendulum oscillates.* (Pl. 8, Fig. 4.)

I measure the length of the water along the axes of the canal and its legs, and make it equal to the sum of those axes; and take no notice of the resistance of the water arising from its attrition by the sides of the canal. Let, therefore, AB, CD, represent the mean height of the water in both legs; and when the water in the leg KL ascends to the height EF, the water will descend in the leg MN to the height GH. Let P be a pendulous body, VP the thread, V the point of suspension, RPQS the cycloid which the pendulum describes, P its lowest point, PQ an arc equal to the height AE. The force with which the motion of the water is accelerated and retarded alternately is the excess of the weight of the water in one leg above the weight in the other; and, therefore, when the water in the leg KL ascends to EF, and in the other leg descends to GH, that force is double the weight of the water EABF, and therefore is to the weight of the whole water as AE or PQ to VP or PR. The force also with which the body P is ac-

celerated or retarded in any place, as Q, of a cycloid, is (by cor. prop. 51) to its whole weight as its diſtance PQ from the loweſt place P to the length PR of the cycloid. Therefore the motive forces of the water and pendulum, deſcribing the equal ſpaces AE, PQ, are as the weights to be moved; and therefore if the water and pendulum are quieſcent at firſt, thoſe forces will move them in equal times, and will cauſe them to go and return together with a reciprocal motion. Q.E.D.

Cor. 1. Therefore the reciprocations of the water in aſcending and deſcending are all performed in equal times, whether the motion be more or leſs intenſe or remiſs.

Cor. 2. If the length of the whole water in the canal be of $6\frac{1}{3}$ feet of *French* meaſure, the water will deſcend in one ſecond of time, and will aſcend in another ſecond, and ſo on by turns *in infinitum*; for a pendulum of $3\frac{1}{18}$ ſuch feet in length will oſcillate in one ſecond of time.

Cor. 3. But if the length of the water be increaſed or diminiſhed, the time of the reciprocation will be increaſed or diminiſhed in the ſubduplicate ratio of the length.

PROPOSITION XLV. THEOREM XXXVI.

The velocity of waves is in the ſubduplicate ratio of the breadths.

This follows from the conſtruction of the following propoſition.

PROPOSITION XLVI. PROBLEM X.

To find the velocity of waves.

Let a pendulum be conſtructed, whoſe length between the point of ſuſpenſion and the centre of oſcillation is equal to the breadth of the waves; and in the time that the pendulum will perform one ſingle oſcillation the waves will advance forward nearly a ſpace equal to their breadth.

That which I call the breadth of the waves is the tranſverſe meaſure lying between the deepeſt part of the hollows, or the tops of the ridges. Let ABCDEF (Pl. 8, Fig. 5) repreſent the ſurface of ſtagnant water aſcending and deſcending in ſucceſſive waves; and let A, C, E, &c. be the tops of the waves; and let B, D, F, &c. be the intermediate hollows.

Fig. 1.

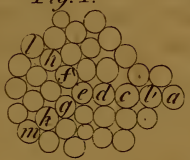


Fig. 2.

Fig. 3.

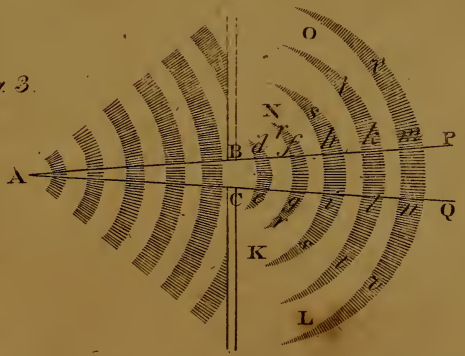
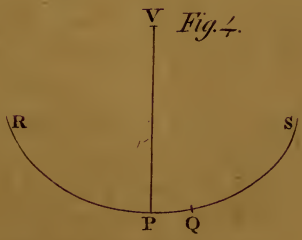


Fig. 4.



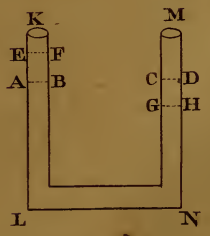
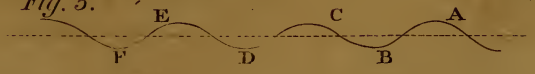


Fig. 5.



Becauſe the motion of the waves is carried on by the ſucceſſive aſcent and deſcent of the water, ſo that the parts thereof, as A, C, E, &c. which are higheſt at one time become loweſt immediately after; and becauſe the motive force, by which the higheſt parts deſcend and the loweſt aſcend, is the weight of the elevated water, that alternate aſcent and deſcent will be analogous to the reciprocal motion of the water in the canal, and obſerve the ſame laws as to the times of its aſcent and deſcent; and therefore (by prop. 44) if the diſtances between the higheſt places of the waves A, C, E, and the loweſt B, D, F, be equal to twice the length of any pendulum, the higheſt parts A, C, E, will become the loweſt in the time of one oſcillation, and in the time of another oſcillation will aſcend again. Therefore between the paſſage of each wave the time of two oſcillations will intervene; that is, the wave will deſcribe its breadth in the time that pendulum will oſcillate twice; but a pendulum of four times that length, and which therefore is equal to the breadth of the waves, will juſt oſcillate once in that time. Q.E.I.

Cor. 1. Therefore waves, whoſe breadth is equal to $3\frac{1}{18}$ *French* feet, will advance through a ſpace equal to their breadth in one ſecond of time; and therefore in one minute will go over a ſpace of $183\frac{1}{3}$ feet; and in an hour a ſpace of 11000 feet, nearly.

Cor. 2. And the velocity of greater or leſs waves will be augmented or diminiſhed in the ſubduplicate ratio of their breadth.

Theſe things are true upon the ſuppoſition that the parts of water aſcend or deſcend in a right line; but, in truth, that aſcent and deſcent is rather performed in a circle; and therefore I propoſe the time defined by this propoſition as only near the truth.

PROPOSITION XLVII. THEOREM XXXVII.

If pulſes are propagated through a fluid, the ſeveral particles of the fluid, going and returning with the ſhorteſt reciprocal motion, are always accelerated or retarded according to the law of the oſcillating pendulum. (Pl. 9, Fig. 1.)

Let AB, BC, CD, &c. repreſent equal diſtances of ſucceſſive pulſes; ABC the line of direction of the motion of the ſucceſſive pulſes propagated from A to B; E, F, G three phyſical points of the quieſcent medium ſituate in the right line AC at equal diſtances from each other; Ee, Ff, Gg, equal ſpaces of extreme ſhortneſs, through which thoſe points go and return with a reciprocal motion in each vibration; ε, ϕ, γ, any intermediate places of the ſame points; EF, FG phyſical lineolæ, or linear parts of the medium lying between thoſe points, and ſucceſſively transfered into the places $\varepsilon\phi$, $\phi\gamma$, and ef, fg. Let there be drawn the right line PS equal to the right line Ee. Biſect the ſame in O, and from the centre O, with the interval OP, deſcribe the circle SIPi. Let the whole time of one vibration, with its proportional parts, be expounded by the whole circumference of this circle and its parts, in ſuch ſort, that, when any time PH or PHSh is completed, if there be let fall to PS the perpendicular HL or hl, and there be taken Eε equal to PL or Pl, the phyſical point E may be found in ε. A point, as E, moving according to this law with a reciprocal motion, in its going from E through ε to e, and returning again through ε to E, will perform its ſeveral vibrations with the ſame degrees of acceleration and retardation with thoſe of an oſcillating pendulum. We are now to prove that the ſeveral phyſical points of the medium will be agitated with ſuch a kind of motion. Let us ſuppoſe, then, that a medium hath ſuch a motion excited in it from any cauſe whatſoever, and conſider what will follow from thence.

In the circumference PHSh let there be taken the equal arcs HI,IK, or hi, ik, having the ſame ratio to the whole circumference as the equal right lines EF, FG have to BC, the whole interval of the pulſes. Let fall the perpendiculars IM, KN, or im, kn; then becauſe the points E, F, G are ſucceſſively agitated with like motions, and perform their entire vibrations compoſed of their going and return, while the pulſe is transfered from B to C; if PH or PHSh be the time elapſed ſince the beginning of the motion of the point E, then will PI or PHSi be the time elapſed ſince the beginning of the

motion of the point F, and PK or PHSk the time elapsed since the beginning of the motion of the point G; and therefore $E\varepsilon$, $F\phi$, $G\gamma$, will be respectively equal to PL, PM, PN, while the points are going, and to Pl, Pm, Pn, when the points are returning. Therefore $\varepsilon\gamma$ or EG + Gγ — Eε will, when the points are going, be equal to EG — LN, and in their return equal to EG + ln. But $\varepsilon\gamma$ is the breadth or expansion of the part EG of the medium in the place $\varepsilon\gamma$; and therefore the expansion of that part in its going is to its mean expansion as EG — LN to EG; and in its return, as EG + ln or EG + LN to EG. Therefore since LN is to KH as IM to the radius OP, and KH to EG as the circumference PHShP to BC; that is, if we put V for the radius of a circle whose circumference is equal to BC the interval of the pulses, as OP to V; and, *ex æquo*, LN to EG as IM to V; the expansion of the part EG, or of the physical point F in the place $\varepsilon\gamma$, to the mean expansion of the same part in its first place EG, will be as V — IM to V in going, and as V + im to V in its return. Hence the elastic force of the point F in the place $\varepsilon\gamma$ to its mean elastic force in the place EG is as $\frac{1}{V-IM}$ to $\frac{1}{V}$ in its going, and as $\frac{1}{V+im}$ to $\frac{1}{V}$ in its return. And by the same reasoning the elastic forces of the physical points E and G in going are as $\frac{1}{V-HL}$ and $\frac{1}{V-KN}$ to $\frac{1}{V}$; and the difference of the forces to the mean elastic force of the medium as $\frac{HL-KN}{VV-V\times HL-V\times KN+HL\times KN}$ to $\frac{1}{V}$; that is, as $\frac{HL-KN}{VV}$ to $\frac{1}{V}$, or as HL—KN to V; if we suppose (by reason of the very short extent of the vibrations) HL and KN to be indefinitely less than the quantity V. Therefore since the quantity V is given, the difference of the forces is as HL — KN; that is (because HL — KN is proportional to HK, and OM to OI or OP; and because HK and OP are given) as OM; that is, if Ff be bisected in Ω, as $\Omega\phi$. And for the same reason the difference of the elastic forces of the physical points ε and γ, in the return of the physical lineolæ $\varepsilon\gamma$, is as

$\Omega\Phi$. But that difference (that is, the exceſs of the elaſtic force of the point ε above the elaſtic force of the point γ) is the very force by which the intervening phyſical lineolæ $\varepsilon\gamma$ of the medium is accelerated in going, and retarded in returning; and therefore the accelerative force of the phyſical lineolæ $\varepsilon\gamma$ is as its diſtance from Ω, the middle place of the vibration. Therefore (by prop. 38, book 1) the time is rightly expounded by the arc PI; and the linear part of the medium $\varepsilon\gamma$ is moved according to the law above-mentioned, that is, according to the law of a pendulum oſcillating; and the caſe is the ſame of all the linear parts of which the whole medium is compounded. Q.E.D.

Cor. Hence it appears that the number of the pulſes propagated is the ſame with the number of the vibrations of the tremulous body, and is not multiplied in their progreſs. For the phyſical lineolæ $\varepsilon\gamma$ as ſoon as it returns to its firſt place is at reſt; neither will it move again, unleſs it receives a new motion either from the impulſe of the tremulous body, or of the pulſes propagated from that body. As ſoon, therefore, as the pulſes ceaſe to be propagated from the tremulous body, it will return to a ſtate of reſt, and move no more.

PROPOSITION XLVIII. THEOREM XXXVIII.

The velocities of pulſes propagated in an elaſtic fluid are in a ratio compounded of the ſubduplicate ratio of the elaſtic force directly, and the ſubduplicate ratio of the denſity inverſely; ſuppoſing the elaſtic force of the fluid to be proportional to its condenſation.

Case 1. If the mediums be homogeneous, and the diſtances of the pulſes in thoſe mediums be equal amongſt themſelves, but the motion in one medium is more intenſe than in the other, the contractions and dilatations of the correſpondent parts will be as thoſe motions: not that this proportion is perfectly accurate. However, if the contractions and dilatations are not exceedingly intenſe, the error will not be ſenſible; and therefore this proportion may be conſidered as phyſically exact. Now the motive elaſtic forces are as the contractions and dilatations; and the velocities generated in the ſame time in equal parts are as the forces.

Therefore equal and correſponding parts of correſponding pulſes will go and return together, through ſpaces proportional to their contractions and dilatations, with velocities that are as thoſe ſpaces; and therefore the pulſes, which in the time of one going and returning advance forwards a ſpace equal to their breadth, and are always ſucceeding into the places of the pulſes that immediately go before them, will, by reaſon of the equality of the diſtances, go forward in both mediums with equal velocity.

Case 2. If the diſtances of the pulſes or their lengths are greater in one medium than in another, let us ſuppoſe that the correſpondent parts deſcribe ſpaces, in going and returning, each time proportional to the breadths of the pulſes; then will their contractions and dilatations be equal; and therefore if the mediums are homogeneous, the motive elaſtic forces, which agitate them with a reciprocal motion, will be equal alſo. Now the matter to be moved by theſe forces is as the breadth of the pulſes; and the ſpace through which they move every time they go and return is in the ſame ratio. And, moreover, the time of one going and returning is in a ratio compounded of the ſubduplicate ratio of the matter, and the ſubduplicate ratio of the ſpace; and therefore is as the ſpace. But the pulſes advance a ſpace equal to their breadths in the times of going once and returning once; that is, they go over ſpaces proportional to the times, and therefore are equally ſwift.

Case 3. And therefore in mediums of equal denſity and elaſtic force, all the pulſes are equally ſwift. Now if the denſity or the elaſtic force of the medium were augmented, then, becauſe the motive force is increaſed in the ratio of the elaſtic force, and the matter to be moved is increaſed in the ratio of the denſity, the time which is neceſſary for producing the ſame motion as before will be increaſed in the ſubduplicate ratio of the denſity, and will be diminiſhed in the ſubduplicate ratio of the elaſtic force. And therefore the velocity of the pulſes will be in a ratio compounded of the ſubduplicate ratio of the denſity of the medium inverſely, and the ſubduplicate ratio of the elaſtic force directly. Q.E.D.

This propoſition will be made more clear from the conſtruction of the following problem.

PROPOSITION XLIX. PROBLEM XI.

The denſity and elaſtic force of a medium being given, to find the velocity of the pulſes.

Suppoſe the medium to be preſſed by an incumbent weight after the manner of our air; and let A be the height of a homogeneous medium, whoſe weight is equal to the incumbent weight, and whoſe denſity is the ſame with the denſity of the compreſſed medium in which the pulſes are propagated. Suppoſe a pendulum to be conſtructed whoſe length between the point of ſuſpenſion and the centre of oſcillation is A: and in the time in which that pendulum will perform one entire oſcillation compoſed of its going and returning, the pulſe will be propagated right onwards through a ſpace equal to the circumference of a circle deſcribed with the radius A.

For, letting thoſe things ſtand which were conſtructed in prop. 47, if any phyſical line, as EF (Pl. 9, Fig. 1), deſcribing the ſpace PS in each vibration, be acted on in the extremities P and S of every going and return that it makes by an elaſtic force that is equal to its weight, it will perform its ſeveral vibrations in the time in which the ſame might oſcillate in a cycloid whoſe whole perimeter is equal to the length PS; and that becauſe equal forces will impel equal corpuſcles through equal ſpaces in the ſame or equal times. Therefore ſince the times of the oſcillations are in the ſubduplicate ratio of the lengths of the pendulums, and the length of the pendulum is equal to half the arc of the whole cycloid, the time of one vibration would be to the time of the oſcillation of a pendulum whoſe length is A in the ſubduplicate ratio of the length $\frac{1}{2}$PS or PO to the length A. But the elaſtic force with which the phyſical lineolæ EG is urged, when it is found in its extreme places P, S, was (in the demonſtration of prop. 47) to its whole elaſtic force as HL — KN to V, that is (ſince the point K now falls upon P), as HK to V: and all that force, or, which is the ſame thing, the incumbent weight by which the lineolæ EG is compreſſed, is to the weight of the lineolæ as the altitude A of the incumbent weight to EG the length

of the lineolæ; and therefore, *ex æquo*, the force with which the lineolæ EG is urged in the places P and S is to the weight of that lineolæ as HK × A to V × EG; or as PO × A to VV; becauſe HK was to EG as PO to V. Therefore ſince the times in which equal bodies are impelled through equal ſpaces are reciprocally in the ſubduplicate ratio of the forces, the time of one vibration, produced by the action of that elaſtic force, will be to the time of a vibration, produced by the impulſe of the weight in a ſubduplicate ratio of VV to PO × A, and therefore to the time of the oſcillation of a pendulum whoſe length is A in the ſubduplicate ratio of VV to PO × A, and the ſubduplicate ratio of PO to A conjunctly; that is, in the entire ratio of V to A. But in the time of one vibration compoſed of the going and returning of the pendulum, the pulſe will be propagated right onwards through a ſpace equal to its breadth BC. Therefore the time in which a pulſe runs over the ſpace BC is to the time of one oſcillation compoſed of the going and returning of the pendulum as V to A, that is, as BC to the circumference of a circle whoſe radius is A. But the time in which the pulſe will run over the ſpace BC is to the time in which it will run over a length equal to that circumference in the ſame ratio; and therefore in the time of ſuch an oſcillation the pulſe will run over a length equal to that circumference. Q.E.D.

Cor. 1. The velocity of the pulſes is equal to that which heavy bodies acquire by falling with an equally accelerated motion, and in their fall deſcribing half the altitude A. For the pulſe will, in the time of this fall, ſuppoſing it to move with the velocity acquired by that fall, run over a ſpace that will be equal to the whole altitude A; and therefore in the time of one oſcillation compoſed of one going and return, will go over a ſpace equal to the circumference of a circle deſcribed with the radius A; for the time of the fall is to the time of oſcillation as the radius of a circle to its circumference.

Cor. 2. Therefore ſince that altitude A is as the elaſtic force of the fluid directly, and the denſity of the ſame inverſely, the velocity of the pulſes will be in a ratio compounded

of the ſubduplicate ratio of the denſity inverſely, and the ſubduplicate ratio of the elaſtic force directly.

PROPOSITION L. PROBLEM XII.

To find the diſtances of the pulſes.

Let the number of the vibrations of the body, by whoſe tremor the pulſes are produced, be found to any given time. By that number divide the ſpace which a pulſe can go over in the ſame time, and the part found will be the breadth of one pulſe. Q.E.I.

SCHOLIUM.

The laſt propoſitions reſpect the motions of light and ſounds; for ſince light is propagated in right lines, it is certain that it cannot conſiſt in action alone (by prop. 41 and 42). As to ſounds, ſince they ariſe from tremulous bodies, they can be nothing elſe but pulſes of the air propagated through it (by prop. 43); and this is confirmed by the tremors which ſounds, if they be loud and deep, excite in the bodies near them, as we experience in the ſound of drums; for quick and ſhort tremors are leſs eaſily excited. But it is well known that any ſounds, falling upon ſtrings in uniſon with the ſonorous bodies, excite tremors in thoſe ſtrings. This is alſo confirmed from the velocity of ſounds; for ſince the ſpecific gravities of rain-water and quickſilver are to one another as about 1 to $13\frac{2}{3}$, and when the mercury in the barometer is at the height of 30 inches of our meaſure, the ſpecific gravities of the air and of rain-water are to one another as about 1 to 870, therefore the ſpecific gravity of air and quickſilver are to each other as 1 to 11890. Therefore when the height of the quickſilver is at 30 inches, a height of uniform air, whoſe weight would be ſufficient to compreſs our air to the denſity we find it to be of, muſt be equal to 356700 inches, or 29725 feet of our meaſure: and this is that very height of the medium, which I have called A in the conſtruction of the foregoing propoſition. A circle whoſe radius is 29725 feet is 186768 feet in circumference. And ſince a pendulum $39\frac{1}{5}$ inches in length completes one oſcillation, compoſed of its going and return, in two ſeconds of time, as is commonly known, it follows that a pendulum

29725 feet, or 356700 inches in length will perform a like oſcillation in 190¾ ſeconds. Therefore in that time a ſound will go right onwards 186768 feet, and therefore in one ſecond 979 feet.

But in this computation we have made no allowance for the craſſitude of the ſolid particles of the air, by which the ſound is propagated inſtantaneouſly. Becauſe the weight of air is to the weight of water as 1 to 870, and becauſe ſalts are almoſt twice as denſe as water; if the particles of air are ſuppoſed to be of near the ſame denſity as thoſe of water or ſalt, and the rarity of the air ariſes from the intervals of the particles; the diameter of one particle of air will be to the interval between the centres of the particles as 1 to about 9 or 10, and to the interval between the particles themſelves as 1 to 8 or 9. Therefore to 979 feet, which, according to the above calculation, a ſound will advance forward in one ſecond of time, we may add $\frac{979}{9}$, or about 109 feet, to compenſate for the craſſitude of the particles of the air: and then a ſound will go forward about 1088 feet in one ſecond of time.

Moreover, the vapours floating in the air being of another ſpring, and a different tone, will hardly, if at all, partake of the motion of the true air in which the ſounds are propagated. Now if theſe vapours remain unmoved, that motion will be propagated the ſwifter through the true air alone, and that in the ſubduplicate ratio of the defect of the matter. So if the atmoſphere conſiſt of ten parts of true air and one part of vapours, the motion of ſounds will be ſwifter in the ſubduplicate ratio of 11 to 10, or very nearly in the entire ratio of 21 to 20, than if it were propagated through eleven parts of true air: and therefore the motion of ſounds above diſcovered muſt be increaſed in that ratio. By this means the ſound will paſs through 1142 feet in one ſecond of time.

Theſe things will be found true in ſpring and autumn, when the air is rarefied by the gentle warmth of thoſe ſeaſons, and by that means its elaſtic force becomes ſomewhat more intenſe. But in winter, when the air is condenſed by the cold, and its elaſtic force is ſomewhat remitted, the motion

of ſounds will be ſlower in a ſubduplicate ratio of the denſity; and, on the other hand, ſwifter in the ſummer.

Now by experiments it actually appears that ſounds do really advance in one ſecond of time about 1142 feet of *Engliſh* meaſure, or 1070 feet of *French* meaſure.

The velocity of ſounds being known, the intervals of the pulſes are known alſo. For M. *Sauveur*, by ſome experiments that he made, found that an open pipe about five *Paris* feet in length gives a ſound of the ſame tone with a viol-ſtring that vibrates a hundred times in one ſecond. Therefore there are near 100 pulſes in a ſpace of 1070 *Paris* feet, which a ſound runs over in a ſecond of time; and therefore one pulſe fills up a ſpace of about $10\frac{7}{10}$ *Paris* feet, that is, about twice the length of the pipe. From whence it is probable that the breadths of the pulſes, in all ſounds made in open pipes, are equal to twice the length of the pipes.

Moreover, from the corollary of prop. 47 appears the reaſon why the ſounds immediately ceaſe with the motion of the ſonorous body, and why they are heard no longer when we are at a great diſtance from the ſonorous bodies than when we are very near them. And beſides, from the foregoing principles, it plainly appears how it comes to paſs that ſounds are ſo mightily increaſed in ſpeaking-trumpets; for all reciprocal motion uſes to be increaſed by the generating cauſe at each return. And in tubes hindering the dilatation of the ſounds, the motion decays more ſlowly, and recurs more forcibly; and therefore is the more increaſed by the new motion impreſſed at each return. And theſe are the principal phænomena of ſounds.

SECTION IX.

Of the circular motion of fluids.

HYPOTHESIS.

The reſiſtance ariſing from the want of lubricity in the parts of a fluid, is, cæteris paribus, *proportional to the velocity with which the parts of the fluid are ſeparated from each other.*

PROPOSITION LI. THEOREM XXXVIII.

If a ſolid cylinder infinitely long, in an uniform and infinite fluid, revolve with an uniform motion about an axis given in poſition, and the fluid be forced round by only this impulſe of the cylinder, and every part of the fluid perſevere uniformly in its motion; I ſay, that the periodic times of the parts of the fluid are as their diſtances from the axis of the cylinder.

Let AFL (Pl. 9, Fig. 2) be a cylinder turning uniformly about the axis S, and let the concentric circles BGM, CHN, DIO, EKP, &c. divide the fluid into innumerable concentric cylindric ſolid orbs of the ſame thickneſs. Then, becauſe the fluid is homogeneous, the impreſſions which the contiguous orbs make upon each other mutually will be (by the hypotheſis) as their tranſlations from each other, and as the contiguous ſuperficies upon which the impreſſions are made. If the impreſſion made upon any orb be greater or leſs on its concave than on its convex ſide, the ſtronger impreſſion will prevail, and will either accelerate or retard the motion of the orb, according as it agrees with, or is contrary to, the motion of the ſame. Therefore, that every orb may perſevere uniformly in its motion, the impreſſions made on both ſides muſt be equal, and their directions contrary. Therefore ſince the impreſſions are as the contiguous ſuperficies, and as their tranſlations from one another, the tranſlations will be inverſely as the ſuperficies, that is, inverſely as the diſtances of the ſuperficies from the axis. But the differences of the angular motions about the axis are as thoſe tranſlations applied to the diſtances, or as the tranſlations directly and the diſtances inverſely; that is, joining theſe ratios together, as the ſquares of the diſtances inverſely. Therefore if there be erected the lines Aa, Bb, Cc, Dd, Ee, &c. perpendicular to the ſeveral parts of the infinite right line SABCDEQ, and reciprocally proportional to the ſquares of SA, SB, SC, SD, SE, &c. and through the extremities of thoſe perpendiculars there be ſuppoſed to paſs an hyperbolic curve, the ſums of the differences, that is, the whole angular motions, will be as the correſpondent ſums of the lines Aa, Bb, Cc, Dd, Ee, that

is (if to conſtitute a medium uniformly fluid the number of the orbs be increaſed and their breadth diminiſhed *in infinitum*), as the hyperbolic areas AaQ, BbQ, CcQ, DdQ, EeQ, &c. analogous to the ſums; and the times, reciprocally proportional to the angular motions, will be alſo reciprocally proportional to thoſe areas. Therefore the periodic time of any particle, as D, is reciprocally as the area DdQ, that is (as appears from the known methods of quadratures of curves), directly as the diſtance SD. Q.E.D.

Cor. 1. Hence the angular motions of the particles of the fluid are reciprocally as their diſtances from the axis of the cylinder, and the abſolute velocities are equal.

Cor. 2. If a fluid be contained in a cylindric veſſel of an infinite length, and contain another cylinder within, and both the cylinders revolve about one common axis, and the times of their revolutions be as their ſemi-diameters, and every part of the fluid perſeveres in its motion, the periodic times of the ſeveral parts will be as the diſtances from the axis of the cylinders.

Cor. 3. If there be added or taken away any common quantity of angular motion from the cylinder and fluid moving in this manner; yet, becauſe this new motion will not alter the mutual attrition of the parts of the fluid, the motion of the parts among themſelves will not be changed; for the tranſlations of the parts from one another depend upon the attrition. Any part will perſevere in that motion, which, by the attrition made on both ſides with contrary directions, is no more accelerated than it is retarded.

Cor. 4. Therefore if there be taken away from this whole ſyſtem of the cylinders and the fluid all the angular motion of the outward cylinder, we ſhall have the motion of the fluid in a quieſcent cylinder.

Cor. 5. Therefore if the fluid and outward cylinder are at reſt, and the inward cylinder revolve uniformly, there will be communicated a circular motion to the fluid, which will be propagated by degrees through the whole fluid; and will go on continually increaſing, till ſuch time as the ſeveral parts of the fluid acquire the motion determined in cor. 4.

Cor. 6. And becauſe the fluid endeavours to propagate its motion ſtill farther, its impulſe will carry the outmoſt cylinder alſo about with it, unleſs the cylinder be violently detained; and accelerate its motion till the periodic times of both cylinders become equal among themſelves. But if the outward cylinder be violently detained, it will make an effort to retard the motion of the fluid; and unleſs the inward cylinder preſerve that motion by means of ſome external force impreſſed thereon, it will make it ceaſe by degrees.

All theſe things will be found true by making the experiment in deep ſtanding water.

PROPOSITION LII. THEOREM XL.

If a ſolid ſphere, in an uniform and infinite fluid, revolves about an axis given in poſition with an uniform motion, and the fluid be forced round by only this impulſe of the ſphere; and every part of the fluid perſeveres uniformly in its motion: I ſay, that the periodic times of the parts of the fluid are as the ſquares of their diſtances from the centre of the ſphere.

Case 1. Let AFL be a ſphere turning uniformly about the axis S, and let the concentric circles BGM, CHN, DIO, EKP, &c. divide the fluid into innumerable concentric orbs of the ſame thickneſs. Suppoſe thoſe orbs to be ſolid; and, becauſe the fluid is homogeneous, the impreſſions which the contiguous orbs make one upon another will be (by the ſuppoſition) as their tranſlations from one another, and the contiguous ſuperficies upon which the impreſſions are made. If the impreſſion upon any orb be greater or leſs upon its concave than upon its convex ſide, the more forcible impreſſion will prevail, and will either accelerate or retard the velocity of the orb, according as it is directed with a conſpiring or contrary motion to that of the orb. Therefore that every orb may perſevere uniformly in its motion, it is neceſſary that the impreſſions made upon both ſides of the orb ſhould be equal, and have contrary directions. Therefore ſince the impreſſions are as the contiguous ſuperficies, and as their tranſlations from one another, the tranſlations will be inverſely as the ſuperficies, that is, inverſely as the ſquares of the diſtances of the ſuperficies from the centre. But the differences of the angular motions about the axis are as thoſe tranſlations ap-

plied to the diſtances, or as the tranſlations directly and the diſtances inverſely; that is, by compounding thoſe ratios, as the cubes of the diſtances inverſely. Therefore if upon the ſeveral parts of the infinite right line SABCDEQ there be erected the perpendiculars Aa, Bb, Cc, Dd, Ee, &c. reciprocally proportional to the cubes of SA, SB, SC, SD, SE, &c. the ſums of the differences, that is, the whole angular motions, will be as the correſponding ſums of the lines Aa, Bb, Cc, Dd, Ee, &c. that is (if to conſtitute an uniformly fluid medium the number of the orbs be increaſed and their thickneſs diminiſhed *in infinitum*), as the hyperbolic areas AaQ, BbQ, CcQ, DdQ, EeQ, &c. analogous to the ſums; and the periodic times being reciprocally proportional to the angular motions, will be alſo reciprocally proportional to thoſe areas. Therefore the periodic time of any orb DIO is reciprocally as the area DdQ, that is (by the known methods of quadratures), directly as the ſquare of the diſtance SD. Which was firſt to be demonſtrated.

Case 2. From the centre of the ſphere let there be drawn a great number of indefinite right lines, making given angles with the axis, exceeding one another by equal differences; and, by theſe lines revolving about the axis, conceive the orbs to be cut into innumerable annuli; then will every annulus have four annuli contiguous to it, that is, one on its inſide, one on its outſide, and two on each hand. Now each of theſe annuli cannot be impelled equally and with contrary directions by the attrition of the interior and exterior annuli, unleſs the motion be communicated according to the law which we demonſtrated in caſe 1. This appears from that demonſtration. And therefore any ſeries of annuli, taken in any right line extending itſelf *in infinitum* from the globe, will move according to the law of caſe 1, except we ſhould imagine it hindered by the attrition of the annuli on each ſide of it. But now in a motion, according to this law, no ſuch attrition is, and therefore cannot be, any obſtacle to the motions perſevering according to that law. If annuli at equal diſtances from the centre revolve either more ſwiftly or more ſlowly near the poles than near the ecliptic, they will be accelerated if ſlow, and retarded if ſwift, by their mutual attri-

tion; and ſo the periodic times will continually approach to equality, according to the law of caſe 1. Therefore this attrition will not at all hinder the motion from going on according to the law of caſe 1, and therefore that law will take place; that is, the periodic times of the ſeveral annuli will be as the ſquares of their diſtances from the centre of the globe. Which was to be demonſtrated in the ſecond place.

Case 3. Let now every annulus be divided by tranſverſe ſections into innumerable particles conſtituting a ſubſtance abſolutely and uniformly fluid; and becauſe theſe ſections do not at all reſpect the law of circular motion, but only ſerve to produce a fluid ſubſtance, the law of circular motion will continue the ſame as before. All the very ſmall annuli will either not at all change their aſperity and force of mutual attrition upon account of theſe ſections, or elſe they will change the ſame equally. Therefore the proportion of the cauſes remaining the ſame, the proportion of the effects will remain the ſame alſo; that is, the proportion of the motions and the periodic times. Q.E.D. But now as the circular motion, and the centrifugal force thence ariſing, is greater at the ecliptic than at the poles, there muſt be ſome cauſe operating to retain the ſeveral particles in their circles; otherwiſe the matter that is at the ecliptic will always recede from the centre, and come round about to the poles by the outſide of the vortex, and from thence return by the axis to the ecliptic with a perpetual circulation.

Cor. 1. Hence the angular motions of the parts of the fluid about the axis of the globe are reciprocally as the ſquares of the diſtances from the centre of the globe, and the abſolute velocities are reciprocally as the ſame ſquares applied to the diſtances from the axis.

Cor. 2. If a globe revolve with a uniform motion about an axis of a given poſition in a ſimilar and infinite quieſcent fluid with an uniform motion, it will communicate a whirling motion to the fluid like that of a vortex, and that motion will by degrees be propagated onwards *in infinitum;* and this motion will be increaſed continually in every part of the fluid,

till the periodical times of the ſeveral parts become as the ſquares of the diſtances from the centre of the globe.

Cor. 3. Becauſe the inward parts of the vortex are by reaſon of their greater velocity continually preſſing upon and driving forwards the external parts, and by that action are perpetually communicating motion to them, and at the ſame time thoſe exterior parts communicate the ſame quantity of motion to thoſe that lie ſtill beyond them, and by this action preſerve the quantity of their motion continually unchanged, it is plain that the motion is perpetually transfered from the centre to the circumference of the vortex, till it is quite ſwallowed up and loſt in the boundleſs extent of that circumference. The matter between any two ſpherical ſuperficies concentrical to the vortex will never be accelerated; becauſe that matter will be always transfering the motion it receives from the matter nearer the centre to that matter which lies nearer the circumference.

Cor. 4. Therefore, in order to continue a vortex in the ſame ſtate of motion, ſome active principle is required from which the globe may receive continually the ſame quantity of motion which it is always communicating to the matter of the vortex. Without ſuch a principle it will undoubtedly come to paſs that the globe and the inward parts of the vortex, being always propagating their motion to the outward parts, and not receiving any new motion, will gradually move ſlower and ſlower, and at laſt be carried round no longer.

Cor. 5. If another globe ſhould be ſwimming in the ſame vortex at a certain diſtance from its centre, and in the mean time by ſome force revolve conſtantly about an axis of a given inclination, the motion of this globe will drive the fluid round after the manner of a vortex; and at firſt this new and ſmall vortex will revolve with its globe about the centre of the other; and in the mean time its motion will creep on farther and farther, and by degrees be propagated *in infinitum*, after the manner of the firſt vortex. And for the ſame reaſon that the globe of the new vortex was carried about before by the motion of the other vortex, the globe of this other will be carried about by the motion of this new vortex,

ſo that the two globes will revolve about ſome intermediate point, and by reaſon of that circular motion mutually fly from each other, unleſs ſome force reſtrains them. Afterwards, if the conſtantly impreſſed forces, by which the globes perſevere in their motions, ſhould ceaſe, and every thing be left to act according to the laws of mechanics, the motion of the globes will languiſh by degrees (for the reaſon aſſigned in cor. 3 and 4), and the vortices at laſt will quite ſtand ſtill.

Cor. 6. If ſeveral globes in given places ſhould conſtantly revolve with determined velocities about axes given in poſition, there would ariſe from them as many vortices going on *in infinitum*. For upon the ſame account that any one globe propagates its motion *in infinitum*, each globe apart will propagate its own motion *in infinitum* alſo; ſo that every part of the infinite fluid will be agitated with a motion reſulting from the actions of all the globes. Therefore the vortices will not be confined by any certain limits, but by degrees run mutually into each other; and by the mutual actions of the vortices on each other, the globes will be perpetually moved from their places, as was ſhewn in the laſt corollary; neither can they poſſibly keep any certain poſition among themſelves, unleſs ſome force reſtrains them. But if thoſe forces, which are conſtantly impreſſed upon the globes to continue theſe motions, ſhould ceaſe, the matter (for the reaſon aſſigned in cor. 3 and 4) will gradually ſtop, and ceaſe to move in vortices.

Cor. 7. If a ſimilar fluid be incloſed in a ſpherical veſſel, and, by the uniform rotation of a globe in its centre, is driven round in a vortex; and the globe and veſſel revolve the ſame way about the ſame axis, and their periodical times be as the ſquares of the ſemi-diameters; the parts of the fluid will not go on in their motions without acceleration or retardation, till their periodical times are as the ſquares of their diſtances from the centre of the vortex. No conſtitution of a vortex can be permanent but this.

Cor. 8. If the veſſel, the incloſed fluid, and the globe, retain this motion, and revolve beſides with a common angular motion about any given axis, becauſe the mutual attri-

tion of the parts of the fluid is not changed by this motion, the motions of the parts among each other will not be changed; for the tranſlations of the parts among themſelves depend upon this attrition. Any part will perſevere in that motion in which its attrition on one ſide retards it juſt as much as its attrition on the other ſide accelerates it.

Cor. 9. Therefore if the veſſel be quieſcent, and the motion of the globe be given, the motion of the fluid will be given. For conceive a plane to paſs through the axis of the globe, and to revolve with a contrary motion; and ſuppoſe the ſum of the time of this revolution and of the revolution of the globe to be to the time of the revolution of the globe as the ſquare of the ſemi-diameter of the veſſel to the ſquare of the ſemi-diameter of the globe; and the periodic times of the parts of the fluid in reſpect of this plane will be as the ſquares of their diſtances from the centre of the globe.

Cor. 10. Therefore if the veſſel move about the ſame axis with the globe, or with a given velocity about a different one, the motion of the fluid will be given. For if from the whole ſyſtem we take away the angular motion of the veſſel, all the motions will remain the ſame among themſelves as before, by cor. 8, and thoſe motions will be given by cor. 9.

Cor. 11. If the veſſel and the fluid are quieſcent, and the globe revolves with an uniform motion, that motion will be propagated by degrees through the whole fluid to the veſſel, and the veſſel will be carried round by it, unleſs violently detained; and the fluid and the veſſel will be continually accelerated till their periodic times become equal to the periodic times of the globe. If the veſſel be either withheld by ſome force, or revolve with any conſtant and uniform motion, the medium will come by little and little to the ſtate of motion defined in cor. 8, 9, 10, nor will it ever perſevere in any other ſtate. But if then the forces, by which the globe and veſſel revolve with certain motions, ſhould ceaſe, and the whole ſyſtem be left to act according to the mechanical laws, the veſſel and globe, by means of the intervening fluid, will act upon each other, and will continue to propagate their motions

through the fluid to each other, till their periodic times become equal among themſelves, and the whole ſyſtem revolves together like one ſolid body.

SCHOLIUM.

In all theſe reaſonings I ſuppoſe the fluid to conſiſt of matter of uniform denſity and fluidity; I mean, that the fluid is ſuch, that a globe placed any where therein may propagate with the ſame motion of its own, at diſtances from itſelf continually equal, ſimilar and equal motions in the fluid in the ſame interval of time. The matter by its circular motion endeavours to recede from the axis of the vortex, and therefore preſſes all the matter that lies beyond. This preſſure makes the attrition greater, and the ſeparation of the parts more difficult; and by conſequence diminiſhes the fluidity of the matter. Again; if the parts of the fluid are in any one place denſer or larger than in the others, the fluidity will be leſs in that place, becauſe there are fewer ſuperficies where the parts can be ſeparated from each other. In theſe caſes I ſuppoſe the defect of the fluidity to be ſupplied by the ſmoothneſs or ſoftneſs of the parts, or ſome other condition; otherwiſe the matter where it is leſs fluid will cohere more, and be more ſluggiſh, and therefore will receive the motion more ſlowly, and propagate it farther than agrees with the ratio above aſſigned. If the veſſel be not ſpherical, the particles will move in lines not circular, but anſwering to the figure of the veſſel; and the periodic times will be nearly as the ſquares of the mean diſtances from the centre. In the parts between the centre and the circumference the motions will be ſlower where the ſpaces are wide, and ſwifter where narrow; but yet the particles will not tend to the circumference at all the more for their greater ſwiftneſs; for they then deſcribe arcs of leſs curvity, and the conatus of receding from the centre is as much diminiſhed by the diminution of this curvature as it is augmented by the increaſe of the velocity. As they go out of narrow into wide ſpaces, they recede a little farther from the centre, but in doing ſo are retarded; and when they come out of wide into narrow ſpaces, they are again accelerated; and ſo each particle is retarded and accelerated by turns for

ever. Theſe things will come to paſs in a rigid veſſel; for the ſtate of vortices in an infinite fluid is known by cor. 6 of this propoſition.

I have endeavoured in this propoſition to inveſtigate the properties of vortices, that I might find whether the celeſtial phænomena can be explained by them; for the phænomenon is this, that the periodic times of the planets revolving about Jupiter are in the ſeſquiplicate ratio of their diſtances from Jupiter's centre; and the ſame rule obtains alſo among the planets that revolve about the ſun. And theſe rules obtain alſo with the greateſt accuracy, as far as has been yet diſcovered by aſtronomical obſervation. Therefore if thoſe planets are carried round in vortices revolving about Jupiter and the ſun, the vortices muſt revolve according to that law. But here we found the periodic times of the parts of the vortex to be in the duplicate ratio of the diſtances from the centre of motion; and this ratio cannot be diminiſhed and reduced to the ſeſquiplicate, unleſs either the matter of the vortex be more fluid the farther it is from the centre, or the reſiſtance ariſing from the want of lubricity in the parts of the fluid ſhould, as the velocity with which the parts of the fluid are ſeparated goes on increaſing, be augumented with it in a greater ratio than that in which the velocity increaſes. But neither of theſe ſuppoſitions ſeem reaſonable. The more groſs and leſs fluid parts will tend to the circumference, unleſs they are heavy towards the centre. And though, for the ſake of demonſtration, I propoſed, at the beginning of this ſection, an hypotheſis that the reſiſtance is proportional to the velocity, nevertheleſs, it is in truth probable that the reſiſtance is in a leſs ratio than that of the velocity; which granted, the periodic times of the parts of the vortex will be in a greater than the duplicate ratio of the diſtances from its centre. If, as ſome think, the vortices move more ſwiftly near the centre, then ſlower to a certain limit, then again ſwifter near the circumference, certainly neither the ſeſquiplicate, nor any other certain and determinate ratio, can obtain in them. Let philoſophers then ſee how that phænomenon of the ſeſquiplicate ratio can be accounted for by vortices.

PROPOSITION LIII. THEOREM XLI.

Bodies carried about in a vortex, and returning in the ſame orb, are of the ſame denſity with the vortex, and are moved according to the ſame law with the parts of the vortex, as to velocity and direction of motion.

For if any ſmall part of the vortex, whoſe particles or phyſical points preſerve a given ſituation among each other, be ſuppoſed to be congealed, this particle will move according to the ſame law as before, ſince no change is made either in its denſity, *vis inſita*, or figure. And again; if a congealed or ſolid part of the vortex be of the ſame denſity with the reſt of the vortex, and be reſolved into a fluid, this will move according to the ſame law as before, except in ſo far as its particles, now become fluid, may be moved among themſelves. Neglect, therefore, the motion of the particles among themſelves as not at all concerning the progreſſive motion of the whole, and the motion of the whole will be the ſame as before. But this motion will be the ſame with the motion of other parts of the vortex at equal diſtances from the centre; becauſe the ſolid, now reſolved into a fluid, is become perfectly like to the other parts of the vortex. Therefore a ſolid, if it be of the ſame denſity with the matter of the vortex, will move with the ſame motion as the parts thereof, being relatively at reſt in the matter that ſurrounds it. If it be more denſe, it will endeavour more than before to recede from the centre; and therefore overcoming that force of the vortex, by which, being, as it were, kept in equilibrio, it was retained in its orbit, it will recede from the centre, and in its revolution deſcribe a ſpiral, returning no longer into the ſame orbit. And, by the ſame argument, if it be more rare, it will approach to the centre. Therefore it can never continually go round in the ſame orbit, unleſs it be of the ſame denſity with the fluid. But we have ſhewn in that caſe that it would revolve according to the ſame law with thoſe parts of the fluid that are at the ſame or equal diſtances from the centre of the vortex.

Cor. 1. Therefore a ſolid revolving in a vortex, and continually going round in the ſame orbit, is relatively quieſcent in the fluid that carries it.

Cor. 2. And if the vortex be of an uniform denſity, the ſame body may revolve at any diſtance from the centre of the vortex.

SCHOLIUM.

Hence it is manifeſt that the planets are not carried round in corporeal vortices; for, according to the *Copernican* hypotheſis, the planets going round the ſun revolve in ellipſes, having the ſun in their common focus; and by radii drawn to the ſun deſcribe areas proportional to the times. But now the parts of a vortex can never revolve with ſuch a motion. Let AD, BE, CF (Pl. 9, Fig. 3), repreſent three orbits deſcribed about the ſun S, of which let the utmoſt circle CF be concentric to the ſun; and let the aphelia of the two innermoſt be A, B; and their perihelia D, E. Therefore a body revolving in the orb CF, deſcribing, by a radius drawn to the ſun, areas proportional to the times, will move with an uniform motion. And, according to the laws of aſtronomy, the body revolving in the orb BE will move ſlower in its aphelion B, and ſwifter in its perihelion E; whereas, according to the laws of mechanics, the matter of the vortex ought to move more ſwiftly in the narrow ſpace between A and C than in the wide ſpace between D and F; that is, more ſwiftly in the aphelion than in the perihelion. Now theſe two concluſions contradict each other. So at the beginning of the ſign of Virgo, where the aphelion of Mars is at preſent, the diſtance between the orbits of Mars and Venus is to the diſtance between the ſame orbits, at the beginning of the ſign of Piſces, as about 3 to 2; and therefore the matter of the vortex between thoſe orbits ought to be ſwifter at the beginning of Piſces than at the beginning of Virgo in the ratio of 3 to 2; for the narrower the ſpace is through which the ſame quantity of matter paſſes in the ſame time of one revolution, the greater will be the velocity with which it paſſes through it. Therefore if the earth being relatively at reſt in this celeſtial matter ſhould be carried round by it, and revolve

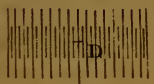
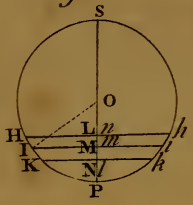


Fig. 1.



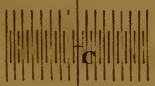


Fig. 2.

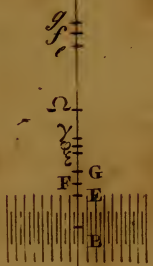


Fig. 3.

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together with it about the ſun, the velocity of the earth at the beginning of Piſces would be to its velocity at the beginning of Virgo in a ſeſquialteral ratio. Therefore the ſun's apparent diurnal motion at the beginning of Virgo ought to be above 70 minutes, and at the beginning of Piſces leſs than 48 minutes; whereas, on the contrary, that apparent motion of the ſun is really greater at the beginning of Piſces than at the beginning of Virgo, as experience teſtifies; and therefore the earth is ſwifter at the beginning of Virgo than at the beginning of Piſces: ſo that the hypotheſis of vortices is utterly irreconcileable with aſtronomical phænomena, and rather ſerves to perplex than explain the heavenly motions. How theſe motions are performed in free ſpaces without vortices, may be underſtood by the firſt book; and I ſhall now more fully treat of it in the following book.

BOOK III.

IN the preceding books I have laid down the principles of philoſophy; principles not philoſophical, but mathematical; ſuch, to wit, as we may build our reaſonings upon in philoſophical enquiries. Theſe principles are the laws and conditions of certain motions, and powers or forces, which chiefly have reſpect to philoſophy; but, leſt they ſhould have appeared of themſelves dry and barren, I have illuſtrated them here and there with ſome philoſophical ſcholiums, giving an account of ſuch things as are of more general nature, and which philoſophy ſeems chiefly to be founded on; ſuch as the denſity and the reſiſtance of bodies, ſpaces void of all bodies, and the motion of light and ſounds. It remains that, from the ſame principles, I now demonſtrate the frame of the Syſtem of the World. Upon this ſubject I had, indeed, compoſed the third book in a popular method, that it might be read by many; but afterwards, conſidering that ſuch as had not ſufficiently entered into the principles could not eaſily diſcern the ſtrength of the conſequences, nor lay aſide the prejudices to which they had been many years accuſtomed, therefore, to prevent the diſputes which might be raiſed upon

ſuch accounts, I choſe to reduce the ſubſtance of this book into the form of propoſitions (in the mathematical way), which ſhould be read by thoſe only who had firſt made themſelves maſters of the principles eſtabliſhed in the preceding books: not that I would adviſe any one to the previous ſtudy of every propoſition of thoſe books; for they abound with ſuch as might coſt too much time, even to readers of good mathematical learning. It is enough if one carefully reads the definitions, the laws of motion, and the firſt three ſections of the firſt book. He may then paſs on to this book, and conſult ſuch of the remaining propoſitions of the firſt two books, as the references in this, and his occaſions, ſhall require.

RULES OF REASONING IN PHILOSOPHY.

RULE I.

We are to admit no more cauſes of natural things than ſuch as are both true and ſufficient to explain their appearances.

To this purpoſe the philoſophers ſay that Nature does nothing in vain, and more is in vain when leſs will ſerve; for Nature is pleaſed with ſimplicity, and affects not the pomp of ſuperfluous cauſes.

RULE II.

Therefore to the ſame natural effects we muſt, as far as poſſible, aſſign the ſame cauſes.

As to reſpiration in a man and in a beaſt; the deſcent of ſtones in *Europe* and in *America;* the light of our culinary fire and of the ſun; the reflection of light in the earth, and in the planets.

RULE III.

The qualities of bodies, which admit neither intenſion nor remiſſion of degrees, and which are found to belong to all bodies within the reach of our experiments, are to be eſteemed the univerſal qualities of all bodies whatſoever.

For ſince the qualities of bodies are only known to us by experiments, we are to hold for univerſal all ſuch as univerſally agree with experiments; and ſuch as are not liable to diminution can never be quite taken away. We are certainly

not to relinquiſh the evidence of experiments for the ſake of dreams and vain fictions of our own deviſing; nor are we to recede from the analogy of Nature, which uſes to be ſimple, and always conſonant to itſelf. We no other way know the extenſion of bodies than by our ſenſes, nor do theſe reach it in all bodies; but becauſe we perceive extenſion in all that are ſenſible, therefore we aſcribe it univerſally to all others alſo. That abundance of bodies are hard, we learn by experience; and becauſe the hardneſs of the whole ariſes from the hardneſs of the parts, we therefore juſtly infer the hardneſs of the undivided particles not only of the bodies we feel but of all others. That all bodies are impenetrable, we gather not from reaſon, but from ſenſation. The bodies which we handle we find impenetrable, and thence conclude impenetrability to be an univerſal property of all bodies whatſoever. That all bodies are moveable, and endowed with certain powers (which we call the *vires inertiæ)* of perſevering in their motion, or in their reſt, we only infer from the like properties obſerved in the bodies which we have ſeen. The extenſion, hardneſs, impenetrability, mobility, and *vis inertiæ* of the whole, reſult from the extenſion, hardneſs, impenetrability, mobility, and *vires inertiæ* of the parts; and thence we conclude the leaſt particles of all bodies to be alſo all extended, and hard, and impenetrable, and moveable, and endowed with their proper *vires inertiæ.* And this is the foundation of all philoſophy. Moreover, that the divided but contiguous particles of bodies may be ſeparated from one another, is matter of obſervation; and, in the particles that remain undivided, our minds are able to diſtinguiſh yet leſſer parts, as is mathematically demonſtrated. But whether the parts ſo diſtinguiſhed, and not yet divided, may, by the powers of Nature, be actually divided and ſeparated from one another, we cannot certainly determine. Yet, had we the proof of but one experiment that any undivided particle, in breaking a hard and ſolid body, ſuffered a diviſion, we might by virtue of this rule conclude that the undivided as well as the divided particles may be divided and actually ſeparated to infinity.

Laſtly, if it univerſally appears, by experiments and aſtronomical obſervations, that all bodies about the earth gravi-

tate towards the earth, and that in proportion to the quantity of matter which they ſeverally contain; that the moon likewiſe, according to the quantity of its matter, gravitates towards the earth; that, on the other hand, our ſea gravitates towards the moon; and all the planets mutually one towards another; and the comets in like manner towards the ſun; we muſt, in conſequence of this rule, univerſally allow that all bodies whatſoever are endowed with a principle of mutual gravitation. For the argument from the appearances concludes with more force for the univerſal gravitation of all bodies than for their impenetrability; of which, among thoſe in the celeſtial regions, we have no experiments, nor any manner of obſervation. Not that I affirm gravity to be eſſential to bodies: by their *vis inſita* I mean nothing but their *vis inertiæ*. This is immutable. Their gravity is diminiſhed as they recede from the earth.

RULE IV.

In experimental philoſophy we are to look upon propoſitions collected by general induction from phænomena as accurately or very nearly true, notwithſtanding any contrary hypotheſes that may be imagined, till ſuch time as other phænomena occur, by which they may either be made more accurate, or liable to exceptions.

This rule we muſt follow, that the argument of induction may not be evaded by hypotheſes.

PHÆNOMENA, OR APPEARANCES.

PHÆNOMENON I.

That the circumjovial planets, by radii drawn to Jupiter's centre, deſcribe areas proportional to the times of deſcription; and that their periodic times, the fixed ſtars being at reſt, are in the ſeſquiplicate proportion of their diſtances from its centre.

This we know from aſtronomical obſervations. For the orbits of theſe planets differ but inſenſibly from circles concentric to Jupiter; and their motions in thoſe circles are found to be uniform. And all aſtronomers agree that their periodic times are in the ſeſquiplicate proportion of the ſemi-diameters of their orbits; and ſo it manifeſtly appears from the following table

The periodic times of the ſatellites of Jupiter.

$1^d.\ 18^h.\ 27'\ 34''.\quad 3^d.\ 13^h.\ 13'\ 42''.\quad 7^d.\ 3^h.\ 42'\ 36''.$
$16^d.\ 16^h.\ 32'\ 9''.$

The diſtances of the ſatellites from Jupiter's centre.

From the obſervations of	1	2	3	4	
Borelli	$5\frac{2}{3}$	$8\frac{2}{3}$	14	$24\frac{2}{3}$	*ſemi-diameter of Jupiter.*
Townley *by the Microm.*	5,52	8,78	13,47	24,72	
Caſſini *by the Teleſcope.*	5	8	13	23	
Caſſini *by the eclip. of the ſatel.*	$5\frac{2}{3}$	9	$14\frac{2\,3}{6\,0}$	$25\frac{3}{10}$	
From the periodic times	5,667	9,017	14,384	25,299	

Mr. *Pound* has determined, by the help of excellent micrometers, the diameters of Jupiter and the elongation of its ſatellites after the following manner. The greateſt heliocentric elongation of the fourth ſatellite from Jupiter's centre was taken with a micrometer in a 15 feet teleſcope, and at the mean diſtance of Jupiter from the earth was found about $8'\ 16''$. The elongation of the third ſatellite was taken with a micrometer in a teleſcope of 123 feet, and at the ſame diſtance of Jupiter from the earth was found $4'\ 42''$. The greateſt elongations of the other ſatellites, at the ſame diſtance of Jupiter from the earth, are found from the periodic times to be $2'\ 56''\ 47'''$, and $1'\ 51''\ 6'''$.

The diameter of Jupiter taken with the micrometer in a 123 feet teleſcope ſeveral times, and reduced to Jupiter's mean diſtance from the earth, proved always leſs than $40''$, never leſs than $38''$, generally $39''$. This diameter in ſhorter teleſcopes is $40''$, or $41''$; for Jupiter's light is a little dilated by the unequal refrangibility of the rays, and this dilatation bears a leſs ratio to the diameter of Jupiter in the longer and more perfect teleſcopes than in thoſe which are ſhorter and leſs perfect. The times in which two ſatellites, the firſt and the third, paſſed over Jupiter's body, were obſerved, from the beginning of the ingreſs to the beginning of the egreſs, and from the complete ingreſs to the complete egreſs, with the long teleſcope. And from the tranſit of the firſt ſatellite, the diameter of Jupiter at its mean diſtance from the earth came forth $37\frac{1}{8}''$, and from the tranſit of the third $37\frac{3}{8}''$. There was obſerved alſo the time in which the ſhadow of the firſt ſatellite paſſed over Jupiter's body, and thence the diameter of Jupiter at its mean diſtance from the earth came out about

37″. Let us ſuppoſe its diameter to be $37\frac{1}{4}$″ very nearly, and then the greateſt elongations of the firſt, ſecond, third, and fourth ſatellite will be reſpectively equal to 5,965, 9,494, 15,141, and 26,63 ſemi-diameters of Jupiter.

PHÆNOMENON II.

That the circumſaturnal planets, by radii drawn to Saturn's centre, deſcribe areas proportional to the times of deſcription; and that their periodic times, the fixed ſtars being at reſt, are in the ſeſquiplicate proportion of their diſtances from its centre.

For, as *Caſſini* from his own obſervations has determined, their diſtances from Saturn's centre and their periodic times are as follow.

The periodic times of the ſatellites of Saturn.

1ᵈ. 21ʰ. 18′ 27″. 2ᵈ. 17ʰ. 41′ 22″. 4ᵈ. 12ʰ. 25′ 12″. 15ᵈ. 22ʰ. 41′ 14″. 79ᵈ. 7ʰ. 48′ 00″.

The diſtances of the ſatellites from Saturn's centre, in ſemi-diameters of its ring.

From obſervations	$1\frac{19}{20}$.	$2\frac{1}{2}$	$3\frac{1}{2}$.	8.	24.
From the periodic times	1,93.	2,47.	3,45.	8.	23,35.

The greateſt elongation of the fourth ſatellite from Saturn's centre is commonly determined from the obſervations to be eight of thoſe ſemi-diameters very nearly. But the greateſt elongation of this ſatellite from Saturn's centre, when taken with an excellent micrometer in Mr. *Huygens*'s teleſcope of 123 feet, appeared to be eight ſemi-diameters and $\frac{1}{16}$ of a ſemi-diameter. And from this obſervation and the periodic times the diſtances of the ſatellites from Saturn's centre in ſemi-diameters of the ring are 2,1. 2,69. 3,75. 8,7. and 25,35. The diameter of Saturn obſerved in the ſame teleſcope was found to be to the diameter of the ring as 3 to 7; and the diameter of the ring, *May* 28-29, 1719, was found to be 43″; and thence the diameter of the ring when Saturn is at its mean diſtance from the earth is 42″, and the diameter of Saturn 18″. Theſe things appear ſo in very long and excellent teleſcopes, becauſe in ſuch teleſcopes the apparent magnitudes of the heavenly bodies bear a greater proportion to the dilatation of light in the extremities of thoſe bodies than in

ſhorter teleſcopes. If we, then, reject all the ſpurious light, the diameter of Saturn will not amount to more than 16″.

PHÆNOMENON III.

That the five primary planets, Mercury, Venus, Mars, Jupiter, and Saturn, with their ſeveral orbits, encompaſs the ſun.

That Mercury and Venus revolve about the ſun, is evident from their moon-like appearances. When they ſhine out with a full face, they are, in reſpect of us, beyond or above the ſun; when they appear half full, they are about the ſame height on one ſide or other of the ſun; when horned, they are below or between us and the ſun; and they are ſometimes, *when directly under,* ſeen like ſpots traverſing the ſun's diſk. That Mars ſurrounds the ſun, is as plain from its full face when near its conjunction with the ſun, and from the gibbous figure which it ſhews in its quadratures. And the ſame thing is demonſtrable of Jupiter and Saturn, from their appearing full in all ſituations; for the ſhadows of their ſatellites that appear ſometimes upon their diſks make it plain that the light they ſhine with is not their own, but borrowed from the ſun.

PHÆNOMENON IV.

That the fixed ſtars being at reſt, the periodic times of the five primary planets, and (whether of the ſun about the earth, or) of the earth about the ſun, are in the ſeſquiplicate proportion of their mean diſtances from the ſun.

This proportion, firſt obſerved by *Kepler,* is now received by all aſtronomers; for the periodic times are the ſame, and the dimenſions of the orbits are the ſame, whether the ſun revolves about the earth, or the earth about the ſun. And as to the meaſures of the periodic times, all aſtronomers are agreed about them. But for the dimenſions of the orbits, *Kepler* and *Bullialdus,* above all others, have determined them from obſervations with the greateſt accuracy; and the mean diſtances correſponding to the periodic times differ but inſenſibly from thoſe which they have aſſigned, and for the moſt part fall in between them; as we may ſee from the following table.

The periodic times, with reſpect to the fixed ſtars, of the planets and earth revolving *about the ſun, in days and decimal parts of a day.*

♄	♃	♂	♁	♀
10759,275.	4332,514.	686,9785.	365,2565.	224,6176.

☿
87,9692.

The mean diſtances of the planets and of the earth from the ſun.

	♄	♃	♂
According to *Kepler*	951000.	519650.	152350.
..... to *Bullialdus*	954198.	522520.	152350.
..... to the periodic times	954006.	520096.	152369.

	♁	♀	☿
According to *Kepler*	100000.	72400.	38806.
..... to *Bullialdus*	100000.	72398.	38585.
..... to the periodic times	100000.	72333.	38710.

As to Mercury and Venus, there can be no doubt about their diſtances from the ſun; for they are determined by the elongations of thoſe planets from the ſun; and for the diſtances of the ſuperior planets, all diſpute is cut off by the eclipſes of the ſatellites of Jupiter. For by thoſe eclipſes the poſition of the ſhadow which Jupiter projects is determined; whence we have the heliocentric longitude of Jupiter. And from its heliocentric and geocentric longitudes compared together, we determine its diſtance.

PHÆNOMENON V.

Then the primary planets, by radii drawn to the earth, deſcribe areas no wiſe proportional to the times; but that the areas which they deſcribe by radii drawn to the ſun are proportional to the times of deſcription.

For to the earth they appear ſometimes direct, ſometimes ſtationary, nay, and ſometimes retrograde. But from the ſun they are always ſeen direct, and to proceed with a motion nearly uniform, that is to ſay, a little ſwifter in the perihelion and a little ſlower in the aphelion diſtances, ſo as to maintain an equality in the deſcription of the areas. This a noted propoſition among aſtronomers, and particularly demonſtra-

ble in Jupiter, from the eclipſes of his ſatellites; by the help of which eclipſes, as we have ſaid, the heliocentric longitudes of that planet, and its diſtances from the ſun, are determined.

PHÆNOMENON VI.

That the moon, by a radius drawn to the earth's centre, deſcribes an area proportional to the time of deſcription.

This we gather from the apparent motion of the moon, compared with its apparent diameter. It is true that the motion of the moon is a little diſturbed by the action of the ſun: but in laying down theſe phænomena, I neglect thoſe ſmall and inconſiderable errors.

PROPOSITIONS.

PROPOSITION I. THEOREM I.

That the forces by which the circumjovial planets are continually drawn off from rectilinear motions, and retained in their proper orbits, tend to Jupiter's centre; and are reciprocally as the ſquares of the diſtances of the places of thoſe planets *from that centre.*

The former part of this propoſition appears from phæn. 1, and prop. 2 or 3, book 1; the latter from phæn. 1, and cor. 6, prop. 4, of the ſame book.

The ſame thing we are to underſtand of the planets which encompaſs Saturn, by phæn. 2.

PROPOSITION II. THEOREM II.

That the forces by which the primary planets are continually drawn off from rectilinear motions, and retained in their proper orbits, tend to the ſun; and are reciprocally as the ſquares of the diſtances of the places of thoſe planets *from the ſun's centre.*

The former part of the propoſition is manifeſt from phæn. 5, and prop. 2, book 1; the latter from phæn. 4, and *cor.* 6, prop. 4, of the ſame book. But this part of the propoſition is, with great accuracy, demonſtrable from the quieſcence of the aphelion points; for a very ſmall aberration from the *reciprocal* duplicate proportion would (by cor. 1, prop. 45, book 1) produce a motion of the apſides ſenſible enough in every ſingle revolution, and in many of them enormouſly great.

PROPOSITION III. THEOREM III.

That the force by which the moon is retained in its orbit tends to the earth; and is reciprocally as the square of the distance of its place from the earth's centre.

The former part of the proposition is evident from phæn. 6, and prop. 2 or 3, book 1; the latter from the very slow motion of the moon's apogee; which in every single revolution amounting but to 3° 3′ *in consequentia*, may be neglected. For (by cor. 1, prop. 45, book 1) it appears, that, if the distance of the moon from the earth's centre is to the semi-diameter of the earth as D to 1, the force, from which such a motion will result, is reciprocally as $D^{2\frac{4}{243}}$, i. e. reciprocally as the power of D, whose exponent is $2\frac{4}{243}$; that is to say, in the proportion of the distance something greater than reciprocally duplicate, but which comes $59\frac{1}{4}$ times nearer to the duplicate than to the triplicate proportion. But in regard that this motion is owing to the action of the sun (as we shall afterwards shew), it is here to be neglected. The action of the sun, attracting the moon from the earth, is nearly as the moon's distance from the earth; and therefore (by what we have shewed in cor. 2, prop. 45, book 1) is to the centripetal force of the moon as 2 to 357,45, or nearly so; that is, as 1 to $178\frac{29}{40}$. And if we neglect so inconsiderable a force of the sun, the remaining force, by which the moon is retained in its orb, will be reciprocally as D^2. This will yet more fully appear from comparing this force with the force of gravity, as is done in the next proposition.

Cor. If we augment the mean centripetal force by which the moon is retained in its orb, first in the proportion of $177\frac{29}{40}$ to $178\frac{29}{40}$, and then in the duplicate proportion of the semi-diameter of the earth to the mean distance of the centres of the moon and earth, we shall have the centripetal force of the moon at the surface of the earth; supposing this force, in descending to the earth's surface, continually to increase in the reciprocal duplicate proportion of the height.

PROPOSITION IV. THEOREM IV.

That the moon gravitates towards the earth, and by the force of gravity is continually drawn off from a rectilinear motion, and retained in its orbit.

The mean diſtance of the moon from the earth in the ſyzygies in ſemi-diameters of the earth, is, according to *Ptolemy* and moſt aſtronomers, 59; according to *Vendelin* and *Huygens*, 60; to *Copernicus*, $60\frac{1}{3}$; to *Street*, $60\frac{2}{3}$; and to *Tycho*, $56\frac{1}{2}$. But *Tycho*, and all that follow his tables of refraction, making the refractions of the ſun and moon (altogether againſt the nature of light) to exceed the refractions of the fixed ſtars, and that by four or five minutes *near the horizon*, did thereby increaſe the moon's *horizontal* parallax by a like number of minutes, that is, by a twelfth or fifteenth part of the whole parallax. Correct this error, and the diſtance will become about $60\frac{1}{2}$ ſemi-diameters of the earth, near to what others have aſſigned. Let us aſſume the mean diſtance of 60 diameters in the ſyzygies; and ſuppoſe one revolution of the moon, in reſpect of the fixed ſtars, to be completed in $27^d.\ 7^h.\ 43'$, as aſtronomers have determined; and the circumference of the earth to amount to 123249600 *Paris* feet, as the *French* have found by menſuration. And now if we imagine the moon, deprived of all motion, to be let go, ſo as to deſcend towards the earth with the impulſe of all that force by which (by cor. prop. 3) it is retained in its orb, it will, in the ſpace of one minute of time, deſcribe in its fall $15\frac{1}{12}$ *Paris* feet. This we gather by a calculus, founded either upon prop. 36, book 1, or (which comes to the ſame thing) upon cor. 9, prop. 4, of the ſame book. For the verſed ſine of that arc, which the moon, in the ſpace of one minute of time, would by its mean motion deſcribe at the diſtance of 60 ſemi-diameters of the earth, is nearly $15\frac{1}{12}$ *Paris* feet, or more accurately 15 feet, 1 inch, and 1 line $\frac{4}{9}$. Wherefore, ſince that force, in approaching to the earth, increaſes in the reciprocal duplicate proportion of the diſtance, and, upon that account, at the ſurface of the earth, is 60×60 times greater than at the moon, a body in our regions, falling with that force, ought, in the ſpace of one minute of time, to deſcribe

60 × 60 × $15\frac{1}{12}$ *Paris* feet; and, in the ſpace of one ſecond of time, to deſcribe $15\frac{1}{12}$ of thoſe feet; or more accurately 15 feet, 1 inch, and 1 line $\frac{4}{9}$. And with this very force we actually find that bodies here upon earth do really deſcend; for a pendulum oſcillating ſeconds in the latitude of *Paris* will be 3 *Paris* feet, and 8 lines $\frac{1}{2}$ in length, as Mr. *Huygens* has obſerved. And the ſpace which a heavy body deſcribes by falling in one ſecond of time is to half the length of this pendulum in the duplicate ratio of the circumference of a circle to its diameter (as Mr. *Huygens* has alſo ſhewn), and is therefore 15 *Paris* feet, 1 inch, 1 line $\frac{7}{9}$. And therefore the force by which the moon is retained in its orbit becomes, at the very ſurface of the earth, equal to the force of gravity which we obſerve in heavy bodies there. And therefore (by rule 1 and 2) the force by which the moon is retained in its orbit is that very ſame force which we commonly call gravity; for, were gravity another force different from that, then bodies deſcending to the earth with the joint impulſe of both forces would fall with a double velocity, and in the ſpace of one ſecond of time would deſcribe $30\frac{1}{6}$ *Paris* feet; altogether againſt experience.

This calculus is founded on the hypotheſis of the earth's ſtanding ſtill; for if both earth and moon move about the ſun, and at the ſame time about their common centre of gravity, the diſtance of the centres of the moon and earth from one another will be $60\frac{1}{2}$ ſemi-diameters of the earth; as may be found by a computation from prop. 60, book 1.

SCHOLIUM.

The demonſtration of this propoſition may be more diffuſely explained after the following manner. Suppoſe ſeveral moons to revolve about the earth, as in the ſyſtem of Jupiter or Saturn; the periodic times of theſe moons (by the argument of induction) would obſerve the ſame law which *Kepler* found to obtain among the planets; and therefore their centripetal forces would be reciprocally as the ſquares of the diſtances from the centre of the earth, by prop. 1, of this book. Now if the loweſt of theſe were very ſmall, and were ſo near the earth as almoſt to touch the tops of the higheſt mountains,

the centripetal force thereof, retaining it in its orb, would be very nearly equal to the weights of any *terreſtrial* bodies that ſhould be found upon the tops of thoſe mountains, as may be known by the foregoing computation. Therefore if the ſame little moon ſhould be deſerted by its centrifugal force that carries it through its orb, and ſo be diſabled from going onwards therein, it would deſcend to the earth; and that with the ſame velocity as heavy bodies do actually fall with upon the tops of thoſe very mountains; becauſe of the equality of the forces that oblige them both to deſcend. And if the force by which that loweſt moon would deſcend were different from gravity, and if that moon were to gravitate towards the earth, as we find terreſtrial bodies do upon the tops of mountains, it would then deſcend with twice the velocity, as being impelled by both theſe forces conſpiring together. Therefore ſince both theſe forces, that is, the gravity of heavy bodies, and the centripetal forces of the moons, reſpect the centre of the earth, and are ſimilar and equal between themſelves, they will (by rule 1 and 2) have one and the ſame cauſe. And therefore the force which retains the moon in its orbit is that very force which we commonly call gravity; becauſe otherwiſe this little moon at the top of a mountain muſt either be without gravity, or fall twice as ſwiftly as heavy bodies uſe to do.

PROPOSITION V. THEOREM V.

That the circumjovial planets gravitate towards Jupiter; the circumſaturnal towards Saturn; the circumſolar towards the ſun; and by the forces of their gravity are drawn off from rectilinear motions, and retained in curvilinear orbits.

For the revolutions of the circumjovial planets about Jupiter, of the circumſaturnal about Saturn, and of Mercury and Venus, and the other circumſolar planets, about the ſun, are appearances of the ſame ſort with the revolution of the moon about the earth; and therefore, by rule 2, muſt be owing to the ſame ſort of cauſes; eſpecially ſince it has been demonſtrated, that the forces upon which thoſe revolutions depend tend to the centres of Jupiter, of Saturn, and of the

ſun; and that thoſe forces, in receding from Jupiter, from Saturn, and from the ſun, decreaſe in the ſame proportion, and according to the ſame law, as the force of gravity does in receding from the earth.

Cor. 1. There is, therefore, a power of gravity tending to all the planets; for, doubtleſs, Venus, Mercury, and the reſt, are bodies of the ſame ſort with Jupiter and Saturn. And ſince all attraction (by law 3) is mutual, Jupiter will therefore gravitate towards all his own ſatellites, Saturn towards his, the earth towards the moon, and the ſun towards all the primary planets.

Cor. 2. The force of gravity which tends to any one planet is reciprocally as the ſquare of the diſtance of places from that planet's centre.

Cor. 3. All the planets do mutually gravitate towards one another, by cor. 1 and 2. And hence it is that Jupiter and Saturn, when near their conjunction, by their mutual attractions ſenſibly diſturb each other's motions. So the ſun diſturbs the motions of the moon; and both ſun and moon diſturb our ſea, as we ſhall hereafter explain.

SCHOLIUM.

The force which retains the celeſtial bodies in their orbits has been hitherto called centripetal force; but it being now made plain that it can be no other than a gravitating force, we ſhall hereafter call it gravity. For the cauſe of that centripetal force which retains the moon in its orbit will extend itſelf to all the planets, by rule 1, 2, and 4.

PROPOSITION VI. THEOREM VI.

That all bodies gravitate towards every planet; and that the weights of bodies towards any the ſame *planet, at equal diſtances from the centre of the planet, are proportional to the quantities of matter which they ſeverally contain.*

It has been, now of a long time, obſerved by others, that all ſorts of heavy bodies (allowance being made for the inequality of retardation which they ſuffer from a ſmall power of reſiſtance in the air) deſcend to the earth *from equal heights* in equal times; and that equality of times we may diſtinguiſh to a great accuracy, by the help of pendulums. I tried the

thing in gold, ſilver, lead, glaſs, ſand, common ſalt, wood, water, and wheat. I provided two wooden boxes, round and equal: I filled the one with wood, and ſuſpended an equal weight of gold (as exactly as I could) in the centre of oſcillation of the other. The boxes hanging by equal threads of 11 feet made a couple of pendulums perfectly equal in weight and figure, and equally receiving the reſiſtance of the air. And, placing the one by the other, I obſerved them to play together forwards and backwards, for a long time, with equal vibrations. And therefore the quantity of matter in the gold (by cor. 1 and 6, prop. 24, book 2) was to the quantity of matter in the wood as the action of the motive force (or *vis motrix)* upon all the gold to the action of the ſame upon all the wood; that is, as the weight of the one to the weight of the other: and the like happened in the other bodies. By theſe experiments, in bodies of the ſame weight, I could manifeſtly have diſcovered a difference of matter leſs than the thouſandth part of the whole, had any ſuch been. But, without all doubt, the nature of gravity towards the planets is the ſame as towards the earth. For, ſhould we imagine our terreſtrial bodies removed to the orb of the moon, and there, together with the moon, deprived of all motion, to be let go, ſo as to fall together towards the earth, it is certain, from what we have demonſtrated before, that, in equal times, they would deſcribe equal ſpaces with the moon, and of conſequence are to the moon, in quantity of matter, as their weights to its weight. Moreover, ſince the ſatellites of Jupiter perform their revolutions in times which obſerve the ſeſquiplicate proportion of their diſtances from Jupiter's centre, their accelerative gravities towards Jupiter will be reciprocally as the ſquares of their diſtances from Jupiter's centre; that is, equal, at equal diſtances. And, therefore, theſe ſatellites, if ſuppoſed to fall *towards Jupiter* from equal heights, would deſcribe equal ſpaces in equal times, in like manner as heavy bodies do on our earth. And, by the ſame argument, if the circumſolar planets were ſuppoſed to be let fall at equal diſtances from the ſun, they would, in their deſcent towards the ſun, deſcribe equal ſpaces in equal times. But forces which

equally accelerate unequal bodies muſt be as thoſe bodies; that is to ſay, the weights of the planets *towards the ſun* muſt be as their quantities of matter. Further, that the weights of Jupiter and of his ſatellites towards the ſun are proportional to the ſeveral quantities of their matter, appears from the exceedingly regular motions of the ſatellites (by cor. 3, prop. 65, book 1). For if ſome of thoſe bodies were more ſtrongly attracted to the ſun in proportion to their quantity of matter than others, the motions of the ſatellites would be diſturbed by that inequality of attraction (by cor. 2, prop. 65, book 1). If, at equal diſtances from the ſun, any ſatellite, in proportion to the quantity of its matter, did gravitate towards the ſun with a force greater than Jupiter in proportion to his, according to any given proportion, ſuppoſe of d to e; then the diſtance between the centres of the ſun and of the ſatellite's orbit would be always greater than the diſtance between the centres of the ſun and of Jupiter nearly in the ſubduplicate of that proportion; as by ſome computations I have found. And if the ſatellite did gravitate towards the ſun with a force, leſſer in the proportion of e to d, the diſtance of the centre of the ſatellite's orb from the ſun would be leſs than the diſtance of the centre of Jupiter from the ſun in the ſubduplicate of the ſame proportion. Therefore if, at equal diſtances from the ſun, the accelerative gravity of any ſatellite towards the ſun were greater or leſs than the accelerative gravity of Jupiter towards the ſun but by one $\frac{1}{1000}$ part of the whole gravity, the diſtance of the centre of the ſatellite's orbit from the ſun would be greater or leſs than the diſtance of Jupiter from the ſun by one $\frac{1}{2000}$ part of the whole diſtance; that is, by a fifth part of the diſtance of the utmoſt ſatellite from the centre of Jupiter; an eccentricity of the orbit which would be very ſenſible. But the orbits of the ſatellites are concentric to Jupiter, and therefore the accelerative gravities of Jupiter, and of all its ſatellites towards the ſun, are equal among themſelves. And by the ſame argument, the weights of Saturn and of his ſatellites towards the ſun, at equal diſtances from the ſun, are as their ſeveral quantities of matter; and the weights of the moon and of the earth towards the ſun are

either none, or accurately proportional to the maſſes of matter which they contain. But ſome they are, by cor. 1 and 3, prop. 5.

But further; the weights of all the parts of every planet towards any other planet are one to another as the matter in the ſeveral parts; for if ſome parts did gravitate more, others leſs, than for the quantity of their matter, then the whole planet, according to the ſort of parts with which it moſt abounds, would gravitate more or leſs than in proportion to the quantity of matter in the whole. Nor is it of any moment whether theſe parts are external or internal; for if, for example, we ſhould imagine the terreſtrial bodies with us to be raiſed up to the orb of the moon, to be there compared with its body; if the weights of ſuch bodies were to the weights of the external parts of the moon as the quantities of matter in the one and in the other reſpectively; but to the weights of the internal parts in a greater or leſs proportion, then likewiſe the weights of thoſe bodies would be to the weight of the whole moon in a greater or leſs proportion; againſt what we have ſhewed above.

Cor. 1. Hence the weights of bodies do not depend upon their forms and textures; for if the weights could be altered with the forms, they would be greater or leſs, according to the variety of forms, in equal matter; altogether againſt experience.

Cor. 2. Univerſally, all bodies about the earth gravitate towards the earth; and the weights of all, at equal diſtances from the earth's centre, are as the quantities of matter which they ſeverally contain. This is the quality of all bodies within the reach of our experiments; and therefore (by rule 3) to be affirmed of all bodies whatſoever. If the *æther*, or any other body, were either altogether void of gravity, or were to gravitate leſs in proportion to its quantity of matter, then, becauſe (according to *Ariſtotle*, *Des Cartes*, and others) there is no difference betwixt that and other bodies but in *mere* form of matter, by a ſucceſſive change from form to form, it might be changed at laſt into a body of the ſame condition with thoſe which gravitate moſt in proportion to their quantity of matter; and, on the other hand, the heavieſt bodies,

acquiring the firſt form of that body, might by degrees quite loſe their gravity. And therefore the weights would depend upon the forms of bodies, and with thoſe forms might be changed: contrary to what was proved in the preceding corollary.

Cor. 3. All ſpaces are not equally full; for if all ſpaces were equally full, then the ſpecific gravity of the fluid which fills the region of the air, on account of the extreme denſity of the matter, would fall nothing ſhort of the ſpecific gravity of quickſilver, or gold, or any other the moſt denſe body; and, therefore, neither gold, nor any other body, could deſcend in air; for bodies do not deſcend in fluids, unleſs they are ſpecifically heavier than the fluids. And if the quantity of matter in a given ſpace can, by any rarefaction, be diminiſhed, what ſhould hinder a diminution to infinity?

Cor. 4. If all the ſolid particles of all bodies are of the ſame denſity, nor can be rarefied without pores, a void, ſpace, or vacuum muſt be granted. By bodies of the ſame denſity, I mean thoſe whoſe *vires inertiæ* are in the proportion of their bulks.

Cor. 5. The power of gravity is of a different nature from the power of magnetiſm; for the magnetic attraction is not as the matter attracted. Some bodies are attracted more by the magnet; others leſs; moſt bodies not at all. The power of magnetiſm in one and the ſame body may be increaſed and diminiſhed; and is ſometimes far ſtronger, for the quantity of matter, than the power of gravity; and in receding from the magnet decreaſes not in the duplicate but almoſt in the triplicate proportion of the diſtance, as nearly as I could judge from ſome rude obſervations.

PROPOSITION VII. THEOREM VII.

That there is a power of gravity tending to all bodies, proportional to the ſeveral quantities of matter which they contain.

That all the planets mutually gravitate one towards another, we have proved before; as well as that the force of gravity towards every one of them, conſidered apart, is reciprocally as the ſquare of the diſtance of places from the centre of the planet. And thence (by prop. 69, book 1, and its corollaries)

it follows, that the gravity tending towards all the planets is proportional to the matter which they contain.

Moreover, ſince all the parts of any planet A gravitate towards any other planet B; and the gravity of every part is to the gravity of the whole as the matter of the part to the matter of the whole; and (by law 3) to every action correſponds an equal re-action; therefore the planet B will, on the other hand, gravitate towards all the parts of the planet A; and its gravity towards any one part will be to the gravity towards the whole as the matter of the part to the matter of the whole. Q.E.D.

Cor. 1. Therefore the force of gravity towards any whole planet ariſes from, and is compounded of, the forces of gravity towards all its parts. Magnetic and electric attractions afford us examples of this; for all attraction towards the whole ariſes from the attractions towards the ſeveral parts. The thing may be eaſily underſtood in gravity, if we conſider a greater planet, as formed of a number of leſſer planets, meeting together in one globe; for *hence it would appear that* the force of the whole muſt ariſe from the forces of the component parts. If it is objected, that, according to this law, all bodies with us muſt mutually gravitate one towards another, whereas no ſuch gravitation any where appears, I anſwer, that ſince the gravitation towards theſe bodies is to the gravitation towards the whole earth as theſe bodies are to the whole earth, the gravitation towards them muſt be far leſs than to fall under the obſervation of our ſenſes.

Cor. 2. The force of gravity towards the ſeveral equal particles of any body is reciprocally as the ſquare of the diſtance of places from the particles; as appears from cor. 3, prop. 74, book 1.

PROPOSITION VIII. THEOREM VIII.

In two ſpheres mutually gravitating each towards the other, if the matter in places on all ſides round about and equidiſtant from the centres is ſimilar, the weight of either ſphere towards the other will be reciprocally as the ſquare of the diſtance between their centres.

After I had found that the force of gravity towards a whole planet did arife from and was compounded of the forces of gravity towards all its parts, and towards every one part was in the reciprocal proportion of the fquares of the diftances from the part, I was yet in doubt whether that reciprocal duplicate proportion did accurately hold, or but nearly fo, in the total force compounded of fo many partial ones; for it might be that the proportion which accurately enough took place in greater diftances fhould be wide of the truth near the furface of the planet, where the diftances of the particles are unequal, and their fituation diffimilar. But by the help of prop. 75 and 76, book 1, and their corollaries, I was at laft fatisfied of the truth of the propofition, as it now lies before us.

Cor. 1. Hence we may find and compare together the weights of bodies towards different planets; for the weights of bodies revolving in circles about planets are (by cor. 2, prop. 4, book 1) as the diameters of the circles directly, and the fquares of their periodic times reciprocally; and their weights at the furfaces of the planets, or at any other diftances from their centres, are (by this prop.) greater or lefs in the reciprocal duplicate proportion of the diftances. Thus from the periodic times of Venus, revolving about the fun, in $224^d.\ 16\frac{3}{4}^h.$ of the utmoft circumjovial fatellite revolving about Jupiter, in $16^d.\ 16\frac{8}{15}^h.$; of the Huygenian fatellite about Saturn in $15^d.\ 22\frac{2}{3}^h.$; and of the moon about the earth in $27^d.\ 7^h.\ 43'$; compared with the mean diftance of Venus from the fun, and with the greateft heliocentric elongations of the outmoft circumjovial fatellite from Jupiter's centre, $8'\ 16''$; of the Huygenian fatellite from the centre of Saturn, $3'\ 4''$; and of the moon from the earth, $10'\ 33''$: by computation I found that the weight of equal bodies, at equal diftances from the centres of the fun, of Jupiter, of Saturn, and of the earth, towards the fun, Jupiter, Saturn, and the earth, were one to another, as $1, \frac{1}{1067}, \frac{1}{3021}$, and $\frac{1}{169282}$ refpectively. Then becaufe as the diftances are increafed or diminifhed, the weights are diminifhed or increafed in a duplicate ratio, the weights of equal bodies towards the fun, Jupiter, Saturn, and

the earth, at the diſtances 10000, 997, 791, and 109 from their centres, that is at their very ſuperficies, will be as 10000, 943, 529, and 435 reſpectively. How much the weights of bodies are at the ſuperficies of the moon, will be ſhewn hereafter.

Cor. 2. Hence likewiſe we diſcover the quantity of matter in the ſeveral planets; for their quantities of matter are as the forces of gravity at equal diſtances from their centres; that is, in the ſun, Jupiter, Saturn, and the earth, as 1, $\frac{1}{1067}$, $\frac{1}{3021}$, and $\frac{1}{169282}$ reſpectively. If the parallax of the ſun be taken greater or leſs than 10″ 30‴, the quantity of matter in the earth muſt be augmented or diminiſhed in the triplicate of that proportion.

Cor. 3. Hence alſo we find the denſities of the planets; for (by prop. 72, book 1) the weights of equal and ſimilar bodies towards ſimilar ſpheres are, at the ſurfaces of thoſe ſpheres, as the diameters of the ſpheres; and therefore the denſities of diſſimilar ſpheres are as thoſe weights applied to the diameters of the ſpheres. But the true diameters of the ſun, Jupiter, Saturn, and the earth, were one to another as 10000, 997, 791, and 109; and the weights towards the ſame as 10000, 943, 529, and 435 reſpectively; and therefore their denſities are as 100, 94½, 67, and 400. The denſity of the earth, which comes out by this computation, does not depend upon the parallax of the ſun, but is determined by the parallax of the moon, and therefore is here truly defined. The ſun, therefore, is a little denſer than Jupiter, and Jupiter than Saturn, and the earth four times denſer than the ſun; for the ſun, by its great heat, is kept in a ſort of a rarefied ſtate. The moon is denſer than the earth, as ſhall appear afterwards.

Cor. 4. The ſmaller the planets are, they are, *cæteris paribus*, of ſo much the greater denſity; for ſo the powers of gravity on their ſeveral ſurfaces come nearer to equality. They are likewiſe, *cæteris paribus*, of the greater denſity, as they are nearer to the ſun. So Jupiter is more denſe than Saturn, and the earth than Jupiter; for the planets were to be placed at different diſtances from the ſun, that, according to their degrees of denſity, they might enjoy a greater or leſs

proportion of the ſun's heat. Our water, if it were removed as far as the orb of Saturn, would be converted into ice, and in the orb of Mecury would quickly fly away in vapour; for the light of the ſun, to which its heat is proportional, is ſeven times denſer in the orb of Mercury than with us: and by the thermometer I have found that a ſevenfold heat of our ſummer ſun will make water boil. Nor are we to doubt that the matter of Mercury is adapted to its heat, and is therefore more denſe than the matter of our earth; ſince, in a denſer matter, the operations of Nature require a ſtronger heat.

PROPOSITION IX. THEOREM IX.

That the force of gravity, conſidered downwards from the ſurface of the planets, decreaſes nearly in the proportion of the diſtances from their centres.

If the matter of the planet were of an uniform denſity, this propoſition would be accurately true (by prop. 73, book 1). The error, therefore, can be no greater than what may ariſe from the inequality of the denſity.

PROPOSITION X. THEOREM X.

That the motions of the planets in the heavens may ſubſiſt an exceedingly long time.

In the ſcholium of prop. 40, book 2, I have ſhewed that a globe of water frozen into ice, and moving freely in our air, in the time that it would deſcribe the length of its ſemi-diameter, would loſe by the reſiſtance of the air $\frac{1}{4\,5\,8\,6}$ part of its motion; and the ſame proportion holds nearly in all globes, how great ſoever, and moved with whatever velocity. But that our globe of earth is of greater denſity than it would be if the whole conſiſted of water only, I thus make out. If the whole conſiſted of water only, whatever was of leſs denſity than water, becauſe of its leſs ſpecific gravity, would emerge and float above. And upon this account, if a globe of terreſtrial matter, covered on all ſides with water, was leſs denſe than water, it would emerge ſomewhere; and, the ſubſiding water falling back, would be gathered to the oppoſite ſide. And ſuch is the condition of our earth, which in a great meaſure is covered with ſeas. The earth, if it wa not for

its greater denſity, would emerge from the ſeas, and, according to its degree of levity, would be raiſed more or leſs above their ſurface, the water of the ſeas flowing backwards to the oppoſite ſide. By the ſame argument, the ſpots of the ſun, which float upon the lucid matter thereof, are lighter than that matter; and, however the planets have been formed while they were yet in fluid maſſes, all the heavier matter ſubſided to the centre. Since, therefore, the common matter of our earth on the ſurface thereof is about twice as heavy as water, and a little lower, in mines, is found about three, or four, or even five times more heavy, it is probable that the quantity of the whole matter of the earth may be five or ſix times greater than if it conſiſted all of water; eſpecially ſince I have before ſhewed that the earth is about four times more denſe than Jupiter. If, therefore, Jupiter is a little more denſe than water, in the ſpace of thirty days, in which that planet deſcribes the length of 459 of its ſemi-diameters, it would, in a medium of the ſame denſity with our air, loſe almoſt a tenth part of its motion. But ſince the reſiſtance of mediums decreaſes in proportion to their weight or denſity, ſo that water, which is $13\frac{3}{5}$ times lighter than quickſilver, reſiſts leſs in that proportion; and air, which is 860 times lighter than water, reſiſts leſs in the ſame proportion; therefore in the heavens, where the weight of the medium in which the planets move is immenſely diminiſhed, the reſiſtance will almoſt vaniſh.

It is ſhewn in the ſcholium of prop. 22, book 2, that at the height of 200 miles above the earth the air is more rare than it is at the ſuperficies of the earth in the ratio of 30 to 0,0000000000003998, or as 75000000000000 to 1 nearly. And hence the planet Jupiter, revolving in a medium of the ſame denſity with that ſuperior air, would not loſe by the reſiſtance of the medium the 1000000th part of its motion in 1000000 years. In the ſpaces near the earth the reſiſtance is produced only by the air, exhalations, and vapours. When theſe are carefully exhauſted by the air-pump from under the receiver, heavy bodies fall within the receiver with perfect freedom, and without the leaſt ſenſible reſiſtance: gold itſelf, and the lighteſt down, let fall together, will deſcend with equal

velocity; and though they fall through a ſpace of four, ſix, and eight feet, they will come to the bottom at the ſame time; as appears from experiments. And therefore the celeſtial regions being perfectly void of air and exhalations, the planets and comets meeting, no ſenſible reſiſtance in thoſe ſpaces, will continue their motions through them for an immenſe tract of time.

HYPOTHESIS I.

That the centre of the ſyſtem of the world is immovable.

This is acknowledged by all, while ſome contend that the earth, others that the ſun, is fixed in that centre. Let us ſee what may from hence follow.

PROPOSITION XI. THEOREM XI.

That the common centre of gravity of the earth, the ſun, and all the planets, is immovable

For (by cor. 4 of the laws) that centre either is at reſt, or moves uniformly forward in a right line; but if that centre moved, the centre of the world would move alſo, againſt the hypotheſis.

PROPOSITION XII. THEOREM XII.

That the ſun is agitated by a perpetual motion, but never recedes far from the common centre of gravity of all the planets.

For ſince (by cor. 2, prop 8) the quantity of matter in the ſun is to the quantity of matter in Jupiter as 1067 to 1; and the diſtance of Jupiter from the ſun is to the ſemi-diameter of the ſun in a proportion but a ſmall matter greater, the common centre of gravity of Jupiter and the ſun will fall upon a point a little without the ſurface of the ſun. By the ſame argument, ſince the quantity of matter in the ſun is to the quantity of matter in Saturn as 3021 to 1, and the diſtance of Saturn from the ſun is to the ſemi-diameter of the ſun in a proportion but a ſmall matter leſs, the common centre of gravity of Saturn and the ſun will fall upon a point a little within the ſurface of the ſun. And, purſuing the principles of this computation, we ſhould find that though the earth and all the planets were placed on one ſide of the ſun, the diſtance of the common centre of gravity of all from the centre of the ſun would ſcarcely amount to one diameter

of the ſun. In other caſes, the diſtances of thoſe centres are always leſs; and, therefore, ſince that centre of gravity is in perpetual reſt, the ſun, according to the various poſitions of the planets, muſt perpetually be moved every way, but will never recede far from that centre.

Cor. Hence the common centre of gravity of the earth, the ſun, and all the planets, is to be eſteemed the centre of the world; for ſince the earth, the ſun, and all the planets, mutually gravitate one towards another, and are therefore, according to their powers of gravity, in perpetual agitation, as the laws of motion require, it is plain that their moveable centres cannot be taken for the immovable centre of the world. If that body were to be placed in the centre, towards which other bodies gravitate moſt (according to common opinion), that privilege ought to be allowed to the ſun; but ſince the ſun itſelf is moved, a fixed point is to be choſen from which the centre of the ſun recedes leaſt, and from which it would recede yet leſs if the body of the ſun were denſer and greater, and therefore leſs apt to be moved.

PROPOSITION XIII. THEOREM XIII.

The planets move in ellipſes which have their common focus in the centre of the ſun; and, by radii drawn to that centre, they deſcribe areas proportional to the times of deſcription.

We have diſcourſed above of theſe motions from the phænomena. Now that we know the principles on which they depend, from thoſe principles we deduce the motions of the heavens *a priori*. Becauſe the weights of the planets towards the ſun are reciprocally as the ſquares of their diſtances from the ſun's centre, if the ſun was at reſt, and the other planets did not mutually act one upon another, their orbits would be ellipſes, having the ſun in their common focus; and they would deſcribe areas proportional to the times *of deſcription*, by prop. 1 and 11, and cor. 1, prop. 13, book 1. But the mutual actions of the planets one upon another are ſo very ſmall, that they may be neglected; and, by prop. 66, book 1, they leſs diſturb the motions of the planets around the ſun in motion than if thoſe motions were performed about the ſun at reſt.

It is true, that the action of Jupiter upon Saturn is not to be neglected; for the force of gravity towards Jupiter is to the force of gravity towards the sun as 1 to 1067; and therefore in the conjunction of Jupiter and Saturn, because the distance of Saturn from Jupiter is to the distance of Saturn from the sun almost as 4 to 9, the gravity of Saturn towards Jupiter will be to the gravity of Saturn towards the sun as 81 to 16 × 1067; or, as 1 to about 211. And hence arises a perturbation of the orb of Saturn in every conjunction of this planet with Jupiter, so sensible, that astronomers are puzzled with it. As the planet is differently situated in these conjunctions, its eccentricity is sometimes augmented, sometimes diminished; its aphelion is sometimes carried forwards, sometimes backwards, and its mean motion is by turns accelerated and retarded; yet the whole error in its motion about the sun, though arising from so great a force, may be almost avoided (except in the mean motion) by placing the lower focus of its orbit in the common centre of gravity of Jupiter and the sun (according to prop. 67, book 1), and therefore that error, when it is greatest, scarcely exceeds two minutes; and the greatest error in the mean motion scarcely exceeds two minutes yearly. But in the conjunction of Jupiter and Saturn, the accelerative forces of gravity of the sun towards Saturn, of Jupiter towards Saturn, and of Jupiter towards the sun, are almost as 16, 81, and $\frac{16 \times 81 \times 3021}{25}$, or 156609; and therefore the difference of the forces of gravity of the sun towards Saturn, and of Jupiter towards Saturn, is to the force of gravity of Jupiter towards the sun as 65 to 156609, or as 1 to 2409. But the greatest power of Saturn to disturb the motion of Jupiter is proportional to this difference; and therefore the perturbation of the orbit of Jupiter is much less than that of Saturn's. The perturbations of the other orbits are yet far less, except that the orbit of the earth is sensibly disturbed by the moon. The common centre of gravity of the earth and moon moves in an ellipsis about the sun in the focus thereof, and, by a radius drawn to the sun, describes areas proportional to the times of description. But the earth

in the mean time by a menſtrual motion is revolved about this common centre.

PROPOSITION XIV. THEOREM XIV.

The aphelions and nodes of the orbits of the planets are fixed.

The aphelions are immovable, by prop. 11, book 1; and ſo are the planes of the orbits, by prop. 1 of the ſame book. And if the planes are fixed, the nodes muſt be ſo too. It is true, that ſome inequalities may ariſe from the mutual actions of the planets and comets in their revolutions; but theſe will be ſo ſmall, that they may be here paſſed by.

Cor. 1. The fixed ſtars are immovable, ſeeing they keep the ſame poſition to the aphelions and nodes of the planets.

Cor. 2. And ſince theſe ſtars are liable to no ſenſible parallax from the annual motion of the earth, they can have no force, becauſe of their immenſe diſtance, to produce any ſenſible effect in our ſyſtem. Not to mention that the fixed ſtars, every where promiſcuouſly diſperſed in the heavens, by their contrary attractions deſtroy their mutual actions, by prop. 70, book 1.

SCHOLIUM.

Since the planets near the ſun (viz. Mercury, Venus, the Earth, and Mars) are ſo ſmall that they can act but with little force upon each other, therefore their aphelions and nodes muſt be fixed, excepting in ſo far as they are diſturbed by the actions of Jupiter and Saturn, and other higher bodies. And hence we may find, by the theory of gravity, that their aphelions move a little *in conſequentia*, in reſpect of the fixed ſtars, and that in the ſeſquiplicate proportion of their ſeveral diſtances from the ſun. So that if the aphelion of Mars, in the ſpace of an hundred years, is carried 33′ 20″ *in conſequentia*, in reſpect of the fixed ſtars, the aphelions of the Earth, of Venus, and of Mercury, will in an hundred years be carried forwards 17′ 40″, 10′ 53″, and 4′ 16″, reſpectively. But theſe motions are ſo inconſiderable, that we have neglected them in this propoſition.

PROPOSITION XV. PROBLEM I.

To find the principal diameters of the orbits of the planets.

They are to be taken in the ſub-ſeſquiplicate proportion of the periodic times, by prop. 15, book 1, and then to be ſeverally augmented in the proportion of the ſum of the maſſes of matter in the ſun and each planet to the firſt of two mean proportionals betwixt that ſum and the quantity of matter in the ſun, by prop. 60, book 1.

PROPOSITION XVI. PROBLEM II.

To find the eccentricities and aphelions of the planets.

This problem is reſolved by prop. 18, book 1.

PROPOSITION XVII. THEOREM XV.

That the diurnal motions of the planets are uniform, and that the libration of the moon ariſes from its diurnal motion.

The propoſition is proved from the firſt law of motion, and cor. 22, prop. 66, book 1. Jupiter, with reſpect to the fixed ſtars, revolves in 9^h. 56′; Mars in 24^h. 39′; Venus in about 23^h; the Earth in 23^h. 56′; the Sun in $25\frac{1}{2}$ days, and the moon in 27 days, 7 hours, 43′. Theſe things appear by the phænomena. The ſpots in the ſun's body return to the ſame ſituation on the ſun's diſk, with reſpect to the earth, in $27\frac{1}{2}$ days; and therefore with reſpect to the fixed ſtars the ſun revolves in about $25\frac{1}{2}$ days. But becauſe the lunar day, ariſing from its uniform revolution about its axis, is menſtrual, *that is, equal to the time of its periodic revolution in its orb,* therefore the ſame face of the moon will be always nearly turned to the upper focus of its orb; but, as the ſituation of that focus requires, will deviate a little to one ſide and to the other from the earth in the lower focus; and this is the libration in longitude; for the libration in latitude ariſes from the moon's latitude, and the inclination of its axis to the plane of the ecliptic. This theory of the libration of the moon, Mr. *N. Mercator,* in his Aſtronomy, publiſhed at the beginning of the year 1676, explained more fully out of the letters I ſent him. The utmoſt ſatellite of Saturn ſeems to revolve about its axis with a motion like this of the moon, reſpecting Saturn continually with the ſame face; for in its revolution round Saturn, as often as it comes to the eaſtern part of its orbit, it is

ſcarcely viſible, and generally quite diſappears; which is like to be occaſioned by ſome ſpots in that part of its body, which is then turned toward the earth, as M. *Caſſini* has obſerved. So alſo the utmoſt ſatellite of Jupiter ſeems to revolve about its axis with a like motion, becauſe in that part of its body which is turned from Jupiter it has a ſpot, which always appears as if it were in Jupiter's own body, whenever the ſatellite paſſes between Jupiter and our eye.

PROPOSITION XVIII. THEOREM XVI.

That the axes of the planets are leſs than the diameters drawn perpendicular to the axes.

The equal gravitation of the parts on all ſides would give a ſpherical figure to the planets, if it was not for their diurnal revolution in a circle. By that circular motion it comes to paſs that the parts receding from the axis endeavour to aſcend about the equator; and therefore if the matter is in a fluid ſtate, by its aſcent towards the equator it will enlarge the diameters there, and by its deſcent towards the poles it will ſhorten the axis. So the diameter of Jupiter (by the concurring obſervations of aſtronomers) is found ſhorter betwixt pole and pole than from eaſt to weſt. And, by the ſame argument, if our earth was not higher about the equator than at the poles, the ſeas would ſubſide about the poles, and, riſing towards the equator, would lay all things there under water.

PROPOSITION XIX. PROBLEM III.

To find the proportion of the axis of a planet to the diameters perpendicular thereto.

Our countryman, Mr. *Norwood,* meaſuring a diſtance of 905751 feet of *London* meaſure between *London* and *York,* in 1635, and obſerving the difference of latitudes to be 2° 28′, determined the meaſure of one degree to be 367196 feet of *London* meaſure, that is 57300 *Paris* toiſes. M. *Picart,* meaſuring an arc of one degree, and 22′ 55″ of the meridian between *Amiens* and *Malvoiſine,* found an arc of one degree to be 57060 *Paris* toiſes. M. *Caſſini,* the father, meaſured the diſtance upon the meridian from the town of *Collioure* in *Rouſſillon* to the Obſervatory of *Paris;* and his ſon added the diſtance from the Obſervatory to the Citadel of *Dunkirk.*

The whole diſtance was 486156½ toiſes, and the difference of the latitudes of *Collioure* and *Dunkirk* was 8 degrees, and 31′ 11⅝″. Hence an arc of one degree appears to be 57061 *Paris* toiſes. And from theſe meaſures we conclude that the circumference of the earth is 123249600, and its ſemi-diameter 19615800 *Paris* feet, upon the ſuppoſition that the earth is of a ſpherical figure.

In the latitude of *Paris* a heavy body falling in a ſecond of time deſcribes 15 *Paris* feet, 1 inch, 1 line, as above, that is, 2173 lines $\frac{7}{9}$. The weight of the body is diminiſhed by the weight of the ambient air. Let us ſuppoſe the weight loſt thereby to be $\frac{1}{11000}$ part of the whole weight; then that heavy body falling *in vacuo* will deſcribe a height of 2174 lines in one ſecond of time.

A body in every ſidereal day of 23^h. 56′ 4″ uniformly revolving in a circle at the diſtance of 19615800 feet from the centre, in one ſecond of time deſcribes an arc of 1433, 46 feet; the verſed ſine of which is 0,05236561 feet, or 7,54064 lines. And therefore the force with which bodies deſcend in the latitude of *Paris* is to the centrifugal force of bodies in the equator ariſing from the diurnal motion of the earth as 2174 to 7,54064.

The centrifugal force of bodies in the equator is to the centrifugal force with which bodies recede directly from the earth in the latitude of *Paris* 48° 50′ 10″ in the duplicate proportion of the radius to the coſine of the latitude, that is, as 7,54064 to 3,267. Add this force to the force with which bodies deſcend by their weight in the latitude of *Paris*, and a body, in the latitude of *Paris*, falling by its whole undiminiſhed force of gravity, in the time of one ſecond, will deſcribe 2177,267 lines, or 15 *Paris* feet, 1 inch, and 5,267 lines. And the total force of gravity in that latitude will be to the centrifugal force of bodies in the equator of the earth as 2177,267 to 7,54064, or as 289 to 1.

Wherefore if APBQ (Pl. 10, Fig. 1) repreſent the figure of the earth, now no longer ſpherical, but generated by the rotation of an ellipſis about its leſſer axis; and ACQqca a canal full of water, reaching from the pole Qq to the centre Cc,

and thence riſing to the equator Aa; the weight of the water in the leg of the canal ACca will be to the weight of water in the other leg QCcq as 289 to 288, becauſe the centrifugal force ariſing from the circular motion ſuſtains and takes off one of the 289 parts of the weight (in the one leg), and the weight of 288 in the other ſuſtains the reſt. But by computation (from cor. 2, prop. 91, book 1) I find, that, if the matter of the earth was all uniform, and without any motion, and its axis PQ were to the diameter AB as 100 to 101, the force of gravity in the place Q towards the earth would be to the force of gravity in the ſame place Q towards a ſphere deſcribed about the centre C with the radius PC, or QC, as 126 to 125. And, by the ſame argument, the force of gravity in the place A towards the ſpheroid generated by the rotation of the ellipſis APBQ about the axis AB is to the force of gravity in the ſame place A, towards the ſphere deſcribed about the centre C with the radius AC, as 125 to 126. But the force of gravity in the place A towards the earth is a mean proportional betwixt the forces of gravity towards that ſpheroid and this ſphere; becauſe the ſphere, by having its diameter PQ diminiſhed in the proportion of 101 to 100, is transformed into the figure of the earth; and this figure, by having a third diameter perpendicular to the two diameters AB and PQ diminiſhed in the ſame proportion, is converted into the ſaid ſpheroid; and the force of gravity in A, in either caſe, is diminiſhed nearly in the ſame proportion. Therefore the force of gravity in A towards the ſphere deſcribed about the centre C with the radius AC, is to the force of gravity in A towards the earth as 126 to $125\frac{1}{2}$. And the force of gravity in the place Q towards the ſphere deſcribed about the centre C with the radius QC, is to the force of gravity in the place A towards the ſphere deſcribed about the centre C, with the radius AC, in the proportion of the diameters (by prop. 72, book 1), that is, as 100 to 101. If, therefore, we compound thoſe three proportions 126 to 125, 126 to $125\frac{1}{2}$, and 100 to 101, into one, the force of gravity in the place Q towards the earth will be to the force of gravity in the

place A towards the earth as $126 \times 126 \times 100$ to $125 \times 125\frac{1}{2} \times 101$; or as 501 to 500.

Now since (by cor. 3, prop. 91, book 1) the force of gravity in either leg of the canal ACca, or QCcq, is as the distance of the places from the centre of the earth, if those legs are conceived to be divided by transverse, parallel, and equidistant surfaces, into parts proportional to the wholes, the weights of any number of parts in the one leg ACca will be to the weights of the same number of parts in the other leg as their magnitudes and the accelerative forces of their gravity conjunctly, that is, as 101 to 100, and 500 to 501, or as 505 to 501. And therefore if the centrifugal force of every part in the leg ACca, arising from the diurnal motion, was to the weight of the same part as 4 to 505, so that from the weight of every part, conceived to be divided into 505 parts, the centrifugal force might take off four of those parts, the weights would remain equal in each leg, and therefore the fluid would rest in an equilibrium. But the centrifugal force of every part is to the weight of the same part as 1 to 289; that is, the centrifugal force, which should be $\frac{4}{505}$ parts of the weight, is only $\frac{1}{289}$ part thereof. And, therefore, I say, by the rule of proportion, that if the centrifugal force $\frac{4}{505}$ make the height of the water in the leg ACca to exceed the height of the water in the leg QCcq by one $\frac{1}{100}$ part of its whole height, the centrifugal force $\frac{1}{289}$ will make the excess of the height in the leg ACca only $\frac{1}{289}$ part of the height of the water in the other leg QCcq; and therefore the diameter of the earth at the equator, is to its diameter from pole to pole as 230 to 229. And since the mean semi-diameter of the earth, according to *Picart*'s mensuration, is 19615800 *Paris* feet, or 3923,16 miles (reckoning 5000 feet to a mile), the earth will be higher atthe equator than at the poles by 85472 feet, or $17\frac{1}{10}$ miles. And its height at the equator will be about 19658600 feet, and at the poles 19573000 feet.

If, the density and periodic time of the diurnal revolution remaining the same, the planet was greater or less than the earth, the proportion of the centrifugal force to that of gravity, and therefore also of the diameter betwixt the poles to the diameter at the equator, would likewise remain the same. But if the diurnal motion was accelerated or retarded in any

proportion, the centrifugal force would be augmented or diminiſhed nearly in the ſame duplicate proportion; and therefore the difference of the diameters will be increaſed or diminiſhed in the ſame duplicate ratio very nearly. And if the denſity of the planet was augmented or diminiſhed in any proportion, the force of gravity tending towards it would alſo be augmented or diminiſhed in the ſame proportion; and the difference of the diameters contrariwiſe would be diminiſhed in proportion as the force of gravity is augmented, and augmented in proportion as the force of gravity is diminiſhed. Wherefore, ſince the earth, in reſpect of the fixed ſtars, revolves in $23^h.\ 56'$, but Jupiter in $9^h.\ 56'$, and the ſquares of their periodic times are as 29 to 5, and their denſities as 400 to $94\frac{1}{2}$, the difference of the diameters of Jupiter will be to its leſſer diameter as $\frac{29}{5} \times \frac{400}{941} \times \frac{229}{1}$ to 1, or as 1 to $9\frac{1}{3}$, nearly. Therefore the diameter of Jupiter from eaſt to weſt is to its diameter from pole to pole nearly as $10\frac{1}{3}$ to $9\frac{1}{3}$. Therefore ſince its greateſt diameter is $37''$, its leſſer diameter lying between the poles will be $33''\ 25'''$. Add thereto about $3''$ for the irregular refraction of light, and the apparent diameters of this planet will become $40''$ and $36''\ 25'''$; which are to each other as $11\frac{1}{6}$ to $10\frac{1}{6}$, very nearly. Theſe things are ſo upon the ſuppoſition that the body of Jupiter is uniformly denſe. But now if its body be denſer towards the plane of the equator than towards the poles, its diameters may be to each other as 12 to 11, or 13 to 12, or perhaps as 14 to 13.

And *Caſſini* obſerved, in year 1691, that the diameter of Jupiter reaching from eaſt to weſt is greater by about a fifteenth part than the other diameter. Mr. *Pound* with his 123 feet teleſcope, and an excellent micrometer, meaſured the diameters of Jupiter in the year 1719, and found them as follow.

The Times.		Greateſt diam.	Leſſer diam.	The diam. to each other.
Day.	Hours.	Parts.	Parts.	
January 28	6	13,40	12,28	As 12 to 11
March 6	7	13,12	12,20	$13\frac{3}{4}$ to $12\frac{3}{4}$
March 9	7	13,12	12,08	$12\frac{2}{3}$ to $11\frac{2}{3}$
April 9	9	12,32	11,48	$14\frac{1}{2}$ to $13\frac{1}{2}$

So that the theory agrees with the phænomena; for the planets are more heated by the ſun's rays towards their equators, and therefore are a little more condenſed by that heat than towards their poles.

Moreover, that there is a diminution of gravity occaſioned by the diurnal rotation of the earth, and therefore the earth riſes higher there than it does at the poles (ſuppoſing that its matter is uniformly denſe), will appear by the experiments of pendulums related under the following propoſition.

PROPOSITION XX. PROBLEM IV.

To find and compare together the weights of bodies in the different regions of our earth.

Becauſe the weights of the unequal legs of the canal of water ACQqca are equal; and the weights of the parts proportional to the whole legs, and alike ſituated in them, are one to another as the weights of the wholes, and therefore equal betwixt themſelves; the weights of equal parts, and alike ſituated in the legs, will be reciprocally as the legs, that is, reciprocally as 230 to 229. And the caſe is the ſame in all homogeneous equal bodies alike ſituated in the legs of the canal. Their weights are reciprocally as the legs, that is, reciprocally as the diſtances of the bodies from the centre of the earth. Therefore if the bodies are ſituated in the uppermoſt parts of the canals, or on the ſurface of the earth, their weights will be one to another reciprocally as their diſtances from the centre. And, by the ſame argument, the weights in all other places round the whole ſurface of the earth are reciprocally as the diſtances of the places from the centre; and, therefore, in the hypotheſis of the earth's being a ſpheroid, are given in proportion.

Whence ariſes this theorem, that the increaſe of weight in paſſing from the equator to the poles is nearly as the verſed ſine of double the latitude; or, which comes to the ſame thing, as the ſquare of the right ſine of the latitude; and the arcs of the degrees of latitude in the meridian increaſe nearly in the ſame proportion. And, therefore, ſince the latitude of *Paris* is 48° 50′, that of places under the equator 00° 00′, and that of places under the poles 90°; and the verſed ſines of double thoſe arcs are 11334,00000 and 20000, the radius be-

ing 10000; and the force of gravity at the pole is to the force of gravity at the equator as 230 to 229; and the excefs of the force of gravity at the pole to the force of gravity at the equator as 1 to 229; the excefs of the force of gravity in the latitude of *Paris* will be to the force of gravity at the equator as $1 \times \frac{11334}{20000}$ to 229, or as 5667 to 2290000. And therefore the whole forces of gravity in thofe places will be one to the other as 2295667 to 2290000. Wherefore fince the lengths of pendulums vibrating in equal times are as the forces of gravity, and, in the latitude of *Paris*, the length of a pendulum vibrating feconds is 3 *Paris* feet, and $8\frac{1}{2}$ lines, or rather, becaufe of the weight of the air, $8\frac{5}{9}$ lines, the length of a pendulum vibrating in the fame time under the equator will be fhorter by 1,087 lines. And by a like calculus the following table is made.

Latitude of the place.	Length of the pendulum.		Meafure of one degree in the meridian.
Deg.	Feet.	Lines.	Toifes.
0	3 .	7,468	56637
5	3 .	7,482	56642
10	3 .	7,526	56659
15	3 .	7,596	56687
20	3 .	7,692	56724
25	3 .	7,812	56769
30	3 .	7,948	56823
35	3 .	8,099	56882
40	3 .	8,261	56945
1	3 .	8,294	56958
2	3 .	8,327	56971
3	3 .	8,361	56984
4	3 .	8,394	56997
45	3 .	8,428	57010
6	3 .	8,461	57022
7	3 .	8,494	57035
8	3 .	8,528	57048
9	3 .	8,561	57061
50	3 .	8,594	57074
55	3 .	8,756	57137
60	3 .	8,907	57196
65	3 .	9,044	57250
70	3 .	9,162	57295
75	3 .	9,258	57332
80	3 .	9,329	57360
85	3 .	9,372	57377
90	3 .	9,387	57382

By this table, therefore, it appears that the inequality of degrees is so small, that the figure of the earth, in geographical matters, may be considered as spherical; especially if the earth be a little denser towards the plane of the equator than towards the poles.

Now several astronomers, sent into remote countries to make astronomical observations, have found that pendulum clocks do accordingly move slower near the equator than in our climates. And, first of all, in the year 1672, M. *Richer* took notice of it in the island of *Cayenne;* for when, in the month of *August,* he was observing the transits of the fixed stars over the meridian, he found his clock to go slower than it ought in respect of the mean motion of the sun at the rate of 2′ 28″ a day. Therefore, fitting up a simple pendulum to vibrate in seconds, which were measured by an excellent clock, he observed the length of that simple pendulum; and this he did over and over every week for ten months together. And upon his return to *France,* comparing the length of that pendulum with the length of the pendulum at *Paris* (which was 3 *Paris* feet and $8\frac{3}{5}$ lines), he found it shorter by $1\frac{1}{4}$ line.

Afterwards, our friend Dr. *Halley,* about the year 1677, arriving at the island of St. *Helen,* found his pendulum clock to go slower there than at *London,* without marking the difference. But he shortened the rod of his clock by more than the $\frac{1}{8}$ of an inch, or $1\frac{1}{2}$ line; and to effect this, because the length of the screw at the lower end of the rod was not sufficient, he interposed a wooden ring betwixt the nut and the ball.

Then, in the year 1682, M. *Varin* and M. *des Hayes* found the length of a simple pendulum vibrating in seconds at the Royal Observatory of *Paris* to be 3 feet and $8\frac{5}{9}$ lines. And by the same method in the island of *Goree,* they found the length of an isochronal pendulum to be 3 feet and $6\frac{5}{9}$ lines, differing from the former by two lines. And in the same year, going to the islands of *Guadaloupe* and *Martinico,* they found that the length of an isochronal pendulum in those islands was 3 feet and $6\frac{1}{2}$ lines.

After this, M. *Couplet*, the ſon, in the month of *July* 1697, at the Royal Obſervatory of *Paris*, ſo fitted his pendulum clock to the mean motion of the ſun, that for a conſiderable time together the clock agreed with the motion of the ſun. In *November* following, upon his arrival at *Liſbon*, he found his clock to go ſlower than before at the rate of 2′ 13″ in 24 hours. And next *March* coming to *Paraiba*, he found his clock to go ſlower there than at *Paris*, and at the rate 4′ 12″ in 24 hours; and he affirms, that the pendulum vibrating in ſeconds was ſhorter at *Liſbon* by $2\frac{1}{2}$ line, and at *Paraiba* by $3\frac{2}{3}$ lines, than at *Paris*. He had done better to have reckoned thoſe differences $1\frac{1}{3}$ and $2\frac{5}{9}$; for theſe differences correſpond to the differences of the times 2′ 13″ and 4′ 12″. But this gentleman's obſervations are ſo groſs, that we cannot confide in them.

In the following years, 1699 and 1700, M. *des Hayes*, making another voyage to *America*, determined that in the iſlands of *Cayenne* and *Granada* the length of the pendulum vibrating in ſeconds was a ſmall matter leſs than 3 feet and $6\frac{1}{2}$ lines; that in the iſland of St. *Chriſtophers* it was 3 feet and $6\frac{1}{4}$ lines; and in the iſland of St. *Domingo* 3 feet and 7 lines.

And in the year 1704, P. *Feuillé*, at *Puerto Bello* in *America*, found that the length of the pendulum vibrating in ſeconds was 3 *Paris* feet, and only $5\frac{7}{12}$ lines, that is, almoſt 3 lines ſhorter than at *Paris*; but the obſervation was faulty. For afterwards, going to the iſland of *Martinico*, he found the length of the iſochronal pendulum there 3 *Paris* feet and $5\frac{10}{12}$ lines.

Now the latitude of *Paraiba* is 6° 38′ ſouth; that of *Puerto Bello* 9° 33′ north; and the latitudes of the iſlands *Cayenne*, *Goree*, *Gaudaloupe*, *Martinico*, *Granada*, St. *Chriſtophers*, and St. *Domingo*, are reſpectively 4° 55′, 14° 40″, 14° 00′, 14° 44′, 12° 06′, 17° 19′, and 19° 48′, north. And the exceſſes of the length of the pendulum at *Paris* above the lengths of the iſochronal pendulums obſerved in thoſe latitudes are a little greater than by the table of the lengths of the pendulum before computed. And therefore the earth is a little higher

under the equator than by the preceding calculus, and a little denſer at the centre than in mines near the ſurface, unleſs, perhaps, the heats of the torrid zone have a little extended the length of the pendulums.

For M. *Picart* has obſerved, that a rod of iron, which in froſty weather in the winter ſeaſon was one foot long, when heated by fire, was lengthened into one foot and $\frac{1}{4}$ line. Afterwards M. *de la Hire* found that a rod of iron, which in the like winter ſeaſon was 6 feet long, when expoſed to the heat of the ſummer ſun, was extended into 6 feet and $\frac{2}{3}$ line. In the former caſe the heat was greater than in the latter; but in the latter it was greater than the heat of the external parts of a human body; for metals expoſed to the ſummer ſun acquire a very conſiderable degree of heat. But the rod of a pendulum clock is never expoſed to the heat of the ſummer ſun, nor ever acquires a heat equal to that of the external parts of a human body; and, therefore, though the 3 feet rod of a pendulum clock will indeed be a little longer in the ſummer than in the winter ſeaſon, yet the difference will ſcarcely amount to $\frac{1}{4}$ line. Therefore the total difference of the lengths of iſochronal pendulums in different climates cannot be aſcribed to the difference of heat; nor, indeed, to the miſtakes of the *French* aſtronomers. For although there is not a perfect agreement betwixt their obſervations, yet the errors are ſo ſmall that they may be neglected; and in this they all agree, that iſochronal pendulums are ſhorter under the equator than at the Royal Obſervatory of *Paris*, by a difference not leſs than $1\frac{1}{4}$ line, nor greater than $2\frac{2}{3}$ lines. By the obſervations of M. *Richer* in the iſland of *Cayenne*, the difference was $1\frac{1}{4}$ line. That difference being corrected by thoſe of M. *des Hayes*, becomes $1\frac{1}{2}$ line or $1\frac{1}{4}$ line. By the leſs accurate obſervations of others, the ſame was made about two lines. And this diſagreement might ariſe partly from the errors of the obſervations, partly from the diſſimilitude of the internal parts of the earth, and the height of mountains; partly from the different heats of the air.

I take an iron rod of 3 feet long to be ſhorter by a ſixth part of one line in winter time with us here in *England* than in the ſummer. Becauſe of the great heats under the

equator, ſubduct this quantity from the difference of one line and a quarter obſerved by M. *Richer*, and there will remain one line $\frac{1}{12}$, which agrees very well with $1\frac{87}{1000}$ line collected, by the theory a little before. M. *Richer* repeated his obſervations, made in the iſland of *Cayenne*, every week for ten months together, and compared the lengths of the pendulum which he had there noted in the iron rods with the lengths thereof which he obſerved in *France*. This diligence and care ſeems to have been wanting to the other obſervers. If this gentleman's obſervations are to be depended on, the earth is higher under the equator than at the poles, and that by an exceſs of about 17 miles; as appeared above by the theory.

PROPOSITION XXI. THEOREM XVII.

That the equinoctial points go backwards, and that the axis of the earth, by a nutation in every annual revolution, twice vibrates towards the ecliptic, and as often returns to its former poſition.

The propoſition appears from cor. 20, prop. 66, book 1; but that motion of nutation muſt be very ſmall, and, indeed, ſcarcely perceptible.

PROPOSITION XXII. THEOREM XVIII.

That all the motions of the moon, and all the inequalities of thoſe motions, follow from the principles which we have laid down.

That the greater planets, while they are carried about the ſun, may in the mean time carry other leſſer planets, revolving about them; and that thoſe leſſer planets muſt move in ellipſes which have their foci in the centres of the greater, appears from prop. 65, book 1. But then their motions will be ſeveral ways diſturbed by the action of the ſun, and they will ſuffer ſuch inequalities as are obſerved in our moon. Thus our moon (by cor. 2, 3, 4, and 5, prop. 66, book 1) moves faſter, and, by a radius drawn to the earth, deſcribes an area greater for the time, and has its orbit leſs curved, and therefore approaches nearer to the earth in the ſyzygies than in the quadratures, excepting in ſo far as theſe effects are hindered by the motion of eccentricity; for (by cor. 9, prop. 66, book 1) the eccentricity is greateſt when the apogeon of the moon

is in the ſyzygies, and leaſt when the ſame is in the quadratures; and upon this account the perigeon moon is ſwifter, and nearer to us, but the apogeon moon ſlower, and farther from us, in the ſyzygies than in the quadratures. Moreover, the apogee goes forwards, and the nodes backwards; and this is done not with a regular but an unequal motion. For (by cor. 7 and 8, prop. 66, book 1) the apogee goes more ſwiftly forwards in its ſyzygies, more ſlowly backwards in its quadratures; and, by the exceſs of its progreſs above its regreſs, advances yearly *in conſequentia*. But, contrariwiſe, the nodes (by cor. 11, prop. 66, book 1) are quieſcent in their ſyzygies, and go faſteſt back in their quadratures. Farther, the greateſt latitude of the moon (by cor. 10, prop. 66, book 1) is greater in the quadratures of the moon than in its ſyzygies. And (by cor. 6, prop. 66, book 1) the mean motion of the moon is ſlower in the perihelion of the earth than in its aphelion. And theſe are the principal inequalities (of the moon) taken notice of by aſtronomers.

But there are yet other inequalities not obſerved by former aſtronomers, by which the motions of the moon are ſo diſturbed, that to this day we have not been able to bring them under any certain rule. For the velocities or horary motions of the apogee and nodes of the moon, and their equations, as well as the difference betwixt the greateſt eccentricity in the ſyzygies, and the leaſt eccentricity in the quadratures, and that inequality which we call the variation, are (by cor. 14, prop. 66, book 1) in the courſe of the year augmented and diminiſhed in the triplicate proportion of the ſun's apparent diameter. And beſides (by cor. 1 and 2, lem. 10, and cor. 16, prop. 66, book 1) the variation is augmented and diminiſhed nearly in the duplicate proportion of the time between the quadratures. But in aſtronomical calculations this inequality is commonly thrown into and confounded with the equation of the moon's centre.

PROPOSITION XXIII. PROBLEM V.

To derive the unequal motions of the ſatellites of Jupiter and Saturn from the motions of our moon.

From the motions of our moon we deduce the correſponding motions of the moons or ſatellites of Jupiter in this man-

ner, by cor. 16, prop. 66, book 1. The mean motion of the nodes of the outmoſt ſatellite of Jupiter is to the mean motion of the nodes of our moon in a proportion compounded of the duplicate proportion of the periodic time of the earth about the ſun to the periodic time of Jupiter about the ſun, and the ſimple proportion of the periodic time of the ſatellite about Jupiter to the periodic time of our moon about the earth; and, therefore, thoſe nodes, in the ſpace of a hundred years, are carried 8° 24′ backwards, or *in antecedentia*. The mean motions of the nodes of the inner ſatellites are to the mean motion of the nodes of the outmoſt as their periodic times to the periodic time of the former, by the ſame corollary, and are thence given. And the motion of the apſis of every ſatellite *in conſequentia* is to the motion of its nodes *in antecedentia* as the motion of the apogee of our moon to the motion of its nodes (by the ſame corollary), and is thence given. But the motions of the apſides thus found muſt be diminiſhed in the proportion of 5 to 9, or of about 1 to 2, on account of a cauſe which I cannot here deſcend to explain. The greateſt equations of the nodes, and of the apſis of every ſatellite, are to the greateſt equations of the nodes, and apogee of our moon reſpectively, as the motions of the nodes and apſides of the ſatellites, in the time of one revolution of the former equations, to the motions of the nodes and apogee of our moon, in the time of one revolution of the latter equations. The variation of a ſatellite ſeen from Jupiter is to the variation of our moon in the ſame proportion as the whole motions of their nodes reſpectively during the times in which the ſatellite and our moon (after parting from) are revolved (again) to the ſun, by the ſame corollary; and therefore in the outmoſt ſatellite the variation does not exceed 5″ 12‴.

PROPOSITION XXIV. THEOREM XIX.

That the flux and reflux of the ſea ariſe from the actions of the ſun and moon.

By cor. 19 and 20, prop. 66, book 1, it appears that the waters of the ſea ought twice to riſe and twice to fall every day, as well lunar as ſolar; and that the greateſt height of the waters in the open and deep ſeas ought to follow the ap-

pulſe of the luminaries to the meridian of the place by a leſs interval than 6 hours; as happens in all that eaſtern tract of the *Atlantic* and *Æthiopic* ſeas between *France* and the *Cape of Good Hope;* and on the coaſts of *Chili* and *Peru* in the *South Sea;* in all which ſhores the flood falls out about the ſecond, third, or fourth hour, unleſs where the motion propagated from the deep ocean is by the ſhallowneſs of the channels, through which it paſſes to ſome particular places, retarded to the fifth, ſixth, or ſeventh hour, and even later. The hours I reckon from the appulſe of each luminary to the meridian of the place, as well under as above the horizon; and by the hours of the lunar day I underſtand the 24th parts of that time which the moon, by its apparent diurnal motion, employs to come about again to the meridian of the place which it left the day before. The force of the ſun or moon in raiſing the ſea is greateſt in the appulſe of the luminary to the meridian of the place; but the force impreſſed upon the ſea at that time continues a little while after the impreſſion, and is afterwards increaſed by a new though leſs force ſtill acting upon it. This makes the ſea riſe higher and higher, till this new force becoming too weak to raiſe it any more, the ſea riſes to its greateſt height. And this will come to paſs, perhaps, in one or two hours, but more frequently near the ſhores in about three hours, or even more, where the ſea is ſhallow.

The two luminaries excite two motions, which will not appear diſtinctly, but between them will ariſe one mixed motion compounded out of both. In the conjunction or oppoſition of the luminaries their forces will be conjoined, and bring on the greateſt flood and ebb. In the quadratures the ſun will raiſe the waters which the moon depreſſes, and depreſs the waters which the moon raiſes, and from the difference of their forces the ſmalleſt of all tides will follow. And becauſe (as experience tells us) the force of the moon is greater than that of the ſun, the greateſt height of the waters will happen about the third lunar hour. Out of the ſyzygies and quadratures, the greateſt tide, which by the ſingle force of the moon ought to fall out at the third lunar hour, and by the ſingle force of the ſun at the third ſolar hour, by the compounded forces of

both muſt fall out in an intermediate time that approaches nearer to the third hour of the moon than to that of the ſun. And, therefore, while the moon is paſſing from the ſyzygies to the quadratures, during which time the 3d hour of the ſun precedes the 3d hour of the moon, the greateſt height of the waters will alſo precede the 3d hour of the moon, and that, by the greateſt interval, a little after the octants of the moon; and, by like intervals, the greateſt tide will follow the 3d lunar hour, while the moon is paſſing from the quadratures to the ſyzygies. Thus it happens in the open ſea; for in the mouths of rivers the greater tides come later to their height.

But the effects of the luminaries depend upon their diſtances from the earth; for when they are leſs diſtant, their effects are greater, and when more diſtant, their effects are leſs, and that in the triplicate proportion of their apparent diameter. Therefore it is that the ſun, in the winter time, being then in its perigee, has a greater effect, and makes the tides in the ſyzygies ſomething greater, and thoſe in the quadratures ſomething leſs than in the ſummer ſeaſon; and every month the moon, while in the perigee, raiſes greater tides than at the diſtance of 15 days before or after, when it is in its apogee. Whence it come to paſs that two higheſt tides do not follow one the other in two immediately ſucceeding ſyzygies.

The effect of either luminary doth likewiſe depend upon its declination or diſtance from the equator; for if the luminary was placed at the pole, it would conſtantly attract all the parts of the waters without any intenſion or remiſſion of its action, and could cauſe no reciprocation of motion. And, therefore, as the luminaries decline from the equator towards either pole, they will, by degrees, loſe their force, and on this account will excite leſſer tides in the ſolſtitial than in the equinoctial ſyzygies. But in the ſolſtitial quadratures they will raiſe greater tides than in the quadratures about the equinoxes; becauſe the force of the moon, then ſituated in the equator, moſt exceeds the force of the ſun. Therefore the greateſt tides fall out in thoſe ſyzygies, and the leaſt in thoſe quadratures, which happen about the time of both equinoxes: and the greateſt tide in the ſyzygies is always ſucceeded by the leaſt tide in

the quadratures, as we find by experience. But, becauſe the ſun is leſs diſtant from the earth in winter than in ſummer, it comes to paſs that the greateſt and leaſt tides more frequently appear before than after the vernal equinox, and more frequently after than before the autumnal.

Moreover, the effects of the luminaries depend upon the latitudes of places. Let ApEP (Pl. 10, Fig. 2) repreſent the earth covered with deep waters; C its centre; P, p its poles; AE the equator; F any place without the equator; Ff the parallel of the place; Dd the correſpondent parallel on the other ſide of the equator; L the place of the moon three hours before; H the place of the earth directly under it; h the oppoſite place; K, k the places at 90 degrees diſtance; CH, Ch, the greateſt heights of the ſea from the centre of the earth; and CK, ck, its leaſt heights: and if with the axes Hh, Kk, an ellipſis is deſcribed, and by the revolution of that ellipſis about its longer axis Hh a ſpheroid HPKhpk is formed, this ſpheroid will nearly repreſent the figure of the ſea; and CF, Cf, CD, Cd, will repreſent the heights of the ſea in the places Ff, Dd. But farther; in the ſaid revolution of the ellipſis any point N deſcribes the circle NM cutting the parallels Ff, Dd, in any places RT, and the equator AE in S; CN will repreſent the height of the ſea in all thoſe places R, S, T, ſituated in this circle. Wherefore, in the diurnal revolution of any place F, the greateſt flood will be in F, at the third hour after the appulſe of the moon to the meridian above the horizon; and afterwards the greateſt ebb in Q, at the third hour after the ſetting of the moon; and then the greateſt flood in f, at the third hour after the appulſe of the moon to the meridian under the horizon; and, laſtly, the greateſt ebb in Q, at the third hour after the riſing of the moon; and the latter flood in f will be leſs than the preceding flood in F. For the whole ſea is divided into two hemiſpherical floods, one in the hemiſphere KHk on the north ſide, the other in the oppoſite hemiſphere Khk, which we may therefore call the northern and the ſouthern floods. Theſe floods, being always oppoſite the one to the other, come by turns to the meridians of all places, after an interval of 12 lunar hours.

And ſeeing the northern countries partake more of the northern flood, and the ſouthern countries more of the ſouthern flood, thence ariſe tides, alternately greater and leſs in all places without the equator, in which the luminaries riſe and ſet. But the greateſt tide will happen when the moon declines towards the vertex of the place, about the third hour after the appulſe of the moon to the meridian above the horizon; and when the moon changes its declination *to the other ſide of the equator,* that which was the greater tide will be changed into a leſſer. And the greateſt difference of the floods will fall out about the times of the ſolſtices; eſpecially if the aſcending node of the moon is about the firſt of Aries. So it is found by experience that the morning tides in winter exceed thoſe of the evening, and the evening tides in ſummer exceed thoſe of the morning; at *Plymouth* by the height of one foot, but at *Briſtol* by the height of 15 inches, according to the obſervations of *Colepreſs* and *Sturmy*.

But the motions which we have been deſcribing ſuffer ſome alteration from that force of reciprocation, which the waters, being once moved, retain a little while *by their vis inſita*. Whence it comes to paſs that the tides may continue for ſome time, though the actions of the luminaries ſhould ceaſe. This power of retaining the impreſſed motion leſſens the difference of the alternate tides, and makes thoſe tides which immediately ſucceed after the ſyzygies greater, and thoſe which follow next after the quadratures leſs. And hence it is that the alternate tides at *Plymouth* and *Briſtol* do not differ much more one from the other than by the height of a foot or 15 inches, and that the greateſt tides of all at thoſe ports are not the firſt but the third after the ſyzygies. And, beſides, all the motions are retarded in their paſſage through ſhallow channels, ſo that the greateſt tides of all, in ſome ſtraits and mouths of rivers, are the fourth or even the fifth after the ſyzygies.

Farther, it may happen that the tide may be propagated from the ocean through different channels towards the ſame port, and may paſs quicker through ſome channels than through others; in which caſe the ſame tide, divided into two

or more ſucceeding one another, may compound new motions of different kinds. Let us ſuppoſe two equal tides flowing towards the ſame port from different places, the one preceding the other by 6 hours; and ſuppoſe the firſt tide to happen at the third hour of the appulſe of the moon to the meridian of the port. If the moon at the time of the appulſe to the meridian was in the equator, every 6 hours alternately there would ariſe equal floods, which, meeting with as many equal ebbs, would ſo balance one the other, that for that day the water would ſtagnate and remain quiet. If the moon then declined from the equator, the tides in the ocean would be alternately greater and leſs, as was ſaid; and from thence two greater and two leſſer tides would be alternately propagated towards that port. But the two greater floods would make the greateſt height of the waters to fall out in the middle time betwixt both; and the greater and leſſer floods would make the waters to riſe to a mean height in the middle time between them, and in the middle time between the two leſſer floods the waters would riſe to their leaſt height. Thus in the ſpace of 24 hours the waters would come, not twice, as commonly, but once only to their greateſt, and once only to their leaſt height; and their greateſt height, if the moon declined towards the elevated pole, would happen at the 6th or 30th hour after the appulſe of the moon to the meridian; and when the moon changed its declination, this flood would be changed into an ebb. An example of all which Dr. *Halley* has given us, from the obſervations of ſeamen in the port of *Batſham*, in the kingdom of *Tunquin*, in the latitude of 20° 50′ north. In that port, on the day which follows after the paſſage of the moon over the equator, the waters ſtagnate: when the moon declines to the north, they begin to flow and ebb, not twice, as in other ports, but once only every day; and the flood happens at the ſetting, and the greateſt ebb at the riſing of the moon. This tide increaſes with the declination of the moon till the 7th or 8th day; then for the 7 or 8 days following it decreaſes at the ſame rate as it had increaſed before, and ceaſes when the moon changes its declination, croſſing over the equator to the ſouth. After which the flood is immediately changed

into an ebb; and thenceforth the ebb happens at the ſetting and the flood at the riſing of the moon; till the moon, again paſſing the equator, changes its declination. There are two inlets to this port and the neighbouring channels, one from the ſeas of *China*, between the continent and the iſland of *Luconia*; the other from the *Indian* ſea, between the continent and the iſland of *Borneo*. But whether there be really two tides propagated through the ſaid channels, one from the *Indian* ſea in the ſpace of 12 hours, and one from the ſea of *China* in the ſpace of 6 hours, which therefore happening at the 3d and 9th lunar hours, by being compounded together, produce thoſe motions; or whether there be any other circumſtances in the ſtate of thoſe ſeas, I leave to be determined by obſervations on the neighbouring ſhores.

Thus I have explained the cauſes of the motions of the moon and of the ſea. Now it is fit to ſubjoin ſomething concerning the quantity of thoſe motions.

PROPOSITION XXV. PROBLEM VI.

To find the forces with which the ſun diſturbs the motions of the moon. (Pl. 10, Fig. 3.)

Let S repreſent the ſun, T the earth, P the moon, CADB the moon's orbit. In SP take SK equal to ST; and let SL be to SK in the duplicate proportion of SK to SP: draw LM parallel to PT; and if ST or SK is ſuppoſed to repreſent the accelerated force of gravity of the earth towards the ſun, SL will repreſent the accelerative force of gravity of the moon towards the ſun. But that force is compounded of the parts SM and LM, of which the force LM, and that part of SM which is repreſented by TM, diſturb the motion of the moon, as we have ſhewn in prop. 66, book 1, and its corollaries. Foraſmuch as the earth and moon are revolved about their common centre of gravity, the motion of the earth about that centre will be alſo diſturbed by the like forces; but we may conſider the ſums both of the forces and of the motions as in the moon, and repreſent the ſum of the forces by the lines TM and ML, which are analogous to them both. The force ML (in its mean quantity) is to the centripetal force by which the moon may be retained in its orbit revolving

about the earth at reſt, at the diſtance PT, in the duplicate proportion of the periodic time of the moon about the earth to the periodic time of the earth about the ſun (by cor. 17, prop. 66, book 1); that is, in the duplicate proportion of $27^d.\ 7^h.\ 43'$ to $365^d.\ 6^h.\ 9'$; or as 1000 to 178725; or as 1 to $178\frac{29}{40}$. But in the 4th prop. of this book we found, that, if both earth and moon were revolved about their common centre of gravity, the mean diſtance of the one from the other would be nearly $60\frac{1}{2}$ mean ſemi-diameters of the earth; and the force by which the moon may be kept revolving in its orbit about the earth in reſt at the diſtance PT of $60\frac{1}{2}$ ſemi-diameters of the earth, is to the force by which it may be revolved in the ſame time, at the diſtance of 60 ſemi-diameters, as $60\frac{1}{2}$ to 60; and this force is to the force of gravity with us very nearly as 1 to 60×60. Therefore the mean force ML is to the force of gravity on the ſurface of our earth as $1 \times 60\frac{1}{2}$ to $60 \times 60 \times 60 \times 178\frac{29}{40}$, or as 1 to 638092,6; whence by the proportion of the lines TM, ML, the force TM is alſo given; and theſe are the forces with which the ſun diſturbs the motions of the moon. Q.E.I.

PROPOSITION XXVI. PROBLEM VII.

To find the horary increment of the area which the moon, by a radius drawn to the earth, deſcribes in a circular orbit.

We have above ſhewn that the area which the moon deſcribes by a radius drawn to the earth is proportional to the time of deſcription, excepting in ſo far as the moon's motion is diſturbed by the action of the ſun; and here we propoſe to inveſtigate the inequality of the moment, or horary increment *of that area or motion ſo diſturbed.* To render the calculus more eaſy, we ſhall ſuppoſe the orbit of the moon to be circular, and neglect all inequalities but that only which is now under conſideration; and, becauſe of the immenſe diſtance of the ſun, we ſhall farther ſuppoſe that the lines SP and ST are parallel. By this means, the force LM (Pl. 10, Fig. 4) will be always reduced to its mean quantity TP, as well as the force TM to its mean quantity 3PK. Theſe forces (by cor. 2 of the laws of motion) compoſe the force TL; and this force, by letting fall the perpendicular LE upon the radius

TP, is resolved into the forces TE, EL; of which the force TE, acting constantly in the direction of the radius TP, neither accelerates nor retards the description of the area TPC made by that radius TP; but EL, *acting on the radius* TP in a perpendicular direction, accelerates or retards *the description of the area* in proportion as it accelerates or retards the moon. That acceleration of the moon, in its passage from the quadrature C to the conjunction A, is in every moment of time as the *generating* accelerative force EL, that is, as $\frac{3PK \times TK}{TP}$.

Let the time be represented by the mean motion of the moon, or (which comes to the same thing) by the angle CTP, or even by the arc CP. At right angles upon CT erect CG equal to CT; and, supposing the quadrantal arc AC to be divided into an infinite number of equal parts Pp, &c. these *parts* may represent the like *infinite* number of the equal parts of time. Let fall pk perpendicular on CT, and draw TG meeting with KP, kp produced in F and f; then will FK be equal to TK, and Kk be to PK as Pp to Tp, that is, in a given proportion; and therefore FK × Kk, or the area FKkf, will be as $\frac{3PK \times TK}{TP}$, that is, as EL; and compounding, the whole area GCKF will be as the sum of all the forces EL impressed upon the moon in the whole time CP; and therefore also as the velocity generated by that sum, that is, as the acceleration of the description of the area CTP, or as the increment of the moment *thereof*. The force by which the moon may in its periodic time CADB of $27^d. 7^h. 43'$ be retained revolving about the earth in rest at the distance TP, would cause a body falling in the time CT to describe the length $\frac{1}{2}$CT, and at the same time to acquire a velocity equal to that with which the moon is moved in its orbit. This appears from cor. 9, prop. 4, book 1. But since Kd, drawn perpendicular on TP, is *but* a third part of EL, and *equal* to the half of TP, or ML, in the octants, the force EL in the octants, where it is greatest, will exceed the force ML in the proportion of 3 to 2; and therefore will be to that force by which the moon in its periodic time may be retained revolving about

the earth at reſt as 100 to $\frac{2}{3} \times 17872\frac{1}{2}$, or 11915; and in the time CT will generate a velocity equal to $\frac{100}{11915}$ parts of the velocity of the moon; but in the time CPA will generate a greater velocity in the proportion of CA to CT or TP. Let the greateſt force EL in the octants be repreſented by the area FK × Kk, or by the rectangle $\frac{1}{2}$TP × Pp, which is equal thereto; and the velocity which that greateſt force can generate in any time CP will be to the velocity which any other leſſer force EL can generate in the ſame time as the rectangle TP × CP to the area KCGF; but the velocities generated in the whole time CPA will be one to the other as the rectangle $\frac{1}{2}$TP × CA to the triangle TCG, or as the quadrantal arc CA to the radius TP; and therefore the latter velocity generated in the whole time will be $\frac{100}{11915}$ parts of the velocity of the moon. To this velocity of the moon, which is proportional to the mean moment of the area (ſuppoſing this mean moment to be repreſented by the number 11915), we add and ſubtract the half of the other velocity; the ſum 11915 + 50, or 11965, will repreſent the greateſt moment of the area in the ſyzygy A; and the difference 11915 — 50, or 11865, the leaſt moment thereof in the quadratures. Therefore the areas which in equal times are deſcribed in the ſyzygies and quadratures are one to the other as 11965 to 11865. And if to the leaſt moment 11865 we add a moment which ſhall be to 100, the difference of the two former moments, as the trapezium FKCG to the triangle TCG, or, which comes to the ſame thing, as the ſquare of the ſine PK to the ſquare of the radius TP (that is, as Pd to TP), the ſum will repreſent the moment of the area when the moon is in any intermediate place P.

But theſe things take place only in the hypotheſis that the ſun and the earth are at reſt, and that the ſynodical revolution of the moon is finiſhed in $27^d.\ 7^h.\ 43'$. But ſince the moon's ſynodical period is really $29^d.\ 12^h.\ 44'$, the increments of the moments muſt be enlarged in the ſame proportion as the time is, that is, in the proportion of 1080853 to 1000000. Upon which account, the whole increment, which was $\frac{100}{11915}$ parts of the mean moment, will now become $\frac{100}{11024}$ parts thereof; and therefore the moment of the area

in the quadrature of the moon will be to the moment thereof in the ſyzygy as 11023 — 50 to 11023 + 50; or as 10973 to 11073; and to the moment thereof, when the moon is in any intermediate place P, as 10973 to 10973 + Pd; that is, ſuppoſing TP = 100.

The area, therefore, which the moon, by a radius drawn to the earth, deſcribes in the ſeveral little equal parts of time, is nearly as the ſum of the number 219,46, and the verſed ſine of the double diſtance of the moon from the neareſt quadrature, conſidered in a circle which hath unity for its radius. Thus it is when the variation in the octants is in its mean quantity. But if the variation there is greater or leſs, that verſed ſine muſt be augmented or diminiſhed in the ſame proportion.

PROPOSITION XXVII. PROBLEM VIII.

From the horary motion of the moon to find its diſtance from the earth.

The area which the moon, by a radius drawn to the earth, deſcribes in every moment of time, is as the horary motion of the moon and the ſquare of the diſtance of the moon from the earth conjunctly. And therefore the diſtance of the moon from the earth is in a proportion compounded of the ſubduplicate proportion of the area directly, and the ſubduplicate proportion of the horary motion inverſely. Q.E.I.

Cor. 1. Hence the apparent diameter of the moon is given; for it is reciprocally as the diſtance of the moon from the earth. Let aſtronomers try how accurately this rule agrees with the phænomena.

Cor. 2. Hence alſo the orbit of the moon may be more exactly defined from the phænomena than hitherto could be done.

PROPOSITION XXVIII. PROBLEM IX.

To find the diameters of the orbit, in which, without eccentricity, the moon would move.

The curvature of the orbit which a body deſcribes, if attracted in lines perpendicular to the orbit, is as the force of attraction directly, and the ſquare of the velocity inverſely. I eſtimate the curvatures of lines compared one with another according to the evaneſcent proportion of the ſines or tan-

gents of their angles of contact to equal radii, ſuppoſing thoſe radii to be infinitely diminiſhed. But the attraction of the moon towards the earth in the ſyzygies is the exceſs of its gravity towards the earth above the force of the ſun 2PK (ſee Fig. prop. 25), by which force the accelerative gravity of the moon towards the ſun exceeds the accelerative gravity of the earth towards the ſun, or is exceeded by it. But in the quadratures that attraction is the ſum of the gravity of the moon towards the earth, and the ſun's force KT, by which the moon is attracted towards the earth. And theſe attractions, putting N for $\frac{AT + CT}{2}$, are nearly as $\frac{178725}{AT^2} + \frac{2000}{CT \times N}$ and $\frac{178725}{CT^2} + \frac{1000}{AT \times N}$, or as $178725N \times CT^2 - 2000AT^2 \times CT$, and $178725N \times AT^2 + 1000CT^2 \times AT$. For if the accelerative gravity of the moon towards the earth be repreſented by the number 178725, the mean force ML, which in the quadratures is PT or TK, and draws the moon towards the earth, will be 1000, and the mean force TM in the ſyzygies will be 3000; from which, if we ſubtract the mean force ML, there will remain 2000, the force by which the moon in the ſyzygies is drawn from the earth, and which we above called 2PK. But the velocity of the moon in the ſyzygies A and B is to its velocity in the quadratures C and D as CT to AT, and the moment of the area, which the moon by a radius drawn to the earth deſcribes in the ſyzygies, to the moment of that area *deſcribed* in the quadratures conjunctly; that is, as 11073CT to 10973AT. Take this ratio twice inverſely, and the former ratio once directly, and the curvature of the orb of the moon in the ſyzygies will be to the curvature thereof in the quadratures as $120406729 \times 178725AT^2 \times CT^2 \times N - 120406729 \times 2000AT^4 \times CT$ to $122611329 \times 178725AT^2 \times CT^2 \times N + 122611329 \times 1000CT^4 \times AT$, that is, as $2151969AT \times CT \times N - 24081AT^3$ to $2191371AT \times CT \times N + 12261CT^3$.

Becauſe the figure of the moon's orbit is unknown, let us, in its ſtead, aſſume the ellipſis DBCA (Pl. 10, Fig. 5), in the centre of which we ſuppoſe the earth to be ſituated, and the

greater axis de to lie between the quadratures as the leſſer AB between the ſyzygies. But ſince the plane of this ellipſis is revolved about the earth by an angular motion, and the orbit, whoſe curvature we now examine, ſhould be deſcribed in a plane void of ſuch motion, we are to conſider the figure which the moon, while it is revolved in that ellipſis, deſcribes in this plane, that is to ſay, the figure Cpa, the ſeveral points p of which are found by aſſuming any point P in the ellipſis, which may repreſent the place of the moon, and drawing Tp equal to TP in ſuch manner that the angle PTp may be equal to the apparent motion of the ſun from the time of the laſt quadrature in C; or (which comes to the ſame thing) that the angle CTp may be to the angle CTP as the time of the ſynodic revolution of the moon to the time of the periodic revolution thereof, or as $29^d. 12^h. 44'$ to $27^d. 7^h. 43'$. If, therefore, in this proportion we take the angle CTa to the right angle CTA, and make Ta of equal length with TA, we ſhall have a the lower and C the upper apſis of this orbit. But, by computation, I find that the difference betwixt the curvature of this orbit Cpa at the vertex a, and the curvature of a circle deſcribed about the centre T with the interval TA, is to the difference betwixt the curvature of the ellipſis at the vertex A, and the curvature of the ſame circle, in the duplicate proportion of the angle CTP to the angle CTp; and that the curvature of the ellipſis in A is to the curvature of that circle in the duplicate proportion of TA to TC; and the curvature of that circle to the curvature of a circle deſcribed about the centre T with the interval TC as TC to TA; but that the curvature of this *laſt arch* is to the curvature of the ellipſis in C in the duplicate proportion of TA to TC; and that the difference betwixt the curvature of the ellipſis in the vertex C, and the curvature of this laſt circle, is to the difference betwixt the curvature of the figure Tpa, at the vertex C, and the curvature of this ſame *laſt* circle, in the duplicate proportion of the angle CTp to the angle CTP; all which proportions are eaſily drawn from the ſines of the angles of contact, and of the differences of thoſe angles. But, by comparing thoſe proportions together, we find the curvature of the figure

Cpa at a to be to its curvature at C as $AT^3 \times \frac{16824}{100000} CT^2 AT$ to $CT^3 + \frac{16824}{100000} AT^2 \times CT$; where the number $\frac{16824}{100000}$ repreſents the difference of the ſquares of the angles CTP and CTp, applied to the ſquare of the leſſer angle CTP; or (which is all one) the difference of the ſquares of the times $27^d. 7^h. 43'$, and $29^d. 12^h. 44'$, applied to the ſquare of the time $27^d. 7^h. 43'$.

Since, therefore, a repreſents the ſyzygy of the moon, and C its quadrature, the proportion now found muſt be the ſame with that proportion of the curvature of the moon's orb in the ſyzygies to the curvature thereof in the quadratures, which we found above. Therefore, in order to find the proportion of CT to AT, let us multiply the extremes and the means, and the terms which come out, applied to $AT \times CT$, become $2062{,}79CT^4 - 2151969N \times CT^3 + 368676N \times AT \times CT^2 + 36342\ AT^2 \times CT^2 - 362047N \times AT^2 \times CT + 2191371N \times AT^3 + 4051{,}4AT^4 = 0$. Now if for the half ſum N of the terms AT and CT we put 1, and x for their half difference, then CT will be $= 1 + x$, and $AT = 1 - x$. And ſubſtituting thoſe values in the equation, after reſolving thereof, we ſhall find $x = 0{,}00719$; and from thence the ſemi-diameter $CT = 1{,}00719$, and the ſemi-diameter $AT = 0{,}99281$, which numbers are nearly as $70\frac{1}{24}$, and $69\frac{1}{24}$. Therefore the moon's diſtance from the earth in the ſyzygies is to its diſtance in the quadratures (ſetting aſide the conſideration of eccentricity) as $69\frac{1}{24}$ to $70\frac{1}{24}$; or, in round numbers, as 69 to 70.

PROPOSITION XXIX. PROBLEM X.

To find the variation of the moon.

This inequality is owing partly to the elliptic figure of the moon's orbit, partly to the inequality of the moments of the area which the moon by a radius drawn to the earth deſcribes. If the moon P revolved in the ellipſis DBCA about the earth quieſcent in the centre of the ellipſis, and by the radius TP, drawn to the earth, deſcribed the area CTP, proportional to the time *of deſcription*; and the greateſt ſemi-diameter CT of the ellipſis was to the leaſt TA as 70 to 69; the tangent of the angle CTP would be to the tangent of the angle of the mean motion, computed from the quadrature C, as

the ſemi-diameter TA of the ellipſis to its ſemi-diameter TC, or as 69 to 70. But the deſcription of the area CTP, as the moon advances from the quadrature to the ſyzygy, ought to be in ſuch manner accelerated, that the moment of the area in the moon's ſyzygy may be to the moment thereof in its quadrature as 11073 to 10973; and that the exceſs of the moment in any intermediate place P above the moment in the quadrature may be as the ſquare of the ſine of the angle CTP; which we may effect with accuracy enough, if we diminiſh the tangent of the angle CTP in the ſubduplicate proportion of the number 10973 to the number 11073, that is, in proportion of the number 68,6877 to the number 69. Upon which account the tangent of the angle CTP will now be to the tangent of the mean motion as 68,6877 to 70; and the angle CTP in the octants, where the mean motion is 45°, will be found 44° 27′ 28″, which ſubtracted from 45°, the angle of the mean motion, leaves the greateſt variation 32′ 32″. Thus it would be, if the moon, in paſſing from the quadrature to the ſyzygy, deſcribed an angle CTA of 90 degrees only. But becauſe of the motion of the earth, by which the ſun is apparently transfered *in conſequentia*, the moon, before it overtakes the ſun, deſcribes an angle CTa, greater than a right angle, in the proportion of the time of the ſynodic revolution of the moon to the time of its periodic revolution, that is, in the proportion of $29^{d}.\ 12^{h}.\ 44'$ to $27^{d}.\ 7^{h}.\ 43'$. Whence it comes to paſs that all the angles about the centre T are dilated in the ſame proportion; and the greateſt variation, which otherwiſe would be *but* 32′ 32″, now augmented in the ſaid proportion, becomes 35′ 10″.

And this is its magnitude in the mean diſtance of the ſun from the earth, neglecting the differences which may ariſe from the curvature of the *orbis magnus*, and the ſtronger action of the ſun upon the moon when horned and new, than when gibbous and full. In other diſtances of the ſun from the earth, the greateſt variation is in a proportion compounded of the duplicate proportion of the time of the ſynodic revolution of the moon (the time of the year being given) directly, and the triplicate proportion of the diſtance of the ſun

from the earth inverſely. And, therefore, in the apogee of the ſun, the greateſt variation is 33′ 14″, and in its perigee 37′ 11″, if the eccentricity of the ſun is to the tranſverſe ſemidiameter of the *orbis magnus* as $16\frac{1}{2}$ to 1000.

Hitherto we have inveſtigated the variation in an orb not eccentric, in which, to wit, the moon in its octants is always in its mean diſtance from the earth. If the moon, on account of its eccentricity, is more or leſs removed from the earth than if placed in this orb, the variation may be ſomething greater, or ſomething leſs, than according to this rule. But I leave the exceſs or defect to the determination of aſtronomers from the phænomena.

PROPOSITION XXX. PROBLEM XI.

To find the horary motion of the nodes of the moon in a circular orbit. (Pl. 11, Fig. 1.)

Let S repreſent the ſun, T the earth, P the moon, NPn the orbit of the moon, Npn the orthographic projection of the orbit upon the plane of the ecliptic; N, n the nodes, nTNm the line of the nodes produced indefinitely; PI, PK perpendiculars upon the lines ST, Qq; Pp a perpendicular upon the plane of the ecliptic; A, B the moon's ſyzygies in the plane of the ecliptic; AZ a perpendicular let fall upon Nn, the line of the nodes; Q, q the quadratures of the moon in the plane of the ecliptic, and pK a perpendicular on the line Qq lying between the quadratures. The force of the ſun to diſturb the motion of the moon (by prop. 25) is twofold, one proportional to the line LM, the other to the line MT, in the ſcheme of that propoſition; and the moon by the former force is drawn towards the earth, by the latter towards the ſun, in a direction parallel to the right line ST joining the earth and the ſun. The former force LM acts in the direction of the plane of the moon's orbit, and therefore makes no change upon the ſituation thereof, and is upon that account to be neglected; the latter force MT, by which the plane of the moon's orbit is diſturbed, is the ſame with the force 3PK or 3IT. And this force (by prop. 25) is to the force by which the moon may, in its periodic time, be uniformly revolved in a circle about the earth at reſt, as 3IT to the radius of the circle

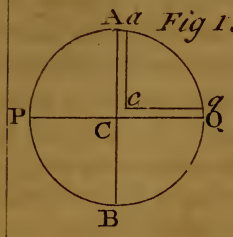


Fig 1.

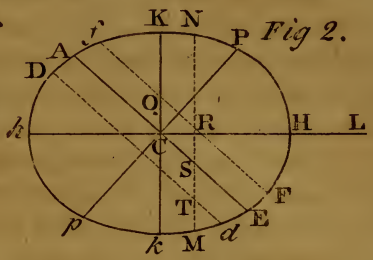


Fig 2.

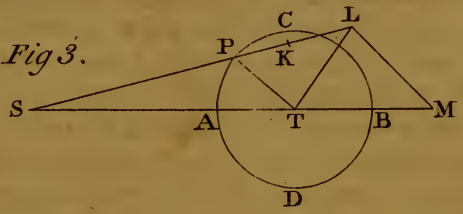


Fig 3.

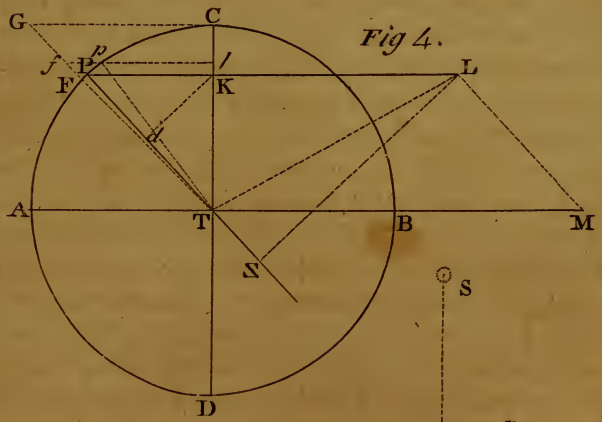


Fig 4.

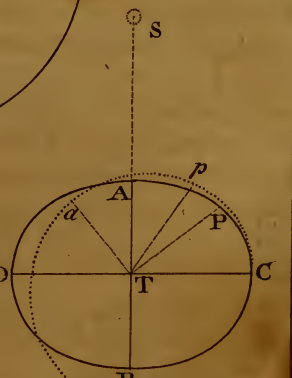


Fig 5.

multiplied by the number 178,725, or as IT to the radius thereof multiplied by 59,575. But in this calculus, and all that follows, I conſider all the lines drawn from the moon to the ſun as parallel to the line which joins the earth and the ſun; becauſe what inclination there is almoſt as much diminiſhes all effects in ſome caſes as it augments them in others; and we are now enquiring after the mean motions of the nodes, neglecting ſuch niceties as are of no moment, and would only ſerve to render the calculus more perplexed.

Now ſuppoſe PM to repreſent an arc which the moon deſcribes in the leaſt moment of time, and ML a little line, the half of which the moon, by the impulſe of the ſaid force 3IT, would deſcribe in the ſame time; and joining PL, MP, let them be produced to m and l, where they cut the plane of the ecliptic, and upon Tm let fall the perpendicular PH. Now, ſince the right line ML is parallel to the plane of the ecliptic, and therefore can never meet with the right line ml which lies in that plane, and yet both thoſe right lines lie in one common plane LMPml, they will be parallel, and upon that account the triangles LMP, lmP will be ſimilar. And ſeeing MPm lies in the plane of the orbit, in which the moon did move while in the place P, the point m will fall upon the line Nn, which paſſes through the nodes N, n, of that orbit. And becauſe the force by which the half of the little line LM is generated, if the whole had been together, and at once impreſſed in the point P, would have generated that whole line, and cauſed the moon to move in the arc whoſe chord is LP; that is to ſay, would have transfered the moon from the plane MPmT into the plane LPlT; therefore the angular motion of the nodes generated by that force will be equal to the angle mTl. But ml is to mP as ML to MP; and ſince MP, becauſe of the time given, is alſo given, ml will be as the rectangle ML × mP, that is, as the rectangle IT × mP. And if Tml is a right angle, the angle mTl will be as $\frac{ml}{Tm}$, and therefore as $\frac{IT \times Pm}{Tm}$, that is (becauſe Tm and mP, TP

and PH are proportional), as $\frac{IT \times PH}{TP}$; and, therefore, because TP is given, as IT × PH. But if the angle Tml or STN is oblique, the angle mTl will be yet leſs, in proportion of the ſine of the angle STN to the radius, or AZ to AT. And therefore the velocity of the nodes is as IT × PH × AZ, or as the ſolid content of the ſines of the three angles TPI, PTN, and STN.

If theſe are right angles, as happens when the nodes are in the quadratures, and the moon in the ſyzygy, the little line ml will be removed to an infinite diſtance, and the angle mTl will become equal to the angle mPl. But in this caſe the angle mPl is to the angle PTM, which the moon in the ſame time by its apparent motion deſcribes about the earth, as 1 to 59,575. For the angle mPl is equal to the angle LPM, that is, to the angle of the moon's deflexion from a rectilinear path; which angle, if the gravity of the moon ſhould have then ceaſed, the ſaid force of the ſun 3IT would by itſelf have generated in that given time; and the angle PTM is equal to the angle of the moon's deflexion from a rectilinear path; which angle, if the force of the ſun 3IT ſhould have then ceaſed, the force alone by which the moon is retained in its orbit would have generated in the ſame time. And theſe forces (as we have above ſhewn) are the one to the other as 1 to 59,575. Since, therefore, the mean horary motion of the moon (in reſpect of the fixed ſtars) is $32' 56'' 27''' 12\frac{1}{2}^{iv}$. the horary motion of the node in this caſe will be $33'' 10''' 33^{iv}. 12^{v}$. But in other caſes the horary motion will be to $33'' 10''' 33^{iv}. 12^{v}$. as the ſolid content of the ſines of the three angles TPI, PTN, and STN (or of the diſtances of the moon from the quadrature, of the moon from the node, and of the node from the ſun) to the cube of the radius. And as often as the ſine of any angle is changed from poſitive to negative, and from negative to poſitive ſo often muſt the regreſſive be changed into a progreſſive, and the progreſſive into a regreſſive motion. Whence it comes to paſs that the nodes are progreſſive as often as the moon happens to be placed between either quadrature, and the node neareſt to that quadrature. In other

caſes they are regreſſive, and, by the exceſs of the regreſs above the progreſs, they are monthly transfered *in antecedentia*.

Cor. 1. Hence if from P and M, the extreme points of a leaſt arc PM (Pl. 11, Fig. 2), on the line Qq joining the quadratures we let fall the perpendiculars PK, Mk, and produce the ſame till they cut the line of the nodes Nn in D and d, the horary motion of the nodes will be as the area MPDd, and the ſquare of the time AZ conjunctly. For let PK, PH, and AZ, be the three ſaid ſines, viz. PK the ſine of the diſtance of the moon from the quadrature, PH the ſine of the diſtance of the moon from the node, and AZ the ſine of the diſtance of the node from the ſun; and the velocity of the node will be as the ſolid content of PK × PH × AZ. But PT is to PK as PM to Kk; and, therefore, becauſe PT and PM are given, Kk will be as PK. Likewiſe AT is to PD as AZ to PH, and therefore PH is as the rectangle PD × AZ; and, by compounding thoſe proportions, PK × PH is as the ſolid content Kk × PD × AZ, and PK × PH × AZ as Kk × PD × AZ^2; that is, as the area PDdM and AZ^2 conjunctly. Q.E.D.

Cor. 2. In any given poſition of the nodes their mean horary motion is half their horary motion in the moon's ſyzygies; and therefore is to $16''\ 35'''\ 16^{iv}.\ 36^{v}.$ as the ſquare of the ſine of the diſtance of the nodes from the ſyzygies to the ſquare of the radius, or as AZ^2 to AT^2. For if the moon, by an uniform motion, deſcribes the ſemi-circle QAq, the ſum of all the areas PDdM, during the time of the moon's paſſage from Q to M, will make up the area QMdE, terminating at the tangent QE of the circle; and by the time that the moon has arrived at the point n, that ſum will make up the whole area EQAn deſcribed by the line PD: but when the moon proceeds from n to q, the line PD will fall without the circle, and will deſcribe the area nqe, terminating at the tangent qe of the circle; which area, becauſe the nodes were before regreſſive, but are now progreſſive, muſt be ſubducted from the former area, and, being itſelf equal to the area QEN, will leave the ſemi-circle NQAn. While, therefore, the moon deſcribes a ſemi-circle, the ſum of all the areas PDdM will be the area of that ſemi-circle; and while

the moon defcribes a complete circle, the fum of thofe areas will be the area of the whole circle. But the area PDdM, when the moon is in the fyzygies, is the rectangle of the arc PM into the radius PT; and the fum of all the areas, *every one* equal to this area, in the time that the moon defcribes a complete circle, is the rectangle of the whole circumference into the radius of the circle; and this rectangle, being double the area of the circle, will be double the quantity of the former fum. If, therefore, the nodes went on with that velocity uniformly continued which they acquire in the moon's fyzygies, they would defcribe a fpace double of that which they defcribe in fact; and, therefore, the mean motion, by which, if uniformly continued, they would defcribe the fame fpace with that which they do in fact defcribe by an unequal motion, is *but* one half of that motion which they are poffeffed of in the moon's fyzygies. Wherefore fince their greateft horary motion, if the nodes are in the quadratures, is $33'' 10''' 33^{iv}. 12^{v}.$ their mean horary motion in this cafe will be $16'' 35''' 16^{iv}. 36^{v}.$ And feeing the horary motion of the nodes is every where as AZ^2 and the area PDdM conjunctly, and, therefore, in the moon's fyzygies, the horary motion of the nodes is as AZ^2 and the area PDdM conjunctly, that is (becaufe the area PDdM defcribed in the fyzygies is given), as AZ^2, therefore the mean motion alfo will be as AZ^2; and, therefore, when the nodes are without the quadratures, this motion will be to $16'' 35''' 16^{iv}. 36^{v}.$ as AZ^2 to AT^2. Q.E.D.

PROPOSITION XXXI. PROBLEM XII.

To find the horary motion of the nodes of the moon in an elliptic orbit. (Pl. 12, Fig. 1.)

Let Qpmaq repreſent an ellipſis defcribed with the greater axis Qq, and the leſſer axis ab; QAqB a circle circumſcribed; T the earth in the common centre of both; S the ſun; p the moon moving in this ellipſis; and pm an arc which it defcribes in the leaſt moment of time; N and n the nodes joined by the line Nn; pK and mk perpendiculars upon the axis Qq, produced both ways till they meet the circle in P and M, and the line of the nodes in D and d. And if the moon, by a radius drawn to the earth, defcribes an area proportional to the

Fig 1.

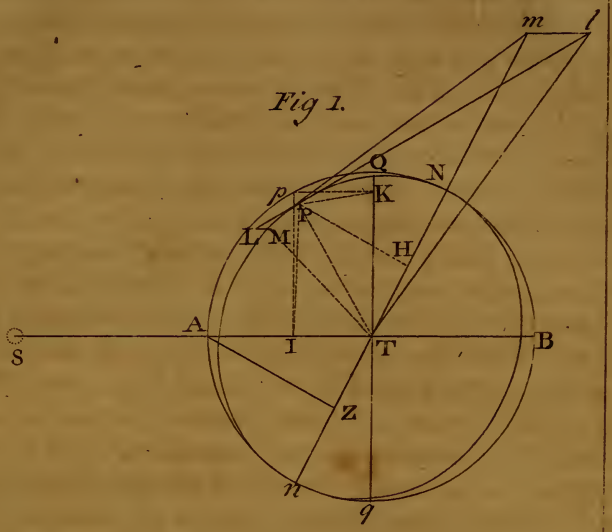
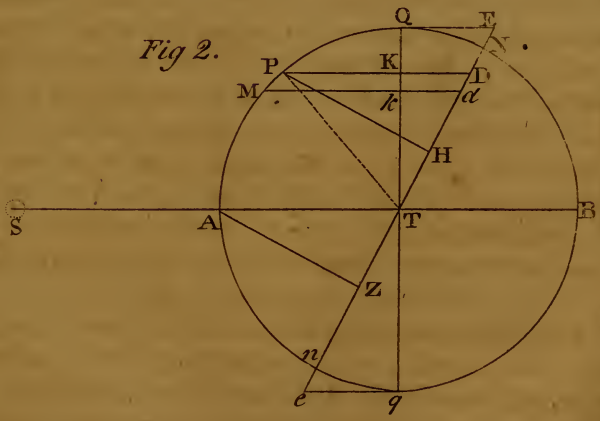


Fig 2.



time *of deſcription*, the horary motion of the node in the ellipſis will be as the area pDdm and AZ^2 conjunctly.

For let PF touch the circle in P, and produced meet TN in F; and pf touch the ellipſis in p, and produced meet the ſame TN in f, and both tangents concur in the axis TQ at Y. And let ML repreſent the ſpace which the moon, by the impulſe of the above-mentioned force 3IT or 3PK, would deſcribe with a tranſverſe motion, in the mean time while revolving in the circle it deſcribes the arc PM; and ml denote the ſpace which the moon revolving in the ellipſis would deſcribe in the ſame time by the impulſe of the ſame force 3IT or 3PK; and let LP and lp be produced till they meet the plane of the ecliptic in G and g, and FG and fg be joined, of which FG produced may cut pf, pg, and TQ, in c, e, and R reſpectively; and fg produced may cut TQ in r. Becauſe the force 3IT or 3PK in the circle is to the force 3IT or 3pK in the ellipſis as PK to pK, or as AT to aT, the ſpace ML generated by the former force will be to the ſpace ml generated by the latter as PK to pK; that is, becauſe of the ſimilar figures PYKp and FYRc, as FR to cR. But (becauſe of the ſimilar triangles PLM, PGF) ML is to FG as PL to PG, that is (on account of the parallels Lk, PK, GR), as pl to pe, that is (becauſe of the ſimilar triangles plm, cpe), as lm to ce; and inverſely as LM is to lm, or as FR is to cR, ſo is FG to ce. And therefore if fg was to ce as fy to cY, that is, as fr to cR (that is, as fr to FR and FR to cR conjunctly, that is, as fT to FT, and FG to ce conjunctly), becauſe the ratio of FG to ce, expunged on both ſides, leaves the ratios fg to FG and fT to FT, fg would be to FG as fT to FT; and, therefore, the angles which FG and fg would ſubtend at the earth T would be equal to each other. But theſe angles (by what we have ſhewn in the preceding propoſition) are the motions of the nodes, while the moon deſcribes in the circle the arc PM, in the ellipſis the arc pm; and therefore the motions of the nodes in the circle and in the ellipſis would be equal to each other. Thus, I ſay, it would be, if fg was to ce as fY to cY, that is, if fg was equal to $\frac{ce \times fY}{cY}$. But becauſe of the ſimilar triangles fgp, cep, fg is to ce as fp to cp; and there-

fore fg is equal to $\frac{ce \times fp}{cp}$; and therefore the angle which fg ſubtends in fact is to the former angle which FG ſubtends, that is to ſay, the motion of the nodes in the ellipſis is to the motion of the ſame in the circle as this fg or $\frac{ce \times fp}{cp}$ to the former fg or $\frac{ce \times fY}{cY}$, that is, as $fp \times cY$ to $fY \times cp$, or as fp to fY, and cY to cp; that is, if ph parallel to TN meet FP in h, as Fh to FY and FY to FP; that is, as Fh to FP or Dp to DP, and therefore as the area Dpmd to the area DPMd. And, therefore, ſeeing (by corol. 1, prop. 30) the latter area and AZ^2 conjunctly are proportional to the horary motion of the nodes in the circle, the former area and AZ^2 conjunctly will be proportional to the horary motion of the nodes in the ellipſis. Q.E.D.

Cor. Since, therefore, in any given poſition of the nodes, the ſum of all the areas pDdm, in the time while the moon is carried from the quadrature to any place m, is the area mpQEd terminated at the tangent of the ellipſis QE; and the ſum of all thoſe areas, in one entire revolution, is the area of the whole ellipſis; the mean motion of the nodes in the ellipſis will be to the mean motion of the nodes in the circle as the ellipſis to the circle; that is, as Ta to TA, or 69 to 70. And, therefore, ſince (by corol. 2, prop. 30) the mean horary motion of the nodes in the circle is to $16'' 35''' 16^{iv}. 36^{v}.$ as AZ^2 to AT^2, if we take the angle $16'' 21''' 3^{iv}. 30^{v}.$ to the angle $16'' 35''' 16^{iv}. 36^{v}.$ as 69 to 70, the mean horary motion of the nodes in the ellipſis will be to $16'' 21''' 3^{iv}. 30^{v}.$ as AZ^2 to AT^2; that is, as the ſquare of the ſine of the diſtance of the node from the ſun to the ſquare of the radius.

But the moon, by a radius drawn to the earth, deſcribes the area in the ſyzygies with a greater velocity than it does that in the quadratures, and upon that account the time is contracted in the ſyzygies, and prolonged in the quadratures; and together with the time the motion of the nodes is likewiſe augmented or diminiſhed. But the moment of the area in the quadrature of the moon was to the moment thereof in the ſyzygies as 10973 to 11073; and therefore the mean moment in the octants is to the exceſs in the ſyzygies, and to the

defect in the quadratures, as 11023, the half sum of those numbers, to their half difference 50. Wherefore since the time of the moon's mora in the several little equal parts of its orbit is reciprocally as its velocity, the mean time in the octants will be to the excess of the time in the quadratures, and to the defect *of the time* in the syzygies arising from this cause, nearly as 11023 to 50. But, reckoning from the quadratures to the syzygies, I find that the excess of the moments of the area, in the several places above the least moment in the quadratures, is nearly as the square of the sine of the moon's distance from the quadratures; and therefore the difference betwixt the moment in any place, and the mean moment in the octants, is as the difference betwixt the square of the sine of the moon's distance from the quadratures, and the square of the sine of 45 degrees, or half the square of the radius; and the increment of the time in the several places between the octants and quadratures, and the decrement thereof between the octants and syzygies, is in the same proportion. But the motion of the nodes, while the moon describes the several little equal parts of its orbit, is accelerated or retarded in the duplicate proportion of the time; for that motion, while the moon describes PM, is *(cæteris paribus)* as ML, and ML is in the duplicate proportion of the time. Wherefore the motion of the nodes in the syzygies, in the time while the moon describes given little parts of its orbit, is diminished in the duplicate proportion of the number 11073 to the number 11023; and the decrement is to the remaining motion as 100 to 10973; but to the whole motion as 100 to 11073 nearly. But the decrement in the places between the octants and syzygies, and the increment in the places between the octants and quadratures, is to this decrement nearly as the whole motion in these places to the whole motion in the syzygies, and the difference betwixt the square of the sine of the moon's distance from the quadrature, and the half square of the radius, to the half square of the radius conjunctly. Wherefore, if the nodes are in the quadratures, and we take two places, one on one side, one on the other, equally distant from the octant and other two distant by the same interval,

one from the ſyzygy, the other from the quadrature, and from the decrements of the motions in the two places between the ſyzygy and octant we ſubtract the increments of the motions in the two other places between the octant and the quadrature, the remaining decrement will be equal to the decrement in the ſyzygy, as will eaſily appear by computation; and therefore the mean decrement, which ought to be ſubducted from the mean motion of the nodes, is the fourth part of the decrement in the ſyzygy. The whole horary motion of the nodes in the ſyzygies (when the moon by a radius drawn to the earth was ſuppoſed to deſcribe an area proportional to the time) was $32'' 42''' 7^{iv}$. And we have ſhewn that the decrement of the motion of the nodes, in the time while the moon, now moving with greater velocity, deſcribes the ſame ſpace, was to this motion as 100 to 11073; and therefore this decrement is $17''' 43^{iv}. 11^{v}$. The fourth part of which $4''' 25^{iv}. 48^{v}$. ſubtracted from the mean horary motion above found, $16'' 21''' 3^{iv}. 30^{v}$. leaves $16'' 16''' 37^{iv}. 42^{v}$. their correct mean horary motion.

If the nodes are without the quadratures, and two places are conſidered, one on one ſide, one on the other, equally diſtant from the ſyzygies, the ſum of the motions of the nodes, when the moon is in thoſe places, will be to the ſum of their motions, when the moon is in the ſame places and the nodes in the quadratures, as AZ^2 to AT^2. And the decrements of the motions ariſing from the cauſes but now explained will be mutually as the motions themſelves, and therefore the remaining motions will be mutually betwixt themſelves as AZ^2 to AT^2; and the mean motions will be as the remaining motions. And, therefore, in any given poſition of the nodes, their correct mean horary motion is to $16'' 16''' 37^{iv}. 42^{v}$. as AZ^2 to AT^2; that is, as the ſquare of the ſine of the diſtance of the nodes from the ſyzygies to the ſquare of the radius.

PROPOSITION XXXIII. PROBLEM XIII.

To find the mean motion of the nodes of the moon. (Pl. 12, Fig. 2.)

The yearly mean motion is the ſum of all the mean horary motions throughout the courſe of the year. Suppoſe that the

node is in N, and that, after every hour is elapſed, it is drawn back again to its former place; ſo that, notwithſtanding its proper motion, it may conſtantly remain in the ſame ſituation with reſpect to the fixed ſtars; while in the mean time the ſun S, by the motion of the earth, is ſeen to leave the node, and to proceed till it completes its apparent annual courſe by an uniform motion. Let Aa repreſent a given leaſt arc, which the right line TS always drawn to the ſun, by its interſection with the circle NAn, deſcribes in the leaſt given moment of time; and the mean horary motion (from what we have above ſhewn) will be as AZ^2, that is (becauſe AZ and ZY are proportional), as the rectangle of AZ into ZY, that is, as the area AZYa; and the ſum of all the mean horary motions from the beginning will be as the ſum of all the areas aYZA, that is, as the area NAZ. But the greateſt AZYa is equal to the rectangle of the arc Aa into the radius of the circle; and therefore the ſum of all theſe rectangles in the whole circle will be to the like ſum of all the greateſt rectangles as the area of the whole circle to the rectangle of the whole circumference into the radius, that is, as 1 to 2. But the horary motion correſponding to that greateſt rectangle was $16'' \; 16''' \; 37^{iv}. \; 42^{v}.$ and this motion in the complete courſe of the ſidereal year, $365^d. \; 6^h. \; 9'$, amounts to $39° \; 38' \; 7'' \; 50'''$, and therefore the half thereof, $19° \; 49' \; 3'' \; 55'''$, is the mean motion of the nodes correſponding to the whole circle. And the motion of the nodes, in the time while the ſun is carried from N to A, is to $19° \; 49' \; 3'' \; 55'''$ as the area NAZ to the whole circle.

Thus it would be if the node was after every hour drawn back again to its former place, that ſo, after a complete revolution, the ſun at the year's end would be found again in the ſame node which it had left when the year begun. But, becauſe of the motion of the node in the mean time, the ſun muſt needs meet the node ſooner; and now it remains that we compute the abbreviation of the time. Since, then, the ſun, in the courſe of the year, travels 360 degrees, and the node in the ſame time by its greateſt motion would be carried $39° \; 38' \; 7'' \; 50'''$, or 39,6355 degrees; and the mean motion of the node

in any place N is to its mean motion in its quadratures as AZ^2 to AT^2; the motion of the sun will be to the motion of the node in N as $360AT^2$ to $39,6355AZ^2$; that is, as $9,0827646AT^2$ to AZ^2. Wherefore if we suppose the circumference NAn of the whole circle to be divided into little equal parts, such as Aa, the time in which the sun would describe the little arc Aa, if the circle was quiescent, will be to the time of which it would describe the same arc, supposing the circle together with the nodes to be revolved about the centre T, reciprocally as $9,0827646AT^2$ to $9,0827646AT^2 + AZ^2$; for the time is reciprocally as the velocity with which the little arc is described, and this velocity is the sum of the velocities of both sun and node. If, therefore, the sector NTA represent the time in which the sun by itself, without the motion of the node, would describe the arc NA, and the indefinitely small part ATa of the sector represent the little moment of the time in which it would describe the least arc Aa; and (letting fall aY perpendicular upon Nn) if in AZ we take dZ of such length that the rectangle of dZ into ZY may be to the least part ATa of the sector as AZ^2 to $9,0827646AT^2 + AZ^2$, that is to say, that dZ may be to $\frac{1}{2}AZ$ as AT^2 to $9,0827646AT^2 + AZ^2$; the rectangle of dZ into ZY will represent the decrement of the time arising from the motion of the node, while the arc Aa is described; and if the curve NdGn is the locus where the point d is always found, the curvilinear area NdZ will be as the whole decrement *of time* while the whole arc NA is described; and, therefore, the excess of the sector NAT above the area NdZ will be as the whole time. But because the motion of the node in a less time is less in proportion of the time, the area AaYZ must also be diminished in the same proportion; which may be done by taking in AZ the line eZ of such length, that it may be to the length of AZ as AZ^2 to $9,0827646AT^2 + AZ^2$; for so the rectangle of eZ into ZY will be to the area AZYa as the decrement of the time in which the arc Aa is described to the whole time in which it would have been described, if the node had been quiescent; and, therefore, that rectangle will be as the decrement of the motion of the node. And if the curve NeFn is the locus of

the point e, the whole area NeZ, which is the ſum of all the decrements *of that motion*, will be as the whole decrement *thereof* during the time in which the arc AN is deſcribed; and the remaining area NAe will be as the remaining motion, which is the true motion of the node, during the time in which the whole arc NA is deſcribed by the joint motions of both ſun and node. Now the area of the ſemi-circle is to the area of the figure NeFn found by the method of infinite ſeries nearly as 793 to 60. But the motion correſponding *or proportional* to the whole circle was 19° 49′ 3″ 55‴; and therefore the motion correſponding to double the figure NeFn is 1° 29′ 58″ 2‴, which taken from the former motion leaves 18° 19′ 5″ 53‴, the whole motion of the node with reſpect to the fixed ſtars in the interval between two of its conjunctions with the ſun; and this motion ſubducted from the annual motion of the ſun 360°, leaves 341° 40′ 54″ 7‴, the motion of the ſun in the interval between the ſame conjunctions. But as this motion is to the annual motion 360°, ſo is the motion of the node but juſt now found 18° 19′ 5″ 53‴ to its annual motion, which will therefore be 19° 18′ 1″ 23‴; and this is the mean motion of the nodes in the ſidereal year. By aſtronomical tables, it is 19° 21′ 21″ 50‴. The difference is leſs than $\frac{1}{360}$ part of the whole motion, and ſeems to ariſe from the eccentricity of the moon's orbit, and its inclination to the plane of the ecliptic. By the eccentricity of this orbit the motion of the nodes is too much accelerated; and, on the other hand, by the inclination of the orbit, the motion of the nodes is ſomething retarded, and reduced to its juſt velocity.

PROPOSITION XXXIII. PROBLEM XIV.

To find the true motion of the nodes of the moon. (Pl. 12, Fig. 3.)

In the time which is as the area NTA — NdZ (in the preceding Fig.) that motion is as the area NAe, and is thence given; but becauſe the calculus is too difficult, it will be better to uſe the following conſtruction of the problem. About the centre C, with any interval CD, deſcribe the circle BEFD; produce DC to A ſo as AB may be to AC as the mean motion to half the mean true motion when the nodes

are in their quadratures (that is, as 19° 18′ 1″ 23‴ to 19° 49′ 3″ 55‴; and therefore BC to AC as the difference of thoſe motions 0° 31′ 2″ 32‴ to the latter motion 19° 49′ 3″ 55‴, that is, as 1 to $38\frac{3}{10}$). Then through the point D draw the indefinite line Gg, touching the circle in D; and if we take the angle BCE, or BCF, equal to the double diſtance of the ſun from the place of the node, as found by the mean motion, and drawing AE or AF cutting the perpendicular DG in G, we take another angle which ſhall be to the whole motion of the node in the interval between its ſyzygies (that is, to 9° 11′ 3″) as the tangent DG to the whole circumference of the circle BED, and add this *laſt* angle (for which the angle DAG may be uſed) to the mean motion of the nodes, while they are paſſing from the quadratures to the ſyzygies, and ſubtract it from their mean motion while they are paſſing from the ſyzygies to the quadratures, we ſhall have their true motion; for the true motion ſo found will nearly agree with the true motion which comes out from aſſuming the times as the area NTA — NdZ, and the motion of the node as the area NAe; as whoever will pleaſe to examine and make the computations will find: and this is the ſemi-menſtrual equation of the motion of the nodes. But there is alſo a menſtrual equation, but which is by no means neceſſary for finding of the moon's latitude; for ſince the variation of the inclination of the moon's orbit to the plane of the ecliptic is liable to a twofold inequality, the one ſemi-menſtrual, the other menſtrual, the menſtrual inequality *of this variation*, and the menſtrual equation of the nodes, ſo moderate and correct each other, that in computing the latitude of the moon both may be neglected.

Cor. From this and the preceding prop. it appears that the nodes are quieſcent in their ſyzygies, but regreſſive in their quadratures, by an hourly motion of 16″ 19‴ 26ⁱᵛ.; and that the equation of the motion of the nodes in the octants is 1° 30′; all which exactly agree with the phænomena of the heavens.

SCHOLIUM.

Mr. *Machin*, Aſtron., Prof. Greſh., and Dr. *Henry Pemberton*, ſeparately found out the motion of the nodes by a different

Fig 1.

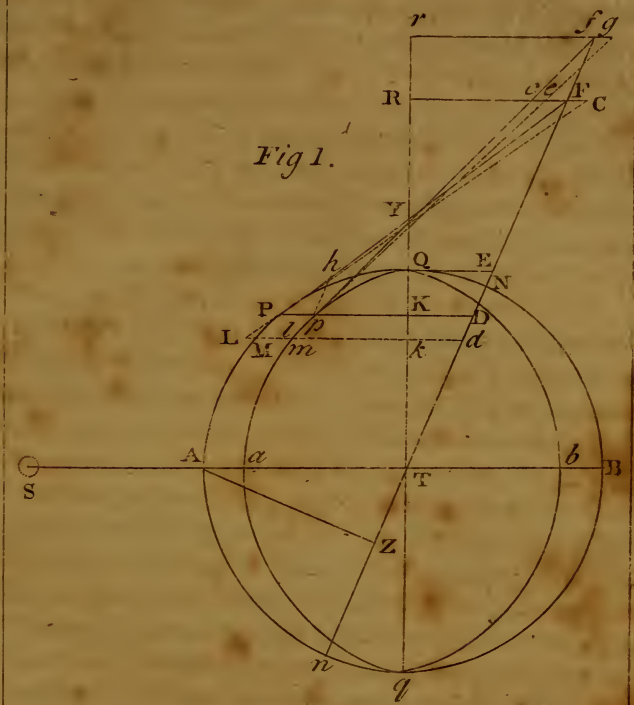


Fig 2.

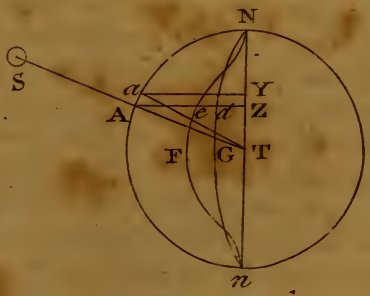
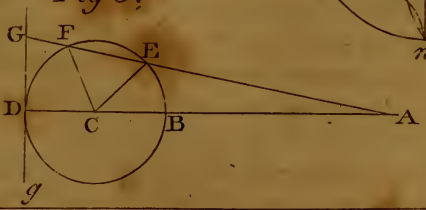


Fig 3.



method. Mention has been made of this method in another place. Their several papers, both of which I have seen, contained two propositions, and exactly agreed with each other in both of them. Mr. *Machin*'s paper coming first to my hands, I shall here insert it.

OF THE MOTION OF THE MOON'S NODES.

" PROPOSITION I.

" *The mean motion of the sun from the node is defined by a*
" *geometric mean proportional between the mean motion of*
" *the sun and that mean motion with which the sun recedes*
" *with the greatest swiftness from the node in the qua-*
" *dratures.*

" Let T (Pl. 13, Fig. 1) be the earth's place, Nn the line of " the moon's nodes at any given time, KTM a perpendicular " thereto, TA a right line revolving about the centre with the " same angular velocity with which the sun and the node " recede from one another, in such sort that the angle be- " tween the quiescent right line Nn and the revolving line " TA may be always equal to the distance of the places " of the sun and node. Now if any right line TK be " divided into parts TS and SK, and those parts be taken " as the mean horary motion of the sun to the mean horary " motion of the node in the quadratures, and there be taken " the right line TH, a mean proportional between the part " TS and the whole TK, this right line will be proportional " to the sun's mean motion from the node.

" For let there be described the circle NKnM from the " centre T and with the radius TK, and about the same cen- " tre, with the semi-axes TH and TN, let there be described " an ellipsis NHnL; and in the time in which the sun recedes " from the node through the arc Na, if there be drawn the " right line Tba, the area of the sector NTa will be the expo- " nent of the sum of the motions of the sun and node in the " same time. Let, therefore, the extremely small arc aA be " that which the right line Tba, revolving according to the " abovesaid law, will uniformly describe in a given particle " of time, and the extremely small sector TAa will be as the " sum of the velocities with which the sun and node are car-

" ried two different ways in that time. Now the ſun's velocity " is almoſt uniform, its inequality being ſo ſmall as ſcarcely " to produce the leaſt inequality in the mean motion of the " nodes. The other part of this ſum, namely, the mean " quantity of the velocity of the node, is increaſed in the re- " ceſs from the ſyzygies in a duplicate ratio of the ſine of its " diſtance from the ſun (by cor. prop. 31, of this book), and, " being greateſt in its quadratures with the ſun in K, is in the " ſame ratio to the ſun's velocity as SK to TS, that is, as (the " difference of the ſquares of TK and TH, or) the rectangle " KHM to TH^2. But the ellipſis NBH divides the ſector " ATa, the exponent of the ſums of theſe two velocities, into " two parts ABba and BTb, proportional to the velocities. " For produce BT to the circle in β, and from the point B let " fall upon the greater axis the perpendicular BG, which be- " ing produced both ways may meet the circle in the points F " and f; and becauſe the ſpace ABba is to the ſector TBb as " the rectangle ABβ to BT^2 (that rectangle being equal to the " difference of the ſquares of TA and TB, becauſe the right " line Aβ is equally cut in T, and unequally in B), therefore " when the ſpace ABba is the greateſt of all in K, this ratio " will be the ſame as the ratio of the rectangle KHM to HT^2. " But the greateſt mean velocity of the node was ſhewn above " to be in that very ratio to the velocity of the ſun; and " therefore in the quadratures the ſector ATa is divided into " parts proportional to the velocities. And becauſe the rect- " angle KHM is to HT^2 as FBf to BG^2, and the rectangle " ABβ is equal to the rectangle FBf, therefore the little area " ABba, where it is greateſt, is to the remaining ſector TBb " as the rectangle ABβ to BG^2. But the ratio of theſe little " areas always was as the rectangle ABβ to BT^2; and there- " fore the little area ABba in the place A is leſs than its " correſpondent little area in the quadratures in the duplicate " ratio of BG to BT, that is, in the duplicate ratio of the ſine " of the ſun's diſtance from the node. And therefore the " ſum of all the little areas ABba, to wit, the ſpace ABN, " will be as the motion of the node in the time in which the " ſun hath been going over the arc NA ſince he left the node;

" and the remaining ſpace, namely, the elliptic ſector NTB, " will be as the ſun's mean motion in the ſame time. And " becauſe the mean annual motion of the node is that mo- " tion which it performs in the time that the ſun completes " one period of its courſe, the mean motion of the node from " the ſun will be to the mean motion of the ſun itſelf as the " area of the circle to the area of the ellipſis; that is, as the " right line TK to the right line TH, which is a mean pro- " portional between TK and TS; or, which comes to the " ſame, as the mean proportional TH to the right line TS.

" PROPOSITION II.

" *The mean motion of the moon's nodes being given, to find* " *their true motion.*

" Let the angle A be the diſtance of the ſun from the " mean place of the node, or the ſun's mean motion from the " node. Then if we take the angle B, whoſe tangent is to " the tangent of the angle A as TH to TK, that is, in the ſub- " duplicate ratio of the mean horary motion of the ſun to the " mean horary motion of the ſun from the node, when the " node is in the quadrature, that angle B will be the diſtance " of the ſun from the node's true place. For join FT, and, " by the demonſtration of the laſt proportion, the angle FTN " will be the diſtance of the ſun from the mean place of the " node, and the angle ATN the diſtance from the true place, " and the tangents of theſe angles are between themſelves as " TK to TH.

" Cor. Hence the angle FTA is the equation of the " moon's nodes; and the ſine of this angle, where it is greateſt " in the octants, is to the radius as KH to TK + TH. But " the ſine of this equation in any other place A is to the " greateſt ſine as the ſine of the ſums of the angles FTN + " ATN to the radius; that is, nearly as the ſine of double " the diſtance of the ſun from the mean place of the node " (namely, 2FTN) to the radius.

" SCHOLIUM.

" If the mean horary motion of the nodes in the quadra- " tures be 16″ 16‴ 37^{iv}. 42^{v}. that is, in a whole ſidereal year, " 39° 38′ 7″ 50‴, TH will be to TK in the ſubduplicate ratio

" of the number 9,0827646 to the number 10,827646, that " is, as 18,6524761 to 19,6524761. And, therefore, TH is " to HK as 18,6524761 to 1; that is, as the motion of the " fun in a fidereal year to the mean motion of the node 19° " 18′ 1″ $23\frac{2}{3}$‴.

" But if the mean motion of the moon's nodes in 20 Julian " years is 386° 50′ 15″, as is collected from the obfervations " made ufe of in the theory of the moon, the mean motion " of the nodes in one fidereal year will be 19° 20′ 31″ 58‴. " And TH will be to HK as 360° to 19° 20′ 31″ 58‴; that is, " as 18,61214 to 1: and from hence the mean horary motion " of the nodes in the quadratures will come out 16″ 18‴ 48^{iv}. " And the greateft equation of the nodes in the octants will " be 1° 29′ 57″."

PROPOSITION XXXIV. PROBLEM XV.

To find the horary variation of the inclination of the moon's orbit to the plane of the ecliptic.

Let A and a (Pl. 13, Fig. 2) repreſent the ſyzygies; Q and q the quadratures; N and n the nodes; P the place of the moon in its orbit; p the orthographic projection of that place upon the plane of the ecliptic; and mTl the momentaneous motion of the nodes as above. If upon Tm we let fall the perpendicular PG, and joining pG we produce it till it meet Tl in g, and join alſo Pg, the angle PGp will be the inclination of the moon's orbit to the plane of the ecliptic when the moon is in P; and the angle Pgp will be the inclination of the ſame after a ſmall moment of time is elapſed; and therefore the angle GPg will be the momentaneous variation of the inclination. But this angle GPg is to the angle GTg as TG to PG and Pp to PG conjunctly. And, therefore, if for the moment of time we aſſume an hour, ſince the angle GTg (by prop. 30) is to the angle 33″ 10‴ 33^{iv}. as IT × PG × AZ to AT^3, the angle GPg (or the horary variation of the inclination) will be to the angle 33″ 10″ 33^{iv}. as IT × AZ × TG × $\frac{Pp}{PG}$ to AT^3. Q.E.I.

And thus it would be if the moon was uniformly revolved in a circular orbit. But if the orbit is elliptical, the mean mo-

tion of the nodes will be diminiſhed in proportion of the leſſer axis to the greater, as we have ſhewn above; and the variation of the inclination will be alſo diminiſhed in the ſame proportion.

Cor. 1. Upon Nn erect the perpendicular TF, and let pM be the horary motion of the moon in the plane of the ecliptic; upon QT let fall the perpendiculars pK, Mk, and produce them till they meet TF in H and h; then IT will be to AT as Kk to Mp; and TG to Hp as TZ to AT; and, therefore, IT × TG will be equal to $\frac{Kk \times Hp \times TZ}{Mp}$, that is, equal to the area HpMh multiplied into the ratio $\frac{TZ}{Mp}$: and therefore the horary variation of the inclination will be to 33″ 10‴ 33^{iv}. as the area HpMh multiplied into AZ × $\frac{TZ}{Mp}$ × $\frac{Pp}{PG}$ to AT^3.

Cor. 2. And, therefore, if the earth and nodes were after every hour drawn back from their new and inſtantly reſtored to their old places, ſo as their ſituation might continue given for a whole periodic month together, the whole variation of the inclination during that month would be to 33″ 10‴ 33^{iv}. as the aggregate of all the areas HpMh, generated in the time of one revolution of the point p (with due regard in ſumming to their proper ſigns + and —), multiplied into AZ × TZ × $\frac{Pp}{PG}$ to Mp × AT^3; that is, as the whole circle QAqa multiplied into AZ × TZ × $\frac{Pp}{PG}$ to Mp × AT^3, that is, as the circumference QAqa multiplied into AZ × TZ × $\frac{Pp}{PG}$ to 2Mp × AT^2.

Cor. 3. And, therefore, in a given poſition of the nodes, the mean horary variation, from which, if uniformly continued through the whole month, that menſtrual variation might be generated, is to 33″ 10‴ 33^{iv}. as AZ × TZ × $\frac{Pp}{PG}$ to 2AT^2, or as Pp × $\frac{AZ \times TZ}{\frac{1}{2}AT}$ to PG × 4AT; that is

(becauſe Pp is to PG as the ſine of the aforeſaid inclination to the radius, and $\frac{AZ \times TZ}{\frac{1}{2}AT}$ to 4AT as the ſine of double the angle ATn to four times the radius), as the ſine of the ſame inclination multiplied into the ſine of double the diſtance of the nodes from the ſun to four times the ſquare of the radius.

Cor. 4. Seeing the horary variation of the inclination, when the nodes are in the quadratures, is (by this prop.) to the angle 33″ 10‴ 33^{iv}. as $IT \times AZ \times TG \times \frac{Pp}{PG}$ to AT^3, that is, as $\frac{IT \times TG}{\frac{1}{2}AT} \times \frac{Pp}{PG}$ to 2AT, that is, as the ſine of double the diſtance of the moon from the quadratures multiplied into $\frac{Pp}{PG}$ to twice the radius, the ſum of all the horary variations during the time that the moon, in this ſituation of the nodes, paſſes from the quadrature to the ſyzygy (that is, in the ſpace of 177$\frac{1}{6}$ hours) will be to the ſum of as many angles 33″ 10‴ 33^{iv}. or 5878″, as the ſum of all the ſines of double the diſtance of the moon from the quadratures multiplied into $\frac{Pp}{PG}$ to the ſum of as many diameters; that is, as the diameter multiplied into $\frac{Pp}{PG}$ to the circumference; that is, if the inclination be 5° 1′, as $7 \times \frac{874}{10000}$ to 22, or as 278 to 10000. And, therefore, the whole variation, compoſed out of the ſum of all the horary variations in the aforeſaid time, is 163″, or 2′ 43″.

PROPOSITION XXXV. PROBLEM XVI.

To a given time to find the inclination of the moon's orbit to the plane of the ecliptic.

Let AD (Pl. 14, Fig. 1) be the ſine of the greateſt inclination, and AB the ſine of the leaſt. Biſect BD in C; and found the centre C, with the interval BC, deſcribe the circle BGD. In AC take CE in the ſame proportion to EB as EB to twice BA. And if to the time given we ſet off the angle AEG equal to double the diſtance of the nodes from the qua

Fig 1.

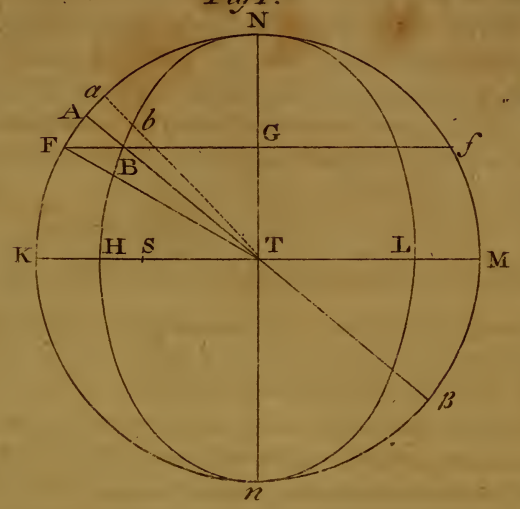
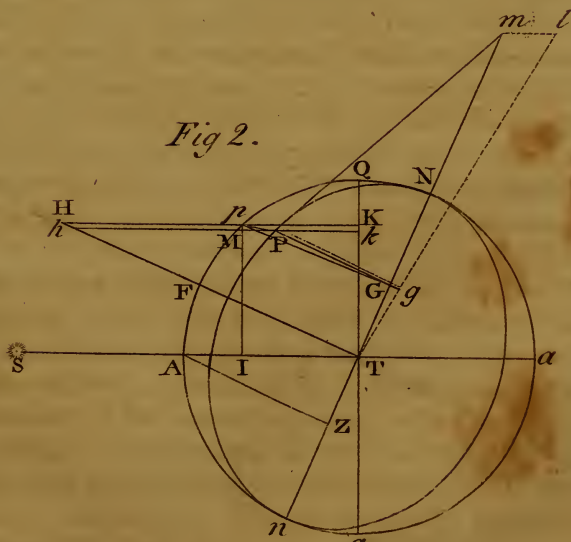


Fig 2.



dratures, and upon AD let fall the perpendicular GH, AH will be the fine of the inclination required.

For GE^2 is equal to $GH^2 + HE^2 = BHD + HE^2 = HBD + HE^2 - BH^2 = HBD + BE^2 - 2BH \times BE = BE^2 + 2EC \times BH = 2EC \times AB + 2EC \times BH = 2EC \times AH$; wherefore fince 2EC is given, GE^2 will be as AH. Now let AEg repreſent double the diſtance of the nodes from the quadratures, in a given moment of time after, and the arc Gg, on account of the given angle GEg, will be as the diſtance GE. But Hh is to Gg as GH to GC, and, therefore, Hh is as the rectangle GH × Gg, or GH × GE, that is, as $\frac{GH}{GE} \times GE^2$, or $\frac{GH}{GE} \times AH$; that is, as AH and the fine of the angle AEG conjunctly. If, therefore, in any one caſe, AH be the fine of inclination, it will increaſe by the ſame increments as the fine of inclination doth, by cor. 3 of the preceding prop. and therefore will always continue equal to that fine. But when the point G falls upon either point B or D, AH is equal to this fine, and therefore remains always equal thereto. Q.E.D.

In this demonſtration I have ſuppoſed that the angle BEG, repreſenting double the diſtance of the nodes from the quadratures, increaſeth uniformly; for I cannot deſcend to every minute circumſtance of inequality. Now ſuppoſe that BEG is a right angle, and that Gg is in this caſe the horary increment of double the diſtance of the nodes from the ſun; then, by cor. 3 of the laſt prop. the horary variation of the inclination in the ſame caſe will be to $33''\ 10'''\ 33^{iv}$. as the rectangle of AH, the fine of the inclination, into the fine of the right angle BEG, double the diſtance of the nodes from the ſun, to four times the ſquare of the radius; that is, as AH, the fine of the mean inclination, to four times the radius; that is, ſeeing the mean inclination is about $5°\ 8\frac{1}{2}$, as its fine 896 to 40000, the quadruple of the radius, or as 224 to 10000. But the whole variation correſponding to BD, the difference of the ſines, is to this horary variation as the diameter BD to the arc Gg, that is, conjunctly as the diameter BD to the ſemi-circumference BGD, and as the time of $2079\frac{7}{10}$ hours, in which

the node proceeds from the quadratures to the ſyzygies, to one hour, that is, as 7 to 11, and $2079\frac{7}{10}$ to 1. Wherefore, compounding all theſe proportions, we ſhall have the whole variation BD to 33″ 10‴ 33ᴵᵛ. as $224 \times 7 \times 2079\frac{7}{10}$ to 110000, that is, as 29645 to 1000; and from thence that variation BD will come out 16′ $23\frac{1}{2}$″.

And this is the greateſt variation of the inclination, abſtracting from the ſituation of the moon in its orbit; for if the nodes are in the ſyzygies, the inclination ſuffers no change from the various poſitions of the moon. But if the nodes are in the quadratures, the inclination is leſs when the moon is in the ſyzygies than when it is in the quadratures by a difference of 2′ 43″, as we ſhewed in cor. 4 of the preceding prop.; and the whole mean variation BD, diminiſhed by 1′ $21\frac{1}{2}$″, the half of this exceſs, becomes 15′ 2″, when the moon is in the quadratures; and increaſed by the ſame, becomes 17′ 45″ when the moon is in the ſyzygies. If, therefore, the moon be in the ſyzygies, the whole variation in the paſſage of the nodes from the quadratures to the ſyzygies will be 17′ 45″; and, therefore, if the inclination be 5° 17′ 20″, when the nodes are in the ſyzygies, it will be 4° 59′ 35″ when the nodes are in the quadratures and the moon in the ſyzygies. The truth of all which is confirmed by obſervations.

Now if the inclination of the orbit ſhould be required when the moon is in the ſyzygies, and the nodes any where between them and the quadratures, let AB be to AD as the ſine of 4° 59′ 35″ to the ſine of 5° 17′ 20″, and take the angle AEG equal to double the diſtance of the nodes from the quadratures; and AH will be the ſine of the inclination deſired. To this inclination of the orbit the inclination of the ſame is equal, when the moon is 90° diſtant from the nodes. In other ſituations of the moon, this menſtrual inequality, to which the variation of the inclination is obnoxious in the calculus of the moon's latitude, is balanced, and in a manner took off, by the menſtrual inequality of the motion of the nodes (as we ſaid before), and therefore may be neglected in the computation of the ſaid latitude.

SCHOLIUM.

By theſe computations of the lunar motions I was willing to ſhew that by the theory of gravity the motions of the moon could be calculated from their phyſical cauſes. By the ſame theory I moreover found that the annual equation of the mean motion of the moon ariſes from the various dilatation which the orbit of the moon ſuffers from the action of the ſun, according to cor. 6, prop. 66, book 1. The force of this action is greater in the perigeon ſun, and dilates the moon's orbit; in the apogeon ſun it is leſs, and permits the orbit to be again contracted. The moon moves ſlower in the dilated and faſter in the contracted orbit; and the annual equation, by which this inequality is regulated, vaniſhes in the apogee and perigee of the ſun. In the mean diſtance of the ſun from the earth it ariſes to about 11′ 50″; in other diſtances of the ſun it is proportional to the equation of the ſun's centre, and is added to the mean motion of the moon, while the earth is paſſing from its aphelion to its perihelion, and ſubducted while the earth is in the oppoſite ſemi-circle. Taking for the radius of the *orbis magnus* 1000, and $16\frac{7}{8}$ for the earth's eccentricity, this equation, when of the greateſt magnitude, by the theory of gravity comes out 11′ 49″. But the eccentricity of the earth ſeems to be ſomething greater, and with the eccentricity this equation will be augmented in the ſame proportion. Suppoſe the eccentricity $16\frac{1}{2}$, and the greateſt equation will be 11′ 51″.

Farther; I found that the apogee and nodes of the moon move faſter in the perihelion of the earth, where the force of the ſun's action is greater, than in the aphelion therof, and that in the reciprocal triplicate proportion of the earth's diſtance from the ſun; and hence ariſe annual equations of thoſe motions proportional to the equation of the ſun's centre. Now the motion of the ſun is in the reciprocal duplicate proportion of the earth's diſtance from the ſun; and the greateſt equation of the centre which this inequality generates is 1° 56′ 20″, correſponding to the above-mentioned eccentricity of the ſun, $16\frac{1}{2}$. But if the motion of the ſun had been in the reciprocal triplicate proportion of the diſtance, this ine-

quality would have generated the greateſt equation 2° 54′ 30″; and therefore the greateſt equations which the inequalities of the motions of the moon's apogee and nodes do generate are to 2° 54′ 30″ as the mean diurnal motion of the moon's apogee and the mean diurnal motion of its nodes are to the mean diurnal motion of the ſun. Whence the greateſt equation of the mean motion of the apogee comes out 19′ 43″, and the greateſt equation of the mean motion of the nodes 9′ 24″. The former equation is added, and the latter ſubducted, while the earth is paſſing from its perihelion to its aphelion, and contrariwiſe when the earth is in the oppoſite ſemi-circle.

By the theory of gravity I likewiſe found that the action of the ſun upon the moon is ſomething greater when the tranſverſe diameter of the moon's orbit paſſeth through the ſun than when the ſame is perpendicular upon the line which joins the earth and the ſun; and therefore the moon's orbit is ſomething larger in the former than in the latter caſe. And hence ariſes another equation of the moon's mean motion, depending upon the ſituation of the moon's apogee in reſpect of the ſun, which is in its greateſt quantity when the moon's apogee is in the octants of the ſun, and vaniſhes when the apogee arrives at the quadratures or ſyzygies; and it is added to the mean motion while the moon's apogee is paſſing from the quadrature of the ſun to the ſyzygy, and ſubducted while the apogee is paſſing from the ſyzygy to the quadrature. This equation, which I ſhall call the ſemi-annual, when greateſt in the octants of the apogee, ariſes to about 3′ 45″, ſo far as I could collect from the phænomena: and this is its quantity in the mean diſtance of the ſun from the earth. But it is increaſed and diminiſhed in the reciprocal triplicate proportion of the ſun's diſtance, and therefore is nearly 3′ 34″ when that diſtance is greateſt, and 3′ 56″ when leaſt. But when the moon's apogee is without the octants, it becomes leſs, and is to its greateſt quantity as the ſine of double the diſtance of the moon's apogee from the neareſt ſyzygy or quadrature to the radius.

By the ſame theory of gravity, the action of the ſun upon the moon is ſomething greater when the line of the moon's

nodes paſſes through the ſun than when it is at right angles with the line which joins the ſun and the earth; and hence ariſes another equation of the moon's mean motion, which I ſhall call the ſecond ſemi-annual; and this is greateſt when the nodes are in the octants of the ſun, and vaniſhes when they are in the ſyzygies or quadratures; and in other poſitions of the nodes is proportional to the ſine of double the diſtance of either node from the neareſt ſyzygy or quadrature. And it is added to the mean motion of the moon, if the ſun is *in antecedentia*, to the node which is neareſt to him, and ſubducted if *in conſequentia*; and in the octants, where it is of the greateſt magnitude, it ariſes to 47″ in the mean diſtance of the ſun from the earth, as I find from the theory of gravity. In other diſtances of the ſun, this equation, greateſt in the octants of the nodes, is reciprocally as the cube of the ſun's diſtance from the earth; and therefore in the ſun's perigee it comes to about 49″, and in its apogee to about 45″.

By the ſame theory of gravity, the moon's apogee goes forward at the greateſt rate when it is either in conjunction with or in oppoſition to the ſun, but in its quadratures with the ſun it goes backward; and the eccentricity comes, in the former caſe, to its greateſt quantity; in the latter to its leaſt, by cor. 7, 8, and 9, prop. 66, book 1. And thoſe inequalities, by the corollaries we have named, are very great, and generate the principal, which I call the ſemi-annual equation of the apogee; and this ſemi-annual equation in its greateſt quantity comes to about 12° 18″, as nearly as I could collect from the phænomena. Our countryman *Horrox* was the firſt who advanced the theory of the moon's moving in an ellipſis about the earth placed in its lower focus. Dr. *Halley* improved the notion, by putting the centre of the ellipſis in an epicycle whoſe centre is uniformly revolved about the earth; and from the motion in this epicycle the mentioned inequalities in the progreſs and regreſs of the apogee, and in the quantity of eccentricity, do ariſe. Suppoſe the mean diſtance of the moon from the earth to be divided into 100000 parts, and let T (Pl. 14, Fig. 2) repreſent the earth, and TC the moon's mean eccentricity of 5505 ſuch parts. Produce TC to B,

ſo as CB may be the ſine of the greateſt ſemi-annual equation 12° 18′ to the radius TC; and the circle BDA deſcribed about the centre C, with the interval CB, will be the epicycle ſpoken of, in which the centre of the moon's orbit is placed, and revolved according to the order of the letters BDA. Set off the angle BCD equal to twice the annual argument, or twice the diſtance of the ſun's true place from the place of the moon's apogee once equated, and CTD will be the ſemi-annual equation of the moon's apogee, and TD the eccentricity of its orbit, tending to the place of the apogee now twice equated, But, having the moon's mean motion, the place of its apogee, and its eccentricity, as well as the longer axis of its orbit 200000, from theſe *data* the true place of the moon in its orbit, together with its diſtance from the earth, may be determined by the methods commonly known.

In the perihelion of the earth, where the force of the ſun is greateſt, the centre of the moon's orbit moves faſter about the centre C than in the aphelion, and that in the reciprocal triplicate proportion of the ſun's diſtance from the earth. But, becauſe the equation of the ſun's centre is included in the annual argument, the centre of the moon's orbit moves faſter in its epicycle BDA, in the reciprocal duplicate proportion of the ſun's diſtance from the earth. Therefore that it may move yet faſter in the reciprocal ſimple proportion of the diſtance, ſuppoſe that from D, the centre of the orbit, a right line DE is drawn, tending towards the moon's apogee once equated, that is, parallel to TC; and ſet off the angle EDF equal to the exceſs of the aforeſaid annual argument above the diſtance of the moon's apogee from the ſun's perigee *in conſequentia;* or, which comes to the ſame thing, take the angle CDF equal to the complement of the ſun's true anomaly to 360°; and let DF be to DC as twice the eccentricity of the *orbis magnus* to the ſun's mean diſtance from the earth, and the ſun's mean diurnal motion from the moon's apogee to the ſun's mean diurnal motion from its own apogee conjunctly, that is, as $33\frac{7}{8}$ to 1000, and 52′ 27″ 16‴ to 59′ 8″ 10‴ conjunctly, or as 3 to 100; and imagine the centre of

the moon's orbit placed in the point F to be revolved in an epicycle whofe centre is D, and radius DF, while the point D moves in the circumference of the circle DABD; for by this means the centre of the moon's orbit comes to defcribe a certain curve line about the centre C, with a velocity which will be almoft reciprocally as the cube of the fun's diftance from the earth, as it ought to be.

The calculus of this motion is difficult, but may be rendered more eafy by the following approximation. Affuming, as above, the moon's mean diftance from the earth of 100000 parts, and the eccentricity TC of 5505 fuch parts, the line CB or CD will be found $1172\frac{3}{4}$, and DF $35\frac{1}{5}$ of thofe parts; and this line DF at the diftance TC fubtends the angle at the earth, which the removal of the centre of the orbit from the place D to the place F generates in the motion of this centre; and double this line DF in a parallel pofition, at the diftance of the upper focus of the moon's orbit from the earth, fubtends at the earth the fame angle as DF did before, which that removal generates in the motion of this upper focus; but at the diftance of the moon from the earth this double line 2DF at the upper focus, in a parallel pofition to the firft line DF, fubtends an angle at the moon, which the faid removal generates in the motion of the moon, which angle may be therefore called the fecond equation of the moon's centre; and this equation, in the mean diftance of the moon from the earth, is nearly as the fine of the angle which that line DF contains with the line drawn from the point F to the moon, and when in its greateft quantity amounts to 2′ 25″. But the angle which the line DF contains with the line drawn from the point F to the moon is found either by fubtracting the angle EDF from the mean anomaly of the moon, or by adding the diftance of the moon from the fun to the diftance of the moon's apogee from the apogee of the fun; and as the radius to the fine of the angle thus found, fo is 2′ 25″ to the fecond equation of the centre: to be added, if the forementioned fum be lefs than a femi-circle; to be fubducted, if greater. And from the moon's place in its orbit thus corrected its longitude may be found in the fyzygies of the luminaries.

The atmosphere of the earth to the height of 35 or 40 miles refracts the sun's light. This refraction scatters and spreads the light over the earth's shadow; and the dissipated light near the limits of the shadow dilates the shadow. Upon which account, to the diameter of the shadow, as it comes out by the parallax, I add 1 or $1\frac{1}{3}$ minute in lunar eclipses.

But the theory of the moon ought to be examined and proved from the phænomena, first in the syzygies, then in the quadratures, and last of all in the octants; and whoever pleases to undertake the work will find it not amiss to assume the following mean motions of the sun and moon at the Royal Observatory of *Greenwich*, to the last day of *December* at noon, *anno* 1700, O.S. viz. The mean motion of the sun ♑ 20° 43′ 40″, and of its apogee ♋ 7° 44′ 30″; the mean motion of the moon ♒ 15° 21′ 00″; of its apogee ♓ 8° 20′ 00″; and of its ascending node ☊ 27° 24′ 20″; and the difference of meridians betwixt the Observatory at *Greenwich* and the Royal Observatory at *Paris*, 0^h. 9′ 20″: but the mean motion of the moon and of its apogee are not yet obtained with sufficient accuracy.

PROPOSITION XXXVI. PROBLEM XVII.

To find the force of the sun to move the sea.

The sun's force ML or PT to disturb the motions of the moon, was (by prop. 25) in the moon's quadratures, to the force of gravity with us, as 1 to 638092,6; and the force TM — LM or 2PK in the moon's syzygies is double that quantity. But, descending to the surface of the earth, these forces are diminished in proportion of the distances from the centre of the earth, that is, in the proportion of $60\frac{1}{2}$ to 1; and therefore the former force on the earth's surface is to the force of gravity as 1 to 38604600; and by this force the sea is depressed in such places as are 90 degrees distant from the sun. But by the other force, which is twice as great, the sea is raised not only in the places directly under the sun, but in those also which are directly opposed to it; and the sum of these forces is to the force of gravity as 1 to 12868200. And because the same force excites the same motion, whether it depresses the waters in those places which are 90 degrees distant from the sun, or raises them in the places which

are directly under and directly oppoſed to the ſun, the aforeſaid ſum will be the total force of the ſun to diſturb the ſea, and will have the ſame effect as if the whole was employed in raiſing the ſea in the places directly under and directly oppoſed to the ſun, and did not act at all in the places which are 90 degrees removed from the ſun.

And this is the force of the ſun to diſturb the ſea in any given place, where the ſun is at the ſame time both vertical, and in its mean diſtance from the earth. In other poſitions of the ſun, its force to raiſe the ſea is as the verſed ſine of double its altitude above the horizon of the place directly, and the cube of the diſtance from the earth reciprocally.

Cor. Since the centrifugal force of the parts of the earth, ariſing from the earth's diurnal motion, which is to the force of gravity as 1 to 289, raiſes the waters under the equator to a height exceeding that under the poles by 85472 *Paris* feet, as above, in prop. 19, the force of the ſun, which we have now ſhewed to be to the force of gravity as 1 to 12868200, and therefore is to that centrifugal force as 289 to 12868200, or as 1 to 44527, will be able to raiſe the waters in the places directly under and directly oppoſed to the ſun to a height exceeding that in the places which are 90 degrees removed from the ſun only by one *Paris* foot and $113\frac{4}{10}$ inches; for this meaſure is to the meaſure of 85472 feet as 1 to 44527.

PROPOSITION XXXVII. PROBLEM XVIII.

To find the force of the moon to move the ſea.

The force of the moon to move the ſea is to be deduced from its proportion to the force of the ſun, and this proportion is to be collected from the proportion of the motions of the ſea, which are the effects of thoſe forces. Before the mouth of the river *Avon*, three miles below *Briſtol*, the height of the aſcent of the water in the vernal and autumnal ſyzygies of the luminaries (by the obſervations of *Samuel Sturmy*) amounts to about 45 feet, but in the quadratures to 25 only. The former of thoſe heights ariſes from the ſum of the aforeſaid forces, the latter from their difference. If, therefore, S and L are ſuppoſed to repreſent reſpectively the forces of the ſun and moon while they are in the equator, as well as in

their mean diſtances from the earth, we ſhall have L + S to L — S as 45 to 25, or as 9 to 5.

At *Plymouth* (by the obſervations of *Samuel Colepreſs*) the tide in its mean height riſes to about 16 feet, and in the ſpring and autumn the height thereof in the ſyzygies may exceed that in the quadratures by more than 7 or 8 feet. Suppoſe the greateſt difference of thoſe heights to be 9 feet, and L + S will be to L — S as $20\frac{1}{2}$ to $11\frac{1}{2}$, or as 41 to 23; a proportion that agrees well enough with the former. But becauſe of the great tide at *Briſtol*, we are rather to depend upon the obſervations of *Sturmy*; and, therefore, till we procure ſomething that is more certain, we ſhall uſe the proportion of 9 to 5.

But becauſe of the reciprocal motions of the waters, the greateſt tides do not happen at the times of the ſyzygies of the luminaries, but, as we have ſaid before, are the third in order after the ſyzygies; or (reckoning from the ſyzygies) follow next after the third appulſe of the moon to the meridian of the place after the ſyzygies; or rather (as *Sturmy* obſerves) are the third after the day of the new or full moon, or rather nearly after the twelfth hour from the new or full moon, and therefore fall nearly upon the forty-third hour after the new or full of the moon. But in this port they fall out about the ſeventh hour after the appulſe of the moon to the meridian of the place; and therefore follow next after the appulſe of the moon to the meridian, when the moon is diſtant from the ſun, or from oppoſition with the ſun by about 18 or 19 degrees *in conſequentia*. So the ſummer and winter ſeaſons come not to their height in the ſolſtices themſelves, but when the ſun is advanced beyond the ſolſtices by about a tenth part of its whole courſe, that is, by about 36 or 37 degrees. In like manner, the greateſt tide is raiſed after the appulſe of the moon to the meridian of the place, when the moon has paſſed by the ſun, *or the oppoſition thereof*, by about a tenth part of the whole motion from *one greateſt* tide to *the next following greateſt* tide. Suppoſe that diſtance about $18\frac{1}{2}$ degrees; and the ſun's force in this diſtance of the moon from the ſyzygies and quadratures will be of leſs mo-

ment to augment and diminiſh that part of the motion of the ſea which proceeds from the motion of the moon than in the ſyzygies and quadratures themſelves in the proportion of the radius to the co-ſine of double this diſtance, or of an angle of 37 degrees; that is, in proportion of 10000000 to 7986355; and, therefore, in the preceding analogy, in place of S we muſt put 0,7986355S.

But farther; the force of the moon in the quadratures muſt be diminiſhed, on account of its declination from the equator; for the moon in thoſe quadratures, or rather in $18\frac{1}{2}$ degrees paſt the quadratures, declines from the equator by about 22° 13′; and the force of either luminary to move the ſea is diminiſhed as it declines from the equator nearly in the duplicate proportion of the co-ſine of the declination; and therefore the force of the moon in thoſe quadratures is only 0,8570327L; whence we have L+0,7986355S to 0,8570327L — 0,7986355S as 9 to 5.

Farther yet; the diameters of the orbit in which the moon ſhould move, ſetting aſide the conſideration of eccentricity, are one to the other as 69 to 70; and therefore the moon's diſtance from the earth in the ſyzygies is to its diſtance in the quadratures, *cæteris paribus*, as 69 to 70; and its diſtances, when $18\frac{1}{2}$ degrees advanced beyond the ſyzygies, where the greateſt tide was excited, and when $18\frac{1}{2}$ degrees paſſed by the quadratures, where the leaſt tide was produced, are to its mean diſtance as 69,098747 and 69,897345 to $69\frac{1}{2}$. But the force of the moon to move the ſea is in the reciprocal triplicate proportion of its diſtance; and therefore its forces, in the greateſt and leaſt of thoſe diſtances, are to its force in its mean diſtance as 0,9830427 and 1,017522 to 1. From whence we have 1,017522L × 0,7986355S to 0,9830427 × 0,8570327L — 0,7986355S as 9 to 5; and S to L as 1 to 4,4815. Wherefore ſince the force of the ſun is to the force of gravity as 1 to 12868200, the moon's force will be to the force of gravity as 1 to 2871400.

Cor. 1. Since the waters excited by the ſun's force riſe to the height of a foot and $11\frac{4}{10}$ inches, the moon's force will raiſe the ſame to the height of 8 feet and $7\frac{5}{12}$ inches; and the

joint forces of both will raiſe the ſame to the height of $10\frac{1}{2}$ feet; and when the moon is in its perigee to the height of $12\frac{1}{2}$ feet, and more, eſpecially when the wind ſets the ſame way as the tide. And a force of that quantity is abundantly ſufficient to excite all the motions of the ſea, and agrees well with the proportion of thoſe motions; for in ſuch ſeas as lie free and open from eaſt to weſt, as in the *Pacific* ſea, and in thoſe tracts of the *Atlantic* and *Ethiopic* ſeas which lie without the tropics, the waters commonly riſe to 6, 9, 12, or 15 feet; but in the *Pacific* ſea, which is of a greater depth, as well as of a larger extent, the tides are ſaid to be greater than in the *Atlantic* and *Ethiopic* ſeas; for to have a full tide raiſed, an extent of ſea from eaſt to weſt is required of no leſs than 90 degrees. In the *Ethiopic* ſea, the waters riſe to a leſs height within the tropics than in the temperate zones, becauſe of the narrowneſs of the ſea between *Africa* and the ſouthern parts of *America*. In the middle of the open ſea the waters cannot riſe without falling together, and at the ſame time, upon both the eaſtern and weſtern ſhores, when notwithſtanding, in our narrow ſeas, they ought to fall on thoſe ſhores by alternate turns; upon which account there is commonly but a ſmall flood and ebb in ſuch iſlands as lie far diſtant from the continent. On the contrary, in ſome ports, where to fill and empty the bays alternately the waters are with great violence forced in and out through ſhallow channels, the flood and ebb muſt be greater than ordinary; as at *Plymouth* and *Chepſtow Bridge* in *England*, at the mountains of St. *Michael*, and the town of *Auranches*, in *Normandy*, and at *Cambaia* and *Pegu* in the *Eaſt Indies*. In theſe places the ſea is hurried in and out with ſuch violence, as ſometimes to lay the ſhores under water, ſometimes to leave them dry, for many miles. Nor is this force of the influx and efflux to be broke till it has raiſed and depreſſed the waters to 30, 40, or 50 feet and above. And a like account is to be given of long and ſhallow channels or ſtraits, ſuch as the *Magellanic* ſtraits, and thoſe channels which environ *England*. The tide in ſuch ports and ſtraits, by the violence of the influx and efflux, is augmented above meaſure. But on ſuch ſhores as lie towards

the deep and open ſea with a ſteep deſcent, where the waters may freely riſe and fall without that precipitation of influx and efflux, the proportion of the tides agrees with the forces of the ſun and moon.

Cor. 2. Since the moon's force to move the ſea is to the force of gravity as 1 to 2871400, it is evident that this force is far leſs than to appear ſenſibly in ſtatical or hydroſtatical experiments, or even in thoſe of pendulums. It is in the tides only that this force ſhews itſelf by any ſenſible effect.

Cor. 3. Becauſe the force of the moon to move the ſea is to the like force of the ſun as 4,4815 to 1, and thoſe forces (by cor. 14, prop. 66, book 1) are as the denſities of the bodies of the ſun and moon and the cubes of their apparent diameters conjunctly, the denſity of the moon will be to the denſity of the ſun as 4,4815 to 1 directly, and the cube of the moon's diameter to the cube of the ſun's diameter inverſely; that is (ſeeing the mean apparent diameters of the moon and ſun are 31′ 16½″, and 32′ 12″), as 4891 to 1000. But the denſity of the ſun was to the denſity of the earth as 1000 to 4000; and therefore the denſity of the moon is to the denſity of the earth as 4891 to 4000, or as 11 to 9. Therefore the body of the moon is more denſe and more earthly than the earth itſelf.

Cor. 4. And ſince the true diameter of the moon (from the obſervations of aſtronomers) is to the true diameter of the earth as 100 to 365, the maſs of matter in the moon will be to the maſs of matter in the earth as 1 to 39,788.

Cor. 5. And the accelerative gravity on the ſurface of the moon will be about three times leſs than the accelerative gravity on the ſurface of the earth.

Cor. 6. And the diſtance of the moon's centre from the centre of the earth will be to the diſtance of the moon's centre from the common centre of gravity of the earth and moon as 40,788 to 39,788.

Cor. 7. And the mean diſtance of the centre of the moon from the centre of the earth will be (in the moon's octants) nearly 60⅔ of the greateſt ſemi-diameters of the earth; for the greateſt ſemi-diameter of the earth was 19658600 *Paris*

feet, and the mean diſtance of the centres of the earth and moon, conſiſting of $60\frac{2}{5}$ ſuch ſemi-diameters, is equal to 1187379440 feet. And this diſtance (by the preceding cor.) is to the diſtance of the moon's centre from the common centre of gravity of the earth and moon as 40,788 to 39,788; which latter diſtance, therefore, is 1158268534 feet. And ſince the moon, in reſpect of the fixed ſtars, performs its revolution in $27^{d}.\ 7^{h}.\ 43\frac{4}{9}'$, the verſed ſine of that angle which the moon in a minute of time deſcribes is 12752341 to the radius 1000,000000,000000; and as the radius is to this verſed ſine, ſo are 1158268534 feet to 14,7706353 feet. The moon, therefore, falling towards the earth by that force which retains it in its orbit, would in one minute of time deſcribe 14,7706353 feet; and if we augment this force in the proportion of $178\frac{29}{40}$ to $177\frac{29}{40}$, we ſhall have the total force of gravity at the orbit of the moon, by cor. prop. 3; and the moon falling by this force, in one minute of time would deſcribe 14,8538067 feet. And at the 60th part of the diſtance of the moon from the earth's centre, that is, at the diſtance of 197896573 feet from the centre of the earth, a body falling by its weight, would, in one ſecond of time, likewiſe deſcribe 14,8538067 feet. And, therefore, at the diſtance of 19615800, which compoſe one mean ſemi-diameter of the earth, a heavy body would deſcribe in falling 15,11175, or 15 feet, 1 inch, and $4\frac{1}{11}$ lines, in the ſame time. This will be the deſcent of bodies in the latitude of 45 degrees. And by the foregoing table, to be found under prop. 20, the deſcent in the latitude of *Paris* will be a little greater by an exceſs of about $\frac{2}{3}$ parts of a line. Therefore, by this computation, heavy bodies in the latitude of *Paris* falling *in vacuo* will deſcribe 15 *Paris* feet, 1 inch, $4\frac{2}{3}\frac{5}{3}$ lines, very nearly, in one ſecond of time. And if the gravity be diminiſhed by taking away a quantity equal to the centrifugal force ariſing in that latitude from the earth's diurnal motion, heavy bodies falling there will deſcribe in one ſecond of time 15 feet, 1 inch, and $1\frac{1}{2}$ line. And with this velocity heavy bodies do really fall in the latitude of *Paris*, as we have ſhewn above in prop. 4 and 19.

Cor. 8. The mean diſtance of the centres of the earth and moon in the ſyzygies of the moon is equal to 60 of the greateſt ſemi-diameters of the earth, ſubducting only about one 30th part of a ſemi-diameter; and in the moon's quadratures the mean diſtance of the ſame centres is $60\frac{5}{6}$ ſuch ſemi-diameters of the earth; for theſe two diſtances are to the mean diſtance of the moon in the octants as 69 and 70 to $69\frac{1}{2}$, by prop. 28.

Cor. 9. The mean diſtance of the centres of the earth and moon in the ſyzygies of the moon is 60 mean ſemi-diameters of the earth, and a 10th part of one ſemi-diameter; and in the moon's quadratures the mean diſtance of the ſame centres is 61 mean ſemi-diameters of the earth, ſubducting one 30th part of one ſemi-diameter.

Cor. 10. In the moon's ſyzygies its mean horizontal parallax in the latitudes of 0, 30, 38, 45, 52, 60, 90 degrees is 57′ 20″, 57′ 16″, 57′ 14″, 57′ 12″, 57′ 10″, 57′ 8″, 57′ 4″, reſpectively.

In theſe computations I do not conſider the magnetic attraction of the earth, whoſe quantity is very ſmall and unknown: if this quantity ſhould ever be found out, and the meaſures of degrees upon the meridian, the lengths of iſochronous pendulums in different parallels, the laws of the motions of the ſea, and the moon's parallax, with the apparent diameters of the ſun and moon, ſhould be more exactly determined from phænomena: we ſhould then be enabled to bring this calculation to a greater accuracy.

PROPOSITION XXXVII. PROBLEM XIX.

To find the figure of the moon's body.

If the moon's body were fluid like our ſea, the force of the earth to raiſe that fluid in the neareſt and remoteſt parts would be to the force of the moon by which our ſea is raiſed in the places under and oppoſite to the moon as the accelerative gravity of the moon towards the earth to the accelerative gravity of the earth towards the moon, and the diameter of the moon to the diameter of the earth conjunctly; that is, as 39,788 to 1, and 100 to 365 conjunctly, or as 1081 to 100. Wherefore, ſince our ſea, by the force of the moon, is raiſed to $8\frac{3}{5}$ feet, the lunar fluid would be raiſed by the

force of the earth to 93 feet; and upon this account the figure of the moon would be a ſpheroid, whoſe greateſt diameter produced would paſs through the centre of the earth, and exceed the diameters perpendicular thereto by 186 feet. Such a figure, therefore, the moon affects, and muſt have put on from the beginning. Q.E.I.

Cor. Hence it is that the ſame face of the moon always reſpects the earth; nor can the body of the moon poſſibly reſt in any other poſition, but would return always by a libratory motion to this ſituation; but thoſe librations, however, muſt be exceedingly ſlow, becauſe of the weakneſs of the forces which excite them; ſo that the face of the moon, which ſhould be always obverted to the earth, may, for the reaſon aſſigned in prop. 17, be turned towards the other focus of the moon's orbit, without being immediately drawn back, and converted again towards the earth.

LEMMA I.

If APEp (Pl. 14, Fig. 3) *repreſent the earth uniformly denſe, marked with the centre* C, *the poles* P, p, *and the equator* AE; *and if about the centre* C, *with the radius* CP, *we ſuppoſe the ſphere* Pape *to be deſcribed, and* QR *to denote the plane on which a right line, drawn from the centre of the ſun to the centre of the earth, inſiſts at right angles; and farther ſuppoſe that the ſeveral particles of the whole exterior earth* PapAPepE, *without the height of the ſaid ſphere, endeavour to recede towards this ſide and that ſide from the plane* QR, *every particle by a force proportional to its diſtance from that plane; I ſay, in the firſt place, that the whole force and efficacy of all the particles that are ſituate in* AE, *the circle of the equator, and diſpoſed uniformly without the globe, encompaſſing the ſame after the manner of a ring, to wheel the earth about its centre, is to the whole force and efficacy of as many particles in that point* A *of the equator which is at the greateſt diſtance from the plane* QR, *to wheel the earth about its centre with a like circular motion, as* 1 *to* 2. *And that circular motion will be performed about an axis lying in the common ſection of the equator and the plane* QR.

For let there be described from the centre K, with the diameter IL, the semi-circle INLK. Suppose the semi-circumference INL to be divided into innumerable equal parts, and from the several parts N to the diameter IL let fall the lines NM. Then the sums of the squares of all the lines NM will be equal to the sums of the squares of the lines KM, and both sums together will be equal to the sums of the squares of as many semi-diameters KN; and therefore the sum of the squares of all the lines NM will be but half so great as the sum of the squares of as many semi-diameters KN.

Suppose now the circumference of the circle AE to be divided into the like number of little equal parts, and from every such part F a perpendicular FG to be let fall upon the plane QR, as well as the perpendicular AH from the point A. Then the force by which the particle F recedes from the plane QR will (by supposition) be as that perpendicular FG; and this force multiplied by the distance CG will represent the power of the particle F to turn the earth round its centre. And, therefore, the power of a particle in the place F will be to the power of a particle in the place A as $FG \times GC$ to $AH \times HC$; that is, as FC^2 to AC^2: and therefore the whole power of all the particles F, in their proper places F, will be to the power of the like number of particles in the place A as the sum of all the FC^2 to the sum of all the AC^2, that is (by what we have demonstrated before), as 1 to 2. Q.E.D.

And because the action of those particles is exerted in the direction of lines perpendicularly receding from the plane QR, and that equally from each side of this plane, they will wheel about the circumference of the circle of the equator, together with the adherent body of the earth, round an axis which lies as well in the plane QR as in that of the equator.

LEMMA II.

The same things still supposed, I say, in the second place, that the total force or power of all the particles situated every

where about the ſphere to turn the earth about the ſaid axis is to the whole force of the like number of particles, uniformly diſpoſed round the whole circumference of the equator AE *in the faſhion of a ring, to turn the whole earth about with the like circular motion, as* 2 *to* 5. (Pl. 14, Fig. 4.)

For let IK be any leſſer circle parallel to the equator AE, and let L, l be any two equal particles in this circle, ſituated without the ſphere Pape; and if upon the plane QR, which is at right angles with a radius drawn to the ſun, we let fall the perpendiculars LM, lm, the total forces by which theſe particles recede from the plane QR will be proportional to the perpendiculars LM, lm. Let the right line Ll be drawn parallel to the plane Pape, and biſect the ſame in X; and through the point X draw Nn parallel to the plane QR, and meeting the perpendiculars LM, lm, in N and n; and upon the plane QR let fall the perpendicular XY. And the contrary forces of the particles L and l to wheel about the earth contrariwiſe are as LM × MC, and lm × mC; that is, as LN × MC + NM × MC, and ln × mC — nm × mC; or LN × MC + NM × MC, and LN × mC — NM × mC, and LN × Mm — NM × $\overline{MC + mC}$, the difference of the two, is the force of both taken together to turn the earth round. The affirmative part of this difference LN × Mm, or 2LN × NX, is to 2AH × HC, the force of two particles of the ſame ſize ſituated in A, as LX^2 to AC^2; and the negative part NM × $\overline{MC + mC}$, or 2XY × CY, is to 2AH × HC, the force of the ſame two particles ſituated in A, as CX^2 to AC^2. And therefore the difference of the parts, that is, the force of the two particles L and l, taken together, to wheel the earth about, is to the force of two particles, equal to the former and ſituated in the place A, to turn in like manner the earth round, as $LX^2 - CX^2$ to AC^2. But if the circumference IK of the circle IK is ſuppoſed to be divided into an infinite number of little equal parts L, all the LX^2 will be to the like number of IX^2 as 1 to 2 (by lem. 1); and to the ſame number of AC^2 as IX^2 to $2AC^2$; and the ſame number of CX^2 to as many AC^2 as $2CX^2$ to $2AC^2$. Wherefore the united forces of all the particles in the circumference of the

circle IK are to the joint forces of as many particles in the place A as $IX^2 - 2CX^2$ to $2AC^2$; and therefore (by lem. 1) to the united forces of as many particles in the circumference of the circle AE as $IX^2 - 2CX^2$ to AC^2.

Now if Pp, the diameter of the ſphere, is conceived to be divided into an infinite number of equal parts, upon which a like number of circles IK are ſuppoſed to infiſt, the matter in the circumference of every circle IK will be as IX^2; and therefore the force of that matter to turn the earth about will be as IX^2 into $IX^2 - 2CX^2$; and the force of the ſame matter, if it was ſituated in the circumference of the circle AE, would be as IX^2 into AC^2. And therefore the force of all the particles of the whole matter ſituated without the ſphere in the circumferences of all the circles is to the force of the like number of particles ſituated in the circumference of the greateſt circle AE as all the IX^2 into $IX^2 - 2CX^2$ to as many IX^2 into AC^2; that is, as all the $AC^2 - CX^2$ into $AC^2 - 3CX^2$ to as many $AC^2 - CX^2$ into AC^2; that is, as all the $AC^4 - 4AC^2 \times CX^2 + 3CX^4$ to as many $AC^4 - AC^2 \times CX^2$; that is, as the whole fluent quantity, whoſe fluxion is $AC^4 - 4AC^2 \times CX^2 + 3CX^4$, to the whole fluent quantity, whoſe fluxion is $AC^4 - AC^2 \times CX^2$; and, therefore, by the method of fluxions, as $AC^4 \times CX - \frac{4}{3}AC^2 \times CX^3 + \frac{1}{5}CX^5$ to $AC^4 \times CX - \frac{1}{3}AC^2 \times CX^3$; that is, if for CX we write the whole Cp, or AC, as $\frac{4}{15}AC^5$ to $\frac{2}{3}AC^5$; that is, as 2 to 5. Q.E.D.

LEMMA III.

The ſame things ſtill ſuppoſed, I ſay, in the third place, that the motion of the whole earth about the axis above-named ariſing from the motions of all the particles, will be to the motion of the aforeſaid ring about the ſame axis in a proportion compounded of the proportion of the matter in the earth to the matter in the ring; and the proportion of three ſquares of the quadrantal arc of any circle to two ſquares of its diameter, that is, in the proportion of the matter to the matter, and of the number 925275 *to the number* 1000000.

For the motion of a cylinder revolved about its quieſcent axis is to the motion of the inſcribed ſphere revolved together with it as any four equal ſquares to three circles inſcribed in

three of thoſe ſquares; and the motion of this cylinder is to the motion of an exceedingly thin ring ſurrounding both ſphere and cylinder in their common contact as double the matter in the cylinder to triple the matter in the ring; and this motion of the ring, uniformly continued about the axis of the cylinder, is to the uniform motion of the ſame about its own diameter performed in the ſame periodic time as the circumference of a circle to double its diameter.

HYPOTHESIS II.

If the other parts of the earth were taken away, and the remaining ring was carried alone about the ſun in the orbit of the earth by the annual motion, while by the diurnal motion it was in the mean time revolved about its own axis inclined to the plane of the ecliptic by an angle of $23\frac{1}{2}$ *degrees, the motion of the equinoctial points would be the ſame, whether the ring were fluid, or whether it conſiſted of a hard and rigid matter.*

PROPOSITION XXXIX. PROBLEM XX.

To find the preceſſion of the equinoxes.

The middle horary motion of the moon's nodes in a circular orbit, when the nodes are in the quadratures, was $16'' \; 35''' \; 16^{iv}. \; 36^{v}.$; the half of which, $8'' \; 17''' \; 38^{iv}. \; 18^{v}.$ (for the reaſons above explained) is the mean horary motion of the nodes in ſuch an orbit, which motion in a whole ſidereal year becomes $20° \; 11' \; 46''$. Becauſe, therefore, the nodes of the moon in ſuch an orbit would be yearly transfered $20° \; 11' \; 46''$ *in antecedentia*; and, if there were more moons, the motion of the nodes of every one (by cor. 16, prop. 66, book 1) would be as its periodic time; if upon the ſurface of the earth a moon was revolved in the time of a ſidereal day, the annual motion of the nodes of this moon would be to $20° \; 11' \; 46''$ as $23^{h}. \; 56'$, the ſidereal day, to $27^{d}. \; 7^{h}. \; 43'$, the periodic time of our moon, that is, as 1436 to 39343. And the ſame thing would happen to the nodes of a ring of moons encompaſſing the earth, whether theſe moons did not mutually touch each the other, or whether they were molten, and formed into a continued ring, or whether that ring ſhould become rigid and inflexible.

Let us, then, ſuppoſe that this ring is in quantity of matter equal to the whole exterior earth PapAPepE, which lies without the ſphere Pape (ſee fig. lem. 2); and becauſe this ſphere is to that exterior earth as aC^2 to $AC^2 - aC^2$, that is (ſeeing PC or aC the leaſt ſemi-diameter of the earth is to AC the greateſt ſemi-diameter of the ſame as 229 to 230), as 52441 to 459; if this ring encompaſſed the earth round the equator, and both together were revolved about the diameter of the ring, the motion of the ring (by lem. 3) would be to the motion of the inner ſphere as 459 to 52441 and 1000000 to 925275 conjunctly, that is, as 4590 to 485223; and therefore the motion of the ring would be to the ſum of the motions of both ring and ſphere as 4590 to 489813. Wherefore if the ring adheres to the ſphere, and communicates its motion to the ſphere, by which its nodes or equinoctial points recede, the motion remaining in the ring will be to its former motion as 4590 to 489813; upon which account the motion of the equinoctial points will be diminiſhed in the ſame proportion. Wherefore the annual motion of the equinoctial points of the body, compoſed of both ring and ſphere, will be to the motion 20° 11′ 46″ as 1436 to 39343 and 4590 to 489813 conjunctly, that is, as 100 to 292369. But the forces by which the nodes of a number of moons (as we explained above), and therefore by which the equinoctial points of the ring recede (that is, the forces 3IT, in fig. prop. 30), are in the ſeveral particles as the diſtances of thoſe particles from the plane QR; and by theſe forces the particles recede from that plane: and therefore (by lem. 2) if the matter of the ring was ſpread all over the ſurface of the ſphere, after the faſhion of the figure PapAPepE, in order to make up that exterior part of the earth, the total force or power of all the particles to wheel about the earth round any diameter of the equator, and therefore to move the equinoctial points, would become leſs than before in the proportion of 2 to 5. Wherefore the annual regreſs of the equinoxes now would be to 20° 11′ 46″ as 10 to 73092; that is, would be 9″ 56‴ 50^{iv}.

But becauſe the plane of the equator is inclined to that of the ecliptic, this motion is to be diminiſhed in the proportion

of the fine 91706 (which is the co-fine of $23\frac{1}{2}$ deg.) to the radius 100000; and the remaining motion will now be $9'' 7''' 20^{iv}$. which is the annual preceſſion of the equinoxes ariſing from the force of the ſun.

But the force of the moon to move the ſea was to the force of the ſun nearly as 4,4815 to 1; and the force of the moon to move the equinoxes is to that of the ſun in the ſame proportion. Whence the annual preceſſion of the equinoxes proceeding from the force of the moon comes out $40'' 52''' 52^{iv}$. and the total annual preceſſion ariſing from the united forces of both will be $50'' 00''' 12^{iv}$. the quantity of which motion agrees with the phænomena; for the preceſſion of the equinoxes, by aſtronomical obſervations, is about $50''$ yearly.

If the height of the earth at the equator exceeds its height at the poles by more than $17\frac{1}{6}$ miles, the matter thereof will be more rare near the ſurface than at the centre; and the preceſſion of the equinoxes will be augmented by the exceſs of height, and diminiſhed by the greater rarity.

And now we have deſcribed the ſyſtem of the ſun, the earth, moon, and planets, it remains that we add ſomething about the comets.

LEMMA IV.

That the comets are higher than the moon, and in the regions of the planets.

As the comets were placed by aſtronomers above the moon, becauſe they were found to have no diurnal parallax, ſo their annual parallax is a convincing proof of their deſcending into the regions of the planets; for all the comets which move in a direct courſe according to the order of the ſigns, about the end of their appearance become more than ordinarily ſlow or retrograde, if the earth is between them and the ſun; and more than ordinarily ſwift, if the earth is approaching to a heliocentric oppoſition with them; whereas, on the other hand, thoſe which move againſt the order of the ſigns, towards the end of their appearance appear ſwifter than they ought to be, if the earth is between them and the ſun; and ſlower, and perhaps retrograde, if the earth is in the other ſide of its orbit. And theſe appearances proceed chiefly from

Fig 1.

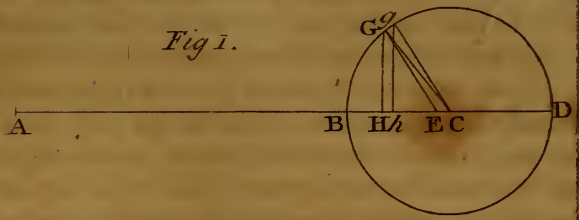


Fig 2.

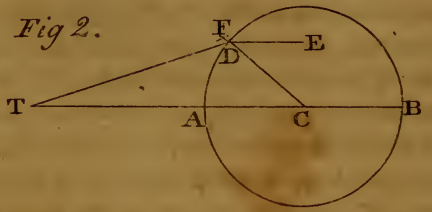
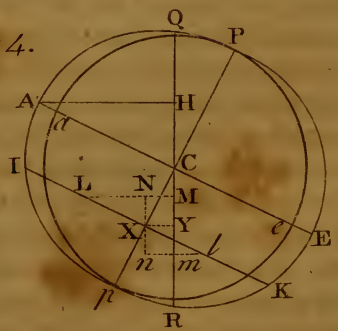


Fig 3.

Fig 4.



the diverſe ſituations which the earth acquires in the courſe of its motion, after the ſame manner as it happens to the planets, which appear ſometimes retrograde, ſometimes more ſlowly, and ſometimes more ſwiftly, progreſſive, according as the motion of the earth falls in with that of the planet, or is directed the contrary way. If the earth move the ſame way with the comet, but, by an angular motion about the ſun, ſo much ſwifter that right lines drawn from the earth to the comet converge towards the parts beyond the comet, the comet ſeen from the earth, becauſe of its ſlower motion, will appear retrograde; and even if the earth is ſlower than the comet, the motion of the earth being ſubducted, the motion of the comet will at leaſt appear retarded; but if the earth tends the contrary way to that of the comet, the motion of the comet will from thence appear accelerated; and from this apparent acceleration, or retardation, or regreſſive motion, the diſtance of the comet may be inferred in this manner. Let TQA, TQB, TQC (Pl. 15, Fig. 1), be three obſerved longitudes of the comet about the time of its firſt appearing, and TQF its laſt obſerved longitude before its diſappearing. Draw the right line ABC, whoſe parts AB, BC, intercepted between the right lines QA and QB, QB and QC, may be one to the other as the two times between the three firſt obſervations. Produce AC to G, ſo as AG may be to AB as the time between the firſt and laſt obſervation to the time between the firſt and ſecond; and join QG. Now if the comet did move uniformly in a right line, and the earth either ſtood ſtill, or was likewiſe carried forwards in a right line by an uniform motion, the angle TQG would be the longitude of the comet at the time of the laſt obſervation. The angle, therefore, FQG, which is the difference of the longitude, proceeds from the inequality of the motions of the comet and the earth; and this angle, if the earth and comet move contrary ways, is added to the angle TQG, and accelerates the apparent motion of the comet; but if the comet move the ſame way with the earth, it is ſubtracted, and either retards the motion of the comet, or perhaps renders it retrograde, as we have but now explained. This angle, therefore, proceeding

chiefly from the motion of the earth, is juſtly to be eſteemed the parallax of the comet; neglecting, to wit, ſome little increment or decrement that may ariſe from the unequal motion of the comet in its orbit; and from this parallax we thus deduce the diſtance of the comet. Let S (Pl. 15, Fig. 2) repreſent the ſun, acT the *orbis magnus*, a the earth's place in the firſt obſervation, c the place of the earth in the third obſervation, T the place of the earth in the laſt obſervation, and TΥ a right line drawn to the beginning of Aries. Set off the angle ΥTV equal to the angle ΥQF, that is, equal to the longitude of the comet at the time when the earth is in T; join ac, and produce it to g, ſo as ag may be to ac as AG to AC; and g will be the place at which the earth would have arrived in the time of the laſt obſervation, if it had continued to move uniformly in the right line ac. Wherefore, if we draw gΥ parallel to TΥ, and make the angle ΥgV equal to the angle ΥQG, this angle ΥgV will be equal to the longitude of the comet ſeen from the place g, and the angle TVg will be the parallax which ariſes from the earth's being transfered from the place g into the place T; and therefore V will be the place of the comet in the plane of the ecliptic. And this place V is commonly lower than the orb of Jupiter.

The ſame thing may be deduced from the incurvation of the way of the comets; for theſe bodies move almoſt in great circles, while their velocity is great; but about the end of their courſe, when that part of their apparent motion which ariſes from the parallax bears a greater proportion to their whole apparent motion, they commonly deviate from thoſe circles, and when the earth goes to one ſide, they deviate to the other; and this deflexion, becauſe of its correſponding with the motion of the earth, muſt ariſe chiefly from the parallax; and the quantity thereof is ſo conſiderable, as, by my computation, to place the diſappearing comets a good deal lower than Jupiter. Whence it follows that when they approach nearer to us in their perigees and perihelions they often deſcend below the orbs of Mars and the inferior planets.

The near approach of the comets is farther confirmed from the light of their heads; for the light of a celeſtial body, illu-

minated by the ſun, and receding to remote parts, is diminiſhed in the quadruplicate proportion of the diſtance; to wit, in one duplicate proportion, on account of the increaſe of the diſtance from the ſun, and in another duplicate proportion, on account of the decreaſe of the apparent diameter. Wherefore if both the quantity of light and the apparent diameter of a comet are given, its diſtance will be alſo given, by taking the diſtance of the comet to the diſtance of a planet in the direct proportion of their diameters and the reciprocal ſubduplicate proportion of their lights. Thus, in the comet of the year 1682, Mr. *Flamſted* obſerved with a teleſcope of 16 feet, and meaſured with a micrometer, the leaſt diameter of its head, 2′ 00; but the nucleus or ſtar in the middle of the head ſcarcely amounted to the tenth part of this meaſure; and therefore its diameter was only 11″ or 12″; but in the light and ſplendor of its head it ſurpaſſed that of the comet in the year 1680, and might be compared with the ſtars of the firſt or ſecond magnitude. Let us ſuppoſe that Saturn with its ring was about four times more lucid; and becauſe the light of the ring was almoſt equal to the light of the globe within, and the apparent diameter of the globe is about 21″, and therefore the united light of both globe and ring would be equal to the light of a globe whoſe diameter is 30″, it follows that the diſtance of the comet was to the diſtance of Saturn as 1 to $\sqrt{4}$ inverſely, and 12″ to 30 directly; that is, as 24 to 30, or 4 to 5. Again; the comet in the month of *April* 1665, as *Hevelius* informs us, excelled almoſt all the fixed ſtars in ſplendor, and even Saturn itſelf, as being of a much more vivid colour; for this comet was more lucid than that other which had appeared about the end of the preceding year, and had been compared to the ſtars of the firſt magnitude. The diameter of its head was about 6′; but the nucleus, compared with the planets by means of a teleſcope, was plainly leſs than Jupiter; and ſometimes judged leſs, ſometimes judged equal, to the globe of Saturn within the ring. Since, then, the diameters of the heads of the comets ſeldom exceed 8′ or 12′, and the diameter of the nucleus or central ſtar is but about a tenth or perhaps fifteenth part of the diameter of

the head, it appears that theſe ſtars are generally of about the ſame apparent magnitude with the planets. But in regard that their light may be often compared with the light of Saturn, yea, and ſometimes exceeds it, it is evident that all comets in their perihelions muſt either be placed below or not far above Saturn; and they are much miſtaken who remove them almoſt as far as the fixed ſtars; for if it was ſo, the comets could receive no more light from our ſun than our planets do from the fixed ſtars.

So far we have gone, without conſidering the obſcuration which comets ſuffer from that plenty of thick ſmoke which encompaſſeth their heads, and through which the heads always ſhew dull, as through a cloud; for by how much the more a body is obſcured by this ſmoke, by ſo much the more near it muſt be allowed to come to the ſun, that it may vie with the planets in the quantity of light which it reflects. Whence it is probable that the comets deſcend far below the orb of Saturn, as we proved before from their parallax. But, above all, the thing is evinced from their tails, which muſt be owing either to the ſun's light reflected by a ſmoke ariſing from them, and diſperſing itſelf through the æther, or to the light of their own heads. In the former caſe, we muſt ſhorten the diſtance of the comets, leſt we be obliged to allow that the ſmoke ariſing from their heads is propagated through ſuch a vaſt extent of ſpace, and with ſuch a velocity and expanſion, as will ſeem altogether incredible; in the latter caſe, the whole light of both head and tail is to be aſcribed to the central nucleus. But, then, if we ſuppoſe all this light to be united and condenſed within the diſk of the nucleus, certainly the nucleus will by far exceed Jupiter itſelf in ſplendor, eſpecially when it emits a very large and lucid tail. If, therefore, under a leſs apparent diameter, it reflects more light, it muſt be much more illuminated by the ſun, and therefore much nearer to it; and the ſame argument will bring down the heads of comets ſometimes within the orb of Venus viz. when, being hid under the ſun's rays, they emit ſuch huge and ſpendid tails, like beams of fire, as ſometimes they do; for if all that light was ſuppoſed to be gathered together into one

ſtar, it would ſometimes exceed not one Venus only, but a great many ſuch united into one.

Laſtly; the ſame thing is inferred from the light of the heads, which increaſes in the receſs of the comets from the earth towards the ſun, and decreaſes in their return from the ſun towards the earth; for ſo the comet of the year 1665 (by the obſervations of *Hevelius*), from the time that it was firſt ſeen, was always loſing of its apparent motion, and therefore had already paſſed its perigee; but yet the ſplendor of its head was daily increaſing, till, being hid under the ſun's rays, the comet ceaſed to appear. The comet of the year 1683 (by the obſervations of the ſame *Hevelius*), about the end of *July*, when it firſt appeared, moved at a very ſlow rate, advancing only about 40 or 45 minutes in its orb in a day's time; but from that time its diurnal motion was continually upon the increaſe, till *September* 4, when it aroſe to about 5 degrees; and therefore, in all this interval of time, the comet was approaching to the earth. Which is likewiſe proved from the diameter of its head, meaſured with a micrometer; for, *Auguſt* 6, *Hevelius* found it only 6′ 05″, including the *coma*, which, *September* 2, he obſerved to be 9′ 07″, and therefore its head appeared far leſs about the beginning than towards the end of the motion; though about the beginning, becauſe nearer to the ſun, it appeared far more lucid than towards the end, as the ſame *Hevelius* declares. Wherefore in all this interval of time, on account of its receſs from the ſun, it decreaſed in ſplendor, notwithſtanding its acceſs towards the earth. The comet of the year 1618, about the middle of *December*, and that of the year 1680, about the end of the ſame month, did both move with their greateſt velocity, and were therefore then in their perigees; but the greateſt ſplendor of their heads was ſeen two weeks before, when they had juſt got clear of the ſun's rays; and the greateſt ſplendor of their tails a little more early, when yet nearer to the ſun. The head of the former comet (according to the obſervations of *Cyſatus*), *December* 1, appeared greater than the ſtars of the firſt magnitude; and, *December* 16 (then in the perigee), it was but little diminiſhed in magnitude, but in the ſplendor and

brightneſs of its light a great deal. *January* 7, *Kepler*, being uncertain about the head, left off obſerving. *December* 12, the head of the latter comet was ſeen and obſerved by Mr. *Flamſted*, when but 9 degrees diſtant from the ſun; which is ſcarcely to be done in a ſtar of the third magnitude. *December* 15 and 17, it appeared as a ſtar of the third magnitude, its luſtre being diminiſhed by the brightneſs of the clouds near the ſetting ſun. *December* 26, when it moved with the greateſt velocity, being almoſt in its perigee, it was leſs than the mouth of *Pegaſus*, a ſtar of the third magnitude. *January* 3, it appeared as a ſtar of the fourth. *January* 9, as one of the fifth. *January* 13, it was hid by the ſplendor of the moon, then in her increaſe. *January* 25, it was ſcarcely equal to the ſtars of the ſeventh magnitude. If we compare equal intervals of time on one ſide and on the other from the perigee, we ſhall find that the head of the comet, which at both intervals of time was far, but yet equally, removed from the earth, and ſhould have therefore ſhone with equal ſplendor, appeared brighteſt on the ſide of the perigee towards the ſun, and diſappeared on the other. Therefore, from the great difference of light in the one ſituation and in the other, we conclude the great vicinity of the ſun and comet in the former; for the light of comets uſes to be regular, and to appear greateſt when the heads move faſteſt, and are therefore in their perigees; excepting in ſo far as it is increaſed by their nearneſs to the ſun.

Cor. 1. Therefore the comets ſhine by the ſun's light, which they reflect.

Cor. 2. From what has been ſaid, we may likewiſe underſtand why comets are ſo frequently ſeen in that hemiſphere in which the ſun is, and ſo ſeldom in the other. If they were viſible in the regions far above Saturn, they would appear more frequently in the parts oppoſite to the ſun; for ſuch as were in thoſe parts would be nearer to the earth, whereas the preſence of the ſun muſt obſcure and hide thoſe that appear in the hemiſphere in which he is. Yet, looking over the hiſtory of comets, I find that four or five times more have been ſeen in the hemiſphere towards the ſun than in the oppoſite

hemiſphere; beſides, without doubt, not a few, which have been hid by the light of the ſun: for comets deſcending into our parts neither emit tails, nor are ſo well illuminated by the ſun, as to diſcover themſelves to our naked eyes, until they are come nearer to us than Jupiter. But the far greater part of that ſpherical ſpace, which is deſcribed about the ſun with ſo ſmall an interval, lies on that ſide of the earth which regards the ſun; and the comets in that greater part are commonly more ſtrongly illuminated, as being for the moſt part nearer to the ſun.

Cor. 3. Hence alſo it is evident that the celeſtial ſpaces are void of reſiſtance; for though the comets are carried in oblique paths, and ſometimes contrary to the courſe of the planets, yet they move every way with the greateſt freedom, and preſerve their motions for an exceeding long time, even where contrary to the courſe of the planets. I am out in my judgment if they are not a ſort of planets revolving in orbits returning into themſelves with a perpetual motion; for, as to what ſome writers contend, that they are no other than meteors, led into this opinion by the perpetual changes that happen to their heads, it ſeems to have no foundation; for the heads of comets are encompaſſed with huge atmoſpheres, and the lowermoſt parts of theſe atmoſpheres muſt be the denſeſt; and therefore it is in the clouds only, not in the bodies of the comets themſelves, that theſe changes are ſeen. Thus the earth, if it was viewed from the planets, would, without all doubt, ſhine by the light of its clouds, and the ſolid body would ſcarcely appear through the ſurrounding clouds. Thus alſo the belts of Jupiter are formed in the clouds of that planet, for they change their poſition one to another, and the ſolid body of Jupiter is hardly to be ſeen through them; and much more muſt the bodies of comets be hid under their atmoſpheres, which are both deeper and thicker.

PROPOSITION XL. THEOREM XX.

That the comets move in ſome of the conic ſections, having their foci in the centre of the ſun; and by radii drawn to the ſun deſcribe areas proportional to the ti mes.

This propoſition appears from cor. 1, prop. 13, book 1, compared with prop. 8, 12, and 13, book 3.

Cor. 1. Hence if comets are revolved in orbits returning into themſelves, thoſe orbits will be ellipſes; and their periodic times be to the periodic times of the planets in the ſeſquiplicate proportion of their principal axes. And therefore the comets, which for the moſt part of their courſe are higher than the planets, and upon that account deſcribe orbits with greater axes, will require a longer time to finiſh their revolutions. Thus if the axis of a comet's orbit was four times greater than the axis of the orbit of Saturn, the time of the revolution of the comet would be to the time of the revolution of Saturn, that is, to 30 years, as 4 $\surd$ 4 (or 8) to 1, and would therefore be 240 years

Cor. 2. But their orbits will be ſo near to parabolas, that parabolas may be uſed for them without ſenſible error.

Cor. 3. And, therefore, by cor. 7, prop. 16, book 1, the velocity of every comet will always be to the velocity of any planet, ſuppoſed to be revolved at the ſame diſtance in a circle about the ſun, nearly in the ſubduplicate proportion of double the diſtance of the planet from the centre of the ſun to the diſtance of the comet from the ſun's centre, very nearly. Let us ſuppoſe the radius of the *orbis magnus*, or the greateſt ſemi-diameter of the ellipſis which the earth deſcribes, to conſiſt of 100000000 parts; and then the earth by its mean diurnal motion will deſcribe 1720212 of thoſe parts, and 71675$\frac{1}{2}$ by its horary motion. And therefore the comet, at the ſame mean diſtance of the earth from the ſun, with a velocity which is to the velocity of the earth as $\surd$ 2 to 1, would by its diurnal motion deſcribe 2432747 parts, and 101364$\frac{1}{2}$ parts by its horary motion. But at greater or leſs diſtances both the diurnal and horary motion will be to this diurnal and horary motion in the reciprocal ſubduplicate proportion of the diſtances, and is therefore given.

Cor. 4. Wherefore if the *latus rectum* of the parabola is quadruple of the radius of the *orbis magnus*, and the ſquare of that radius is ſuppoſed to conſiſt of 100000000 parts, the

area which the comet will daily describe by a radius drawn to the sun will be 1216373½ parts, and the horary area will be 50682¼ parts. But, if the *latus rectum* is greater or less in any proportion, the diurnal and horary area will be less or greater in the subduplicate of the same proportion reciprocally.

LEMMA V.

To find a curve line of the parabolic kind which shall pass through any given number of points. (Pl. 15, Fig. 3.)

Let those points be A, B, C, D, E, F, &c. and from the same to any right line HN, given in position, let fall as many perpendiculars AH, BI, CK, DL, EM, FN, &c.

Case 1. If HI, IK, KL, &c. the intervals of the points H, I, K, L, M, N, &c. are equal, take b, 2b, 3b, 4b, 5b, &c. the first differences of the perpendiculars AH, BI, CK, &c.; their second differences c, 2c, 3c, 4c. &c.; their third, d, 2d, 3d, &c. that is to say, so as AH — BI may be = b, BI — CK = 2b, CK — DL = 3b, DL + EM = 4b, — EM + FN = 5b, &c.; then b — 2b = c, &c. and so on to the last difference, which is here f. Then, erecting any perpendicular RS, which may be considered as an ordinate of the curve required, in order to find the length of this ordinate, suppose the intervals HI, IK, KL, LM, &c. to be units, and let AH = a, — HS = p, ½p into — IS = q, ⅓q into + SK = r, ¼r into + SL = s, ⅕s into + SM = t; proceeding, to wit, to ME, the last perpendicular but one, and prefixing negative signs before the terms HS, IS, &c. which lie from S towards A; and affirmative signs before the terms SK, SL, &c. which lie on the other side of the point S; and, observing well the signs, RS will be = a + bp + cq + dr + es + ft, + &c.

Case 2. But if HI, IK, &c. the intervals of the points H, I, K, L, &c. are unequal, take b, 2b, 3b, 4b, 5b, &c. the first differences of the perpendiculars AH, BI, CK, &c. divided by the intervals between those perpendiculars; c, 2c, 3c, 4c, &c. their second differences, divided by the intervals between every two; d, 2d, 3d, &c. their third differences, divided by the intervals between every three; e, 2e, &c. their fourth differences, divided by the intervals between every

four; and so forth; that is, in such manner, that b may be $= \frac{AH - BI}{HI}$, $2b = \frac{BI - CK}{IK}$, $3b = \frac{CK - DL}{KL}$, &c. then $c = \frac{b - 2b}{HK}$, $2c = \frac{2b - 3b}{IL}$, $3c = \frac{3b - 4b}{KM}$, &c. then $d = \frac{c - 2c}{HL}$, $2d = \frac{2c - 3c}{IM}$, &c. And those differences being found, let AH be $= a$, $- HS = p$, p into $- IS = q$, q into $+ SK = r$, r into $+ SL = s$, s into $+ SM = t$; proceeding, to wit, to ME, the last perpendicular but one; and the ordinate RS will be $= a + bp + cq + dr + es + ft$, + &c.

Cor. Hence the areas of all curves may be nearly found; for if some number of points of the curve to be squared are found, and a parabola be supposed to be drawn through those points, the area of this parabola will be nearly the same with the area of the curvilinear figure proposed to be squared: but the parabola can be always squared geometrically by methods vulgarly known.

LEMMA VI.

Certain observed places of a comet being given, to find the place of the same to any intermediate given time.

Let HI, IK, KL, LM (in the preceding Fig.), represent the times between the observations; HA, IB, KC, LD, ME, five observed longitudes of the comet; and HS the given time between the first observation and the longitude required. Then if a regular curve ABCDE is supposed to be drawn through the points A, B, C, D, E, and the ordinate RS is found out by the preceding lemma, RS will be the longitude required.

After the same method, from five observed latitudes, we may find the latitude to a given time.

If the differences of the observed longitudes are small, suppose of 4 or 5 degrees, three or four observations will be sufficient to find a new longitude and latitude; but if the differences are greater, as of 10 or 20 degrees, five observations ought to be used.

LEMMA VII.

Through a given point P (Pl. 15, Fig. 4) *to draw a right line* BC, *whose parts* PB, PC, *cut off by two right lines* AB, AC,

Fig. 1.

Fig 2

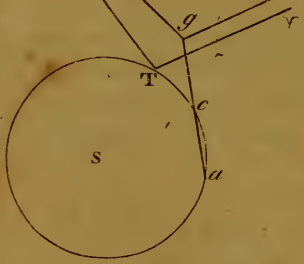


Fig. 3.

b 2b 3b 4b 5b

c 2c 3c 4c

d 2d 3d

e 2e

f

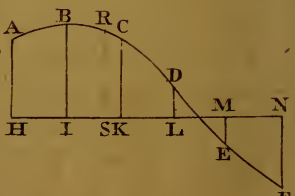
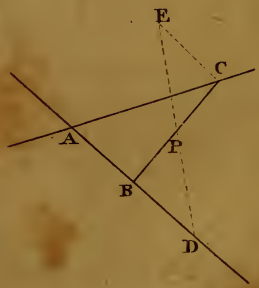


Fig. 4.



given in position, may be one to the other in a given proportion.

From the given point P suppose any right line PD to be drawn to either of the right lines given, as AB; and produce the same towards AC, the other given right line, as far as E, so as PE may be to PD in the given proportion. Let EC be parallel to AD. Draw CPB, and PC will be to PB as PE to PD. Q.E.F.

LEMMA VIII.

Let ABC (Pl. 16, Fig. 1) *be a parabola, having its focus in* S. *By the chord* AC *bisected in* I *cut off the segment* ABCI, *whose diameter is* Iμ *and vertex* μ. *In* Iμ *produced take* μO *equal to one half of* Iμ. *Join* OS, *and produce it to* ξ, *so as* Sξ *may be equal to* 2SO. *Now, supposing a comet to revolve in the arc* CBA, *draw* ξB, *cutting* AC *in* E; *I say, the point* E *will cut off from the chord* AC *the segment* AE, *nearly proportional to the time.*

For if we join EO, cutting the parabolic arc ABC in Y, and draw μX touching the same arc in the vertex μ, and meeting EO in X, the curvilinear area AEXμA will be to the curvilinear area ACYμA as AE to AC; and, therefore, since the triangle ASE is to the triangle ASC in the same proportion, the whole area ASEXμA will be to the whole area ASCYμA as AE to AC. But, because ξO is to SO as 3 to 1, and EO to XO in the same proportion, SX will be parallel to EB; and, therefore, joining BX, the triangle SEB will be equal to the triangle XEB. Wherefore if to the area ASEXμA we add the triangle EXB, and from the sum subduct the triangle SEB, there will remain the area ASBXμA, equal to the area ASEXμA, and therefore in proportion to the area ASCYμA as AE to AC. But the area ASBYμA is nearly equal to the area ASBXμA; and this area ASBYμA is to the area ASCYμA as the time of description of the arc AB to the time of description of the whole arc AC; and, therefore, AE is to AC nearly in the proportion of the times. Q.E.D.

Cor. When the point B falls upon the vertex μ of the parabola, AE is to AC accurately in the proportion of the times.

SCHOLIUM.

If we join $\mu\xi$ cutting AC in δ, and in it take ξn in proportion to μB as $27MI$ to $16M\mu$, and draw Bn, this Bn will cut the chord AC, in the proportion of the times, more accurately than before; but the point n is to be taken beyond or on this ſide the point ξ, according as the point B is more or leſs diſtant from the principal vertex of the parabola than the point μ.

LEMMA IX.

The right lines $I\mu$ *and* μM, *and the length* $\frac{AIC}{4S\mu}$, *are equal among themſelves.*

For $4S\mu$ is the latus rectum of the parabola belonging to the vertex μ.

LEMMA X.

Produce $S\mu$ *to* N *and* P (Pl. 16, Fig. 2), *ſo as* μN *may be one third of* μI, *and* SP *may be to* SN *as* SN *to* $S\mu$; *and in the time that a comet would deſcribe the arc* $A\mu C$, *if it was ſuppoſed to move always forwards with the velocity which it hath in a height equal to* SP, *it would deſcribe a length equal to the chord* AC.

For if the comet with the velocity which it hath in μ was in the ſaid time ſuppoſed to move uniformly forwards in the right line which touches the parabola in μ, the area which it would deſcribe by a radius drawn to the point S would be equal to the parabolic area $ACS\mu A$; and therefore the ſpace contained under the length deſcribed in the tangent and the length $S\mu$ would be to the ſpace contained under the lengths AC and SM as the area $ASC\mu A$ to the triangle ASC, that is, as SN to SM. Wherefore AC is to the length deſcribed in the tangent as $S\mu$ to SN. But ſince the velocity of the comet in the height SP (by cor. 6, prop. 16, book 1) is to the velocity of the ſame in the height $S\mu$ in the reciprocal ſubduplicate proportion of SP to $S\mu$, that is, in the proportion of $S\mu$ to SN, the length deſcribed with this velocity will be to the length in the ſame time deſcribed in the tangent as $S\mu$ to SN. Wherefore ſince AC, and the length deſcribed with this new velocity, are in the ſame proportion to the length

described in the tangent, they must be equal betwixt themselves. Q.E.D.

Cor. Therefore a comet, with that velocity which it hath in the height $S\mu + \frac{2}{3}I\mu$, would in the same time describe the chord AC nearly.

LEMMA XI.

If a comet void of all motion was let fall from the height SN, *or* $S\mu + \frac{1}{3}I\mu$, *towards the sun, and was still impelled to the sun by the same force uniformly continued by which it was impelled at first, the same, in one half of that time in which it might describe the arc* AC *in its own orbit, would in descending describe a space equal to the length* $I\mu$.

For in the same time that the comet would require to describe the parabolic arc AC, it would (by the last lemma), with that velocity which it hath in the height SP, describe the chord AC; and, therefore (by cor. 7, prop. 16, book 1), if it was in the same time supposed to revolve by the force of its own gravity in a circle whose semi-diameter was SP, it would describe an arc of that circle, the length of which would be to the chord of the parabolic arc AC in the subduplicate proportion of 1 to 2. Wherefore if with that weight, which in the height SP it hath towards the sun, it should fall from that height towards the sun, it would (by cor. 9, prop. 4, book 1) in half the said time describe a space equal to the square of half the said chord applied to quadruple the height SP, that is, it would describe the space $\frac{AI^2}{4SP}$. But since the weight of the comet towards the sun in the height SN is to the weight of the same towards the sun in the height SP as SP to $S\mu$, the comet, by the weight which it hath in the height SN, in falling from that height towards the sun, would in the same time describe the space $\frac{AI^2}{4S\mu}$; that is, a space equal to the length $I\mu$ or μM. Q.E.D.

PROPOSITION XLI. PROBLEM XXI.

From three observations given to determine the orbit of a comet moving in a parabola.

This being a problem of very great difficulty, I tried many methods of reſolving it; and ſeveral of thoſe problems, the compoſition whereof I have given in the firſt book, tended to this purpoſe. But afterwards I contrived the following ſolution, which is ſomething more ſimple.

Select three obſervations diſtant one from another by intervals of time nearly equal; but let that interval of time in which the comet moves more ſlowly be ſomewhat greater than the other; ſo, to wit, that the difference of the times may be to the ſum of the times as the ſum of the times to about 600 days; or that the point E (Pl. 16, Fig. 1) may fall upon M nearly, and may err therefrom rather towards I than towards A. If ſuch direct obſervations are not at hand, a new place of the comet muſt be found, by lem. 6.

Let S (Pl. 16, Fig. 3) repreſent the ſun; T, t, τ, three places of the earth in the *orbis magnus;* TA, tB, τC, three obſerved longitudes of the comet; V the time between the firſt obſervation and the ſecond; W the time between the ſecond and the third; X the length which in the whole time V + W the comet might deſcribe with that velocity which it hath in the mean diſtance of the earth from the ſun, which length is to be found by cor. 3, prop. 40, book 3; and tV a perpendicular upon the chord Tτ. In the mean obſerved longitude tB take at pleaſure the point B, for the place of the comet in the plane of the ecliptic; and from thence, towards the ſun S, draw the line BE, which may be to the perpendicular tV as the content under SB and St^2 to the cube of the hypothenuſe of the right angled triangle, whoſe ſides are SB, and the tangent of the latitude of the comet in the ſecond obſervation to the radius tB. And through the point E (by lemma 7) draw the right line AEC, whoſe parts AE and EC, terminating in the right lines TA and τC, may be one to the other as the times V and W: then A and C will be nearly the places of the comet in the plane of the ecliptic in the firſt and third obſervations, if B was its place rightly aſſumed in the ſecond.

Upon AC, biſected in I, erect the perpendicular Ii. Through B draw the obſcure line Bi parallel to AC. Join the obſcure

line Si, cutting AC in λ, and complete the parallelogram iI λμ. Take Iσ equal to 3Iλ; and through the ſun S draw the obſcure line σξ equal to 3Sσ + 3 iλ. Then, cancelling the letters A, E, C, I, from the point B towards the point ξ, draw the new obſcure line BE, which may be to the former BE in the duplicate proportion of the diſtance BS to the quantity Sμ + ⅓ iλ. And through the point E draw again the right line AEC by the ſame rule as before, that is, ſo as its parts AE and EC may be one to the other as the times V and W between the obſervations. Thus A and C will be the places of the comet more accurately.

Upon AC, biſected in I, erect the perpendiculars AM, CN, IO, of which AM and CN may be the tangents of the latitudes in the firſt and third obſervations, to the radii TA and τC. Join MN, cutting IO in O. Draw the rectangular parallelogram iIλμ, as before. In IA produced take ID equal to Sμ + ⅔ iλ. Then in MN, towards N, take MP, which may be to the above found length X in the ſubduplicate proportion of the mean diſtance of the earth from the ſun (or of the ſemidiameter of the *orbis magnus)* to the diſtance OD. If the point P fall upon the point N; A, B, and C, will be three places of the comet, through which its orbit is to be deſcribed in the plane of the ecliptic. But if the point P falls not upon the point N, in the right line AC take CG equal to NP, ſo as the points G and P may lie on the ſame ſide of the line NC.

By the ſame method as the points E, A, C, G, were found from the aſſumed point B, from other points b and β aſſumed at pleaſure, find out the new points e, a, c, g; and ε, α, κ, γ. Then through G, g, and γ, draw the circumference of a circle Ggγ, cutting the right line τC in Z: and Z will be one place of the comet in the plane of the ecliptic. And in AC, ac, ακ, taking AF, af, αφ, equal reſpectively to CG, cg, κγ; through the points F, f, and φ, draw the circumference of a circle Ffφ, cutting the right line AT in X; and the point X will be another place of the comet in the plane of the ecliptic. And at the points X and Z, erecting the tangents of the latitudes of the comet to the radii TX and τZ, two places of the

comet in its own orbit will be determined. Laſtly, if (by prop. 19, book 1) to the focus S a parabola is deſcribed paſſing through thoſe two places, this parabola will be the orbit of the comet. Q.E.I.

The demonſtration of this conſtruction follows from the preceding lemmas, becauſe the right line AC is cut in E in the proportion of the times, by lem. 7, as it ought to be, by lem. 8; and BE, by lem. 11, is a portion of the right line BS or Bξ in the plane of the ecliptic, intercepted between the arc ABC and the chord AEC; and MP (by cor. lem. 10) is the length of the chord of that arc, which the comet ſhould deſcribe in its proper orbit between the firſt and third obſervation, and therefore is equal to MN, providing B is a true place of the comet in the plane of the ecliptic.

But it will be convenient to aſſume the points B, b, β, not at random, but nearly true. If the angle AQt, at which the projection of the orbit in the plane of the ecliptic cuts the right line tB, is rudely known, at that angle with Bt draw the obſcure line AC, which may be to $\frac{4}{3}$Tτ in the ſubduplicate proportion of SQ to St; and, drawing the right line SEB ſo as its part EB may be equal to the length Vt, the point B will be determined, which we are to uſe for the firſt time. Then, cancelling the right line AC, and drawing anew AC according to the preceding conſtruction, and, moreover, finding the length MP, in tB take the point b, by this rule, that, if TA and τC interſect each other in Y, the diſtance Yb may be to the diſtance YB in a proportion compounded of the proportion of MP to MN, and the ſubduplicate proportion of SB to Sb. And by the ſame method you may find the third point β, if you pleaſe to repeat the operation the third time; but if this method is followed, two operations generally will be ſufficient; for if the diſtance Bb happens to be very ſmall, after the points F, f, and G, g, are found, draw the right lines Ff and Gg, and they will cut TA and τC in the points required, X and Z.

EXAMPLE.

Let the comet of the year 1680 be propoſed. The following table ſhews the motion thereof, as obſerved by *Flamſted*,

Fig. 1.

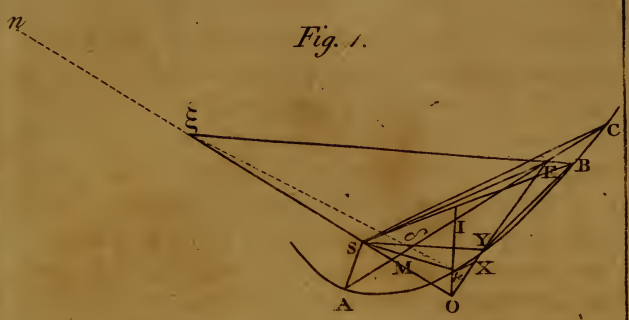


Fig. 2.

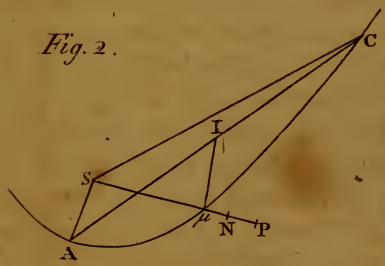
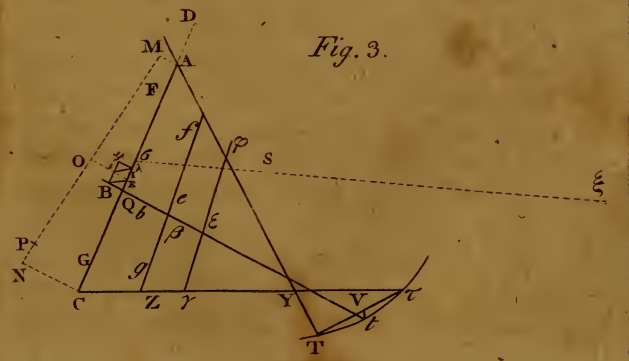


Fig. 3.



and calculated afterwards by him from his obſervations, and corrected by Dr. *Halley* from the ſame obſervations.

	Time		Sun's	Comet's	
	Appar.	True	Longitude	Longitude.	Lat. N.
	h.	h.	° ′ ″	° ′ ″	° ′ ″
1680, *Dec.* 12	4.46	4.46. 0	♑ 1.51.23	♑ 6.32.30	8.28. 0
21	6.32½	6.36.59	11.06.44	♒ 5.08.12	21.42.13
24	6.12	6.17.52	14.09.26	18.49.23	25.23. 5
26	5.14	5.20.44	16.09.22	28.24.13	27.00.52
29	7.55	8.03.02	19.19.43	♓ 13.10.41	28.09.58
30	8.02	8.10.26	20.21.09	17.38.20	28.11.53
1681, *Jan.* 5	5.51	6.01.38	26.22.18	♈ 8.48.53	26.15. 7
9	6.49	7.00.53	♒ 0.29.02	18.44.04	24.11.56
10	5.54	6.06.10	1.27.43	20.40.50	23.43.52
13	6.56	7.08.55	4.33.20	25.59.48	22.17.28
25	7.44	7.58.42	16.45.36	♉ 9.35. 0	17.56.30
30	8.07	8.21.53	21.49.58	13.19.51	16.42.18
Feb. 2	6.20	6.34.51	24.46.59	15.13.53	16.04. 1
5	6.50	7.04.41	27.49.51	16.59.06	15.27. 3

To theſe you may add ſome obſervations of mine.

	Ap. Time.	Comet's	
		Longitude.	Lat. N.
	h. ′	° ′ ″	° ′ ″
1681, *Feb.* 25	8.30	♉ 26.18.35	12.46.46
27	8.15	27.04.20	22.36.12
Mar. 1	11. 0	27.52.42	12.23.40
2	8. 0	28.12.48	12.19.38
5	11.30	29.18. 0	12.03.16
7	9.30	♊ 0. 4. 0	11.57. 0
9	8.30	0.43. 4	11.45.52

Theſe obſervations were made by a teleſcope of 7 feet, with a micrometer and threads placed in the focus of the teleſcope; by which inſtruments we determined the poſitions both of the fixed ſtars among themſe , and of the comet in reſpect of the fixed ſtars. Let A (Pl. 17) repreſent the ſtar of the fourth magnitude in the left heel of *Perſeus* (*Bayer's* o), B the following ſtar of the third magnitude in the left foot (*Bayer's* ζ), C a ſtar of the ſixth magnitude (*Bayer's* n) in the heel of the ſame foot, and D, E, F, G, H, I, K, L, M, N, O, Z, α, β, γ, δ, other ſmaller ſtars in the ſame foot; and let p, P, Q, R, S, T, V, X, repreſent the places of the comet in the obſervations above ſet down; and,

reckoning the distance AB of $80\frac{7}{12}$ parts, AC was $52\frac{1}{4}$ of those parts; BC, $58\frac{5}{6}$; AD, $57\frac{5}{12}$; BD, $82\frac{6}{11}$; CD, $23\frac{2}{3}$; AE, $29\frac{4}{7}$; CE, $57\frac{1}{2}$; DE, $49\frac{11}{12}$; AI, $27\frac{7}{12}$; BI, $52\frac{1}{6}$; CI, $36\frac{7}{12}$; DI, $53\frac{5}{11}$; AK, $38\frac{2}{3}$; BK, 43; CK, $31\frac{5}{9}$; FK, 29; FB, 23; FC, $36\frac{1}{4}$; AH, $18\frac{6}{7}$; DH, $50\frac{7}{8}$; BN, $46\frac{5}{12}$; CN, $31\frac{1}{3}$; BL, $45\frac{5}{12}$; NL, $31\frac{5}{7}$. HO was to HI as 7 to 6, and, produced, did pass between the stars D and E, so as the distance of the star D from this right line was $\frac{1}{6}$CD. LM was to LN as 2 to 9, and, produced, did pass through the star H. Thus were the positions of the fixed stars determined in respect of one another.

Mr. *Pound* has since observed a second time the positions of these fixed stars amongst themselves, and collected their longitudes and latitudes according to the following table.

The fixed stars.	Their Longitudes.	Latitude North.	The fixed stars.	Their Longitudes.	Latitude North.
	° ′ ″	° ′ ″		° ′ ″	° ′ ″
A	♉ 26.41.50	12. 8.36	L	♉ 29.33.34	12. 7.48
B	28.40.23	11.17.54	M	29.18.54	12. 7.20
C	27.58.30	12.40.25	N	28.48.29	12.31. 9
E	26.27.17	12.52. 7	Z	29.44.48	11.57.13
F	28.28.37	11.52.22	α	29.52. 3	11.55.48
G	26.56. 8	12. 4.58	β	♊ 0. 8.23	11.48.56
H	27.11.45	12. [illegible] 1	γ	0.40.10	11.55.18
I	27.25. 2	11.53.11	δ	1. 3.20	11.30.42
K	27.42. 7	11.53.26			

The positions of the comet to these fixed stars were observed to be as follow:

Friday, *February* 25, O.S. at $8\frac{1}{2}$h. P. M. the distance of the comet in p from the star E was less than $\frac{3}{13}$AE, and greater than $\frac{1}{5}$AE, and therefore nearly equal to $\frac{3}{14}$AE; and the angle ApE was a little obtuse, but almost right. For from A, letting fall a perpendicular on pE, the distance of the comet from that perpendicular was $\frac{1}{5}$pE.

The same night, at $9\frac{1}{2}$h. the distance of the comet in P from the star E was greater than $\frac{1}{4\frac{1}{2}}$ AE, and less than $\frac{1}{5\frac{1}{4}}$ AE, and therefore nearly equal to $\frac{10}{47}$ of AE, or $\frac{8}{39}$AE. But

p
P
A
Q
G
O
H
I
D
K
R
C
S
F
B
N
M
T
L
Z
a
V
B
X
Y

the diſtance of the comet from the perpendicular let fall from the ſtar A upon the right line PE was $\frac{4}{5}$PE.

Sunday, *February* 27, $8\frac{1}{4}^{h}$. P. M. the diſtance of the comet in Q from the ſtar O was equal to the diſtance of the ſtars O and H; and the right line QO produced paſſed between the ſtars K and B. I could not, by reaſon of intervening clouds, determine the poſition of the ſtar to greater accuracy.

Tueſday, *March* 1, 11^{h}. P. M. the comet in R lay exactly in a line between the ſtars K and C, ſo as the part CR of the right line CRK was a little greater than $\frac{1}{3}$CK, and a little leſs than $\frac{1}{3}$CK + $\frac{1}{8}$CR, and therefore = $\frac{1}{3}$CK + $\frac{1}{16}$CR, or $\frac{16}{45}$CK.

Wedneſday, *March* 2, 8^{h}. P. M. the diſtance of the comet in S from the ſtar C was nearly $\frac{4}{9}$FC; the diſtance of the ſtar F from the right line CS produced was $\frac{1}{24}$FC; and the diſtance of the ſtar B from the ſame right line was five times greater than the diſtance of the ſtar F; and the right line NS produced paſſed between the ſtars H and I five or ſix times nearer to the ſtar H than to the ſtar I.

Saturday, *March* 5, $11\frac{1}{2}^{h}$. P. M. when the comet was in T, the right line MT was equal to $\frac{1}{2}$ML, and the right line LT produced paſſed between B and F four or five times nearer to F than to B, cutting off from BF a fifth or ſixth part thereof towards F: and MT produced paſſed on the outſide of the ſpace BF towards the ſtar B four times nearer to the ſtar B than to the ſtar F. M was a very ſmall ſtar, ſcarcely to be ſeen by the teleſcope; but the ſtar L was greater, and of about the eighth magnitude.

Monday, *March* 7, $9\frac{1}{2}^{h}$. P. M. the comet being in V, the right line Vα produced did paſs between B and F, cutting off, from BF towards F, $\frac{1}{10}$ of BF, and was to the right line Vβ as 5 to 4. And the diſtance of the comet from the right line $\alpha\beta$ was $\frac{1}{2}$Vβ.

Wedneſday, *March* 9, $8\frac{1}{2}^{h}$. P. M. the comet being in X, the right line γX was equal to $\frac{1}{4}\gamma\delta$; and the perpendicular let fall from the ſtar δ upon the right γX was $\frac{2}{5}$ of $\gamma\delta$.

The ſame night, at 12^{h}. the comet being in Y, the right line γY was equal to $\frac{1}{3}$ of $\gamma\delta$, or a little leſs, as perhaps $\frac{5}{16}$ of $\gamma\delta$; and a perpendicular let fall from the ſtar δ on the right

line γY was equal to about $\frac{1}{6}$ or $\frac{1}{7}\ \gamma\delta$. But the comet being then extremely near the horizon, was ſcarcely diſcernible, and therefore its place could not be determined with that certainty as in the foregoing obſervations.

From theſe obſervations, by conſtructions of figures and calculations, I deduced the longitudes and latitudes of the comet; and Mr. *Pound*, by correcting the places of the fixed ſtars, hath determined more correctly the places of the comet, which correct places are ſet down above. Though my micrometer was none of the beſt, yet the errors in longitude and latitude (as derived from my obſervations) ſcarcely exceed one minute. The comet (according to my obſervations), about the end of its motion, began to decline ſenſibly towards the north, from the parallel which it deſcribed about the end of *February*.

Now, in order to determine the orbit of the comet out of the obſervations above deſcribed, I ſelected thoſe three which *Flamſted* made, *Dec.* 21, *Jan.* 5, and *Jan.* 25; from which I found St of 9842,1 parts, and Vt of 455, ſuch as the ſemi-diameter of the *orbis magnus* contains 10000. Then for the firſt obſervation, aſſuming tB of 5657 of thoſe parts, I found SB 9747, BE for the firſt time 412, Sμ 9503, iλ 413, BE for the ſecond time 421, OD 10186, X 8528,4, PM 8450, MN 8475, NP 25; from whence, by the ſecond operation, I collected the diſtance tb 5640; and by this operation I at laſt deduced the diſtances TX 4775 and τZ 11322. From which, limiting the orbit, I found its deſcending node in $\simeq$, and aſcending node in $\curlyvee$ 1° 53′; the inclination of its plane to the plane of the ecliptic 61° 20$\frac{1}{3}$′; the vertex thereof (or the perihelion of the comet) diſtant from the node 8° 38′, and in $\sagittarius$ 27° 43′, with latitude 7° 34′ ſouth; its *latus rectum* 236,8; and the diurnal area deſcribed by a radius drawn to the ſun 93585, ſuppoſing the ſquare of the ſemi-diameter of the *orbis magnus* 100000000; that the comet in this orbit moved directly according to the order of the ſigns, and on *Dec.* 8^{d}. 00^{h}. 04′ P. M. was in the vertex or perihelion of its orbit. All which I determined by ſcale and compaſs, and the chords of angles, taken from the table of natural ſines, in a pretty large figure,

in which, to wit, the radius of the *orbis magnus* (conſiſting of 10000 parts) was equal to $16\frac{1}{3}$ inches of an *Engliſh* foot.

Laſtly, in order to diſcover whether the comet did truly move in the orbit ſo determined, I inveſtigated its places in this orbit partly by arithmetical operations, and partly by ſcale and compaſs, to the times of ſome of the obſervations, as may be ſeen in the following table.

		The Comet's					
	Diſt. from ſun.	Longitude computed.	Latitud. comput-ed.	Longitude obſerved.	Latitude obſerved.	Dif Lo.	Dif. Lat.
Dec. 12	2792	♑ 6°.32′	8°.18½	♑ 6°.31½	8°.26	+ 1	— 7½
29	8403	♐ 13 .13⅔	28. 00	♐ 13. 11¾	28. 10½	+ 2	—10½
Febr. 5	16669	♉ 17.00	15. 29⅔	♉ 16. 59⅞	15. 27⅖	+ 0	+ 2¼
Mar. 5	21737	29 .19¾	12. 4	29. 20⅙	12. 3½	—1	+ ½

But afterwards Dr. *Halley* did determine the orbit to a greater accuracy by an arithmetical calculus than could be done by linear deſcriptions; and, retaining the place of the nodes in ♋ and ♑ 1° 53′, and the inclination of the plane of the orbit to the ecliptic 61° 20⅓′, as well as the time of the comet's being in perihelio, *Dec.* 8ᵈ. 00ʰ. 04′, he found the diſtance of the perihelion from the aſcending node meaſured in the comet's orbit 9° 20′, and the *latus rectum* of the parabola 2430 parts, ſuppoſing the mean diſtance of the ſun from the earth to be 100000 parts: and from theſe *data*, by an accurate arithmetical calculus, he computed the places of the comet to the times of the obſervations as follows.

True time.				Dist. from the sun.	The Comet's Longitude computed.			Latitude computed.			Errors in Long.			Errors in Lat.		
	d.	h.	′			°	′	″	°	′	″	′	″		′	″
Dec.	12	4	46	28028	♑	6	29	25	8	26	0 Bor.	—	3	5	—	2	0
	21	6	37	61076	♒	5	6	30	21	43	20	—	1	42	+	1	7
	24	6	18	70008		18	48	20	25	22	40		1	3	—	0	25
	26	5	20	75576		28	22	45	27	1	36		1	28	+	0	44
	29	8	3	84021	♓	13	12	40	28	10	10	+	1	59	+	0	12
	30	8	10	86661		17	40	5	28	11	20	+	1	45	—	0	33
Jan.	5	6	1	101440	♈	8	49	49	26	15	15	+	0	56	+	0	8
	9	7	0	110959		18	44	36	24	12	54	+	0	32	+	0	58
	10	6	6	113162		20	41	0	23	44	10	+	0	10	+	0	18
	13	7	9	120000		26	0	21	22	17	30	+	0	33	+	0	2
	25	7	59	145370	♉	9	33	40	17	57	55	—	1	20	+	1	25
	30	8	22	155303		13	17	41	16	42	7	—	2	10	—	0	11
Febr.	2	6	35	160951		15	11	11	16	4	15	—	2	42	+	0	14
	5	7	4	166686		16	58	55	15	29	13	—	0	41	+	2	0
	25	8	41	202570		26	15	46	12	48	0	—	2	49	+	1	10
Mar.	5	11	39	216205		29	18	35	12	5	40	+	0	35	+	2	14

This comet alſo appeared in the *November* before, and at *Coburg*, in *Saxony*, was obſerved by Mr. *Gottfried Kirch*, on the 4th of that month, on the 6th and 11th O. S; from its poſitions to the neareſt fixed ſtars obſerved with ſufficient accuracy, ſometimes with a two feet, and ſometimes with a ten feet teleſcope; from the difference of longitudes of *Coburg* and *London*, 11°; and from the places of the fixed ſtars obſerved by Mr. *Pound*, Dr. *Halley* has determined the places of the comet as follows.

Nov. 3, 17^h. 2′, apparent time at *London*, the comet was in ☊ 29 deg. 51′, with 1 deg. 17′ 45″ latitude north.

November 5, 15^h 58′ the comet was in ♏ 3° 23′, with 1° 6′ north lat.

November 10, 16^h. 31′, the comet was equally diſtant from two ſtars in ☊, which are σ and τ in *Bayer*; but it had not quite touched the right line that joins them, but was very little diſtant from it. In *Flamſted*'s catalogue this ſtar σ was then in ♏ 14°. 15′, with 1 deg. 41′ lat. north nearly, and τ in ♏ 17° 3½′ with 0 deg. 34′ lat. ſouth; and the middle point between thoſe ſtars was ♏ 15° 39¼′, with 0° 33½′ lat. north. Let the diſtance of the comet from that right line be about 10′ or 12′; and the difference of the longitude of the comet and that middle point will be 7′; and the difference of the latitude nearly 7½′; and thence it follows that the comet was in ♏ 15° 32′, with about 26′ lat. north.

The firſt obſervation from the poſition of the comet with reſpect to certain ſmall fixed ſtars had all the exactneſs that could be deſired; the ſecond alſo was accurate enough. In the third obſervation, which was the leaſt accurate, there might be an error of 6 or 7 minutes, but hardly greater. The longitude of the comet, as found in the firſt and moſt accurate obſervation, being computed in the aforeſaid parabolic orbit, comes out ☊ 29° 30′ 22″, its latitude north 1° 25′ 7″, and its diſtance from the ſun 115546.

Moreover, Dr. *Halley*, obſerving that a remarkable comet had appeared four times at equal intervals of 575 years (that is, in the month of *September* after *Julius Cæſar* was killed; *An. Chr.* 531, in the conſulate of *Lampadius* and *Oreſtes*; *An. Chr.* 1106, in the month of *February*; and at the end of the year 1680; and that with a long and remarkable tail, except when it was ſeen after *Cæſar*'s death, at which time, by reaſon of the inconvenient ſituation of the earth, the tail was not ſo conſpicuous), ſet himſelf to find out an elliptic orbit whoſe greater axis ſhould be 1382957 parts, the mean diſtance of the earth from the ſun containing 10000 ſuch; in which orbit a comet might revolve in 575 years; and, placing the aſcending node in ♋ 2° 2′, the inclination of the plane of the

orbit to the plane of the ecliptic in an angle of 61° 6′ 48″, the perihelion of the comet in this plane in ♐ 22° 44′ 25″, the equal time of the perihelion *December* 7^{d}. 23^{h}. 9′, the distance of the perihelion from the ascending node in the plane of the ecliptic 9° 17′ 35″, and its conjugate axis 18481,2, he computed the motions of the comet in this elliptic orbit. The places of the comet, as deduced from the observations, and as arising from computation made in this orbit, may be seen in the following table.

True time.	Longitude obs.	Latitude Nor. obs.	Longitude comp.	Latitude comp.	Errors in Longitude	Errors in Latitude.
d. h. ′	° ′ ″	° ′ ″	° ′ ″	° ′ ″	′ ″	′ ″
Nov. . 3 . 16 . 47	♌ 29 . 51 . 0	1 . 17 . 45	♌ 29 . 51 . 22	1 . 17 . 32 N	+ 0 . 22	— 0 . 13
..... 5 . 15 . 37	♍ 3 . 23 . 0	1 . 6 . 0	♍ 3 . 24 . 32	1 . 6 . 9	+ 1 . 32	+ 0 . 9
.....10 . 16 . 18	15 . 32 . 0	0 . 27 . 0	15 . 33 . 2	0 . 25 . 7	+ 1 . 2	— 1 . 53
.....16 . 17 . 00	.....	.....	♎ 8 . 16 . 45	0 . 53 . 7 S	.....	.....
.....18 . 21 . 34	.....	.....	.. 18 . 52 . 15	1 . 26 . 54	.....	.....
.....20 . 17 . 0	.....	.....	.. 28 . 10 . 36	1 . 53 . 35	.....	.....
.....23 . 17 . 5	.....	.....	♏ 13 . 22 . 42	2 . 29 . 0	.....	.....
Dec. .12 . 4 . 46	♑ 6 . 32 . 30	8 . 28 . 0	♑ 6 . 31 . 20	8 . 29 . 6 N	— 1 . 10	+ 1 . 6
..... 21 . 6 . 37	♒ 5 . 8 . 12	21 . 42 . 13	♒ 5 . 6 . 14	21 . 44 . 42	— 1 . 58	+ 2 . 29
.....24 . 6 . 18	18 . 49 . 23	25 . 23 . 5	18 . 47 . 30	25 . 23 . 35	— 1 . 53	+ 0 . 30
.....26 . 5 . 21	28 . 24 . 13	27 . 0 . 52	28 . 21 . 42	27 . 2 . 1	— 2 . 31	+ 1 . 9
.....29 . 8 . 3	♓ 13 . 10 . 41	28 . 9 . 58	♓ 13 . 11 . 14	28 . 10 . 38	+ 0 . 33	+ 0 . 40
.....30 . 8 . 10	17 . 38 . 0	28 . 11 . 53	17 . 38 . 27	28 . 11 . 37	+ 0 . 7	— 0 . 16
Jan. . 5 . 6 . 1½	♈ 8 . 48 . 53	26 . 15 . 7	♈ 8 . 48 . 51	26 . 14 . 57	— 0 . 2	— 0 . 10
..... 9 . 7 . 1	18 . 44 . 4	24 . 11 . 56	18 . 43 . 51	24 . 12 . 17	— 0 . 13	+ 0 . 21
.....10 . 6 . 6	20 . 40 . 50	23 . 43 . 32	20 . 40 . 23	23 . 43 . 25	— 0 . 27	— 0 . 7
.....13 . 7 . 9	25 . 59 . 48	22 . 17 . 28	26 . 0 . 8	22 . 16 . 32	+ 0 . 20	— 0 . 56
.....25 . 7 . 59	♉ 9 . 35 . 0	17 . 56 . 30	♉ 9 . 34 . 11	17 . 56 . 6	— 0 . 49	— 0 . 24
.....30 . 8 . 22	13 . 19 . 51	16 . 42 . 18	13 . 18 . 28	16 . 40 . 5	— 1 . 23	— 2 . 13
Feb. . 2 . 6 . 35	15 . 13 . 53	16 . 4 . 1	15 . 11 . 59	16 . 2 . 7	— 1 . 54	— 1 . 54
..... 5 . 7 . 4½	16 . 59 . 6	15 . 27 . 3	16 . 59 . 17	15 . 27 . 0	+ 0 . 11	— 0 . 3
.....25 . 8 . 41	26 . 18 . 35	12 . 46 . 46	26 . 16 . 59	12 . 45 . 22	— 1 . 36	— 1 . 24
Mar. . 1 . 11 . 10	27 . 52 . 42	12 . 23 . 40	27 . 51 . 47	12 . 22 . 28	— 0 . 55	— 1 . 12
..... 5 . 11 . 39	29 . 18 . 0	12 . 3 . 16	29 . 20 . 11	12 . 2 . 50	+ 2 . 11	— 0 . 26
..... 9 . 8 . 38	♊ 0 . 43 . 4	11 . 45 . 52	♊ 0 . 42 . 43	11 . 45 . 35	— 0 . 21	— 0 . 17

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The obſervations of this comet from the beginning to the end agree as perfectly with the motion of the comet in the orbit juſt now deſcribed as the motions of the planets do with the theories from whence they are calculated; and by this agreement plainly evince that it was one and the ſame comet that appeared all that time, and alſo that the orbit of that comet is here rightly defined.

In the foregoing table we have omitted the obſervations of *Nov.* 16, 18, 20, and 23, as not ſufficiently accurate, for at thoſe times ſeveral perſons had obſerved the comet. *Nov.* 17, O. S. *Ponthæus* and his companions, at 6^h. in the morning at *Rome* (that is, 5^h. 10′ at *London)*, by threads directed to the fixed ſtars, obſerved the comet in ♎ 8° 30′, with latitude 0° 40′ ſouth. Their obſervations may be ſeen in a treatiſe which *Ponthæus* publiſhed concerning this comet. *Cellius,* who was preſent, and communicated his obſervations in a letter to *Caſſini,* ſaw the comet at the ſame hour in ♎ 8° 30′, with latitude 0° 30′ ſouth. It was likewiſe ſeen by *Galletius* at the ſame hour at *Avignon* (that is, at 5^h. 42′ morning at *London)* in ♎ 8° without latitude. But by the theory the comet was at that time in ♎ 8° 16′ 45″, and its latitude was 0° 53′ 7″ ſouth.

Nov. 18, at 6^h. 30′ in the morning at *Rome* (that is, at 5^h. 40′ at *London)*, *Ponthæus* obſerved the comet in ♎ 13° 30′, with latitude 1° 20′ ſouth; and *Cellius* in ♎ 13° 30′, with latitude 1° 00′ ſouth. But at 5^h. 30′ in the morning at *Avignon, Galletius* ſaw it in ♎ 13° 00′, with latitude 1° 00′ ſouth. In the Univerſity of *La Fleche,* in *France,* at 5^h. in the morning (that is, at 5^h 9′ at *London*), it was ſeen by *P. Ango,* in the middle between two ſmall ſtars, one of which is the middle of the three which lie in a right line in the ſouthern hand of Virgo, *Bayers* ψ; and the other is the outmoſt of the wing, *Bayer's* θ. Whence the comet was then in ♎ 12° 46′, with latitude 50′ ſouth. And I was informed by Dr. *Halley,* that on the ſame day at *Boſton* in *New England,* in the latitude of 42½ deg. at 5^h. in the morning (that is, at 9^h. 44′ in the morning at *London*), the comet was ſeen near ♎ 14°, with latitude 1° 30′ ſouth.

Nov. 19, at $4\frac{1}{2}^h$. at *Cambridge*, the comet (by the obſervation of a young man) was diſtant from *Spica* ♍ about 2° towards the north weſt. Now the ſpike was at that time in ♎ 19° 23′ 47″, with latitude 2° 1′ 59″ ſouth. The ſame day, at 5^h. in the morning, at *Boſton* in *New England*, the comet was diſtant from *Spica* ♍ 1°, with the difference of 40′ in latitude. The ſame day, in the iſland of *Jamaica*, it was about 1° diſtant from *Spica* ♍. The ſame day, Mr. *Arthur Storer*, at the river *Patuxent*, near *Hunting Creek*, in *Maryland*, in the confines of *Virginia*, in lat $38\frac{1}{2}°$, at 5 in the morning (that is, at 10^h. at *London*), ſaw the comet above *Spica* ♍, and very nearly joined with it, the diſtance between them being about $\frac{3}{4}$ of one deg. And from theſe obſervations compared, I conclude, that at 9^h 44′ at *London* the comet was in ♎ 18° 50′, with about 1° 25′ latitude ſouth. Now by the theory the comet was at that time in ♎ 18° 52′ 15″, with 1° 26′ 54″ lat. ſouth.

Nov. 20, *Montenari*, profeſſor of aſtronomy at *Padua*, at 6^h. in the morning at *Venice* (that is, 5^h. 10′ at *London*), ſaw the comet in ♎ 23°, with latitude 1° 30′ ſouth. The ſame day, at *Boſton*, it was diſtant from *Spica* ♍ by about 4° of longitude eaſt, and therefore was in ♎ 23° 24′ nearly.

Nov. 21, *Ponthæus* and his companions, at $7\frac{1}{4}^h$. in the morning, obſerved the comet in ♎ 27° 50′, with latitude 1° 16′ ſouth; *Cellius*, in ♎ 28°; *P. Ango* at 5^h. in the morning, in ♎ 27° 45′; *Montenari* in ♎ 27° 51′. The ſame day, in the iſland of *Jamaica*, it was ſeen near the beginning of ♏, and of abour the ſame latitude with *Spica* ♍, that is, 2° 2′. The ſame day, at 5^h. morning, at *Ballaſore*, in the *Eaſt Indies* (that is, at 11^h. 20′ of the night preceding at *London*), the diſtance of the comet from *Spica* ♍ was taken 7° 35′ to the eaſt. It was in a right line between the ſpike and the balance, and therefore was then in ♎ 26° 58′, with about 1° 11′ lat. ſouth; and after 5^h. 40′ (that is, at 5^h .morning at *London*), it was in ♎ 28° 12′, with 1° 16′ lat. ſouth. Now by the theory the comet was then in ♎ 28° 10′ 36″, with 1° 53′ 35″ lat. ſouth.

Nov. 22, the comet was ſeen by *Montenari* in ♏ 2° 33′; but at *Boſton* in *New England*, it was found in about ♏ 3°, and with almoſt the ſame latitude as before, that is, 1° 30′.

The ſame day, at 5^h. morning at *Ballaſore*, the comet was obſerved in ♏ 1° 50′; and therefore, at 5^h. morning at *London*, the comet was in ♏ 3° 5′ nearly. The ſame day, at $6\frac{1}{2}^h$. in the morning at *London*, Dr. *Hook* obſerved it in about ♏ 3° 30′, and that in the right line which paſſeth through *Spica* ♍ and *Cor Leonis*; not, indeed, exactly, but deviating a little from that line towards the north. *Montenari* likewiſe obſerved, that this day, and ſome days after, a right line drawn from the comet through *Spica* paſſed by the ſouth ſide of *Cor Leonis* at a very ſmall diſtance therefrom. The right line through *Cor Leonis* and *Spica* ♍ did cut the ecliptic in ♍ 3° 46′ at an angle of 2° 51′; and if the comet had been in this line and in ♏ 3°, its latitude would have been 2° 26′; but ſince *Hook* and *Montenari* agree that the comet was at ſome ſmall diſtance from this line towards the north, its latitude muſt have been ſomething leſs. On the 20th, by the obſervation of *Montenari*, its latitude was almoſt the ſame with that of *Spica*, that is, about 1° 30′. But by the agreement of *Hook*, *Montenari*, and *Ango*, the latitude was continually increaſing, and therefore muſt now, on the 22d, be ſenſibly greater than 1° 30′; and, taking a mean between the extreme limits but now ſtated, 2° 26′ and 1° 30′, the latitude will be about 1° 58′. *Hook* and *Montenari* agree that the tail of the comet was directed towards *Spica* ♍, declining a little from that ſtar towards the ſouth according to *Hook*, but towards the north according to *Montenari*; and, therefore, that declination was ſcarcely ſenſible; and the tail, lying nearly parallel to the equator, deviated a little from the oppoſition of the ſun towards the north.

Nov. 23, O. S. at 5^h morning, at *Nuremberg* (that is, at $4\frac{1}{2}^h$ at *London*), Mr. *Zimmerman* ſaw the comet in ♏ 8° 8′, with 2° 31′ ſouth lat. its place being collected by taking its diſtances from fixed ſtars.

Nov. 24, before ſun-riſing, the comet was ſeen by *Montenari* in ♏ 12° 52′ on the north ſide of the right line through *Cor Leonis* and *Spica* ♍, and therefore its latitude was ſomething leſs than 2° 38′; and ſince the latitude, as we ſaid, by the concurring obſervations of *Montenari*, *Ango*, and *Hook*,

was continually increasing, therefore, it was now, on the 24th, something greater than 1° 58′; and, taking the mean quantity, may be reckoned 2° 18′, without any considerable error. *Ponthæus* and *Galletius* will have it that the latitude was now decreasing; and *Cellius*, and the observer in *New England*, that it continued the same, viz. of about 1°, or $1\frac{1}{2}$°. The observations of *Ponthæus* and *Cellius* are more rude, especially those which were made by taking the azimuths and altitudes; as are also the observations of *Galletius*. Those are better which were made by taking the position of the comet to the fixed stars by *Montenari*, *Hook*, *Angо*, and the observer in *New England*, and sometimes by *Ponthæus* and *Cellius*. The same day, at 5^h morning, at *Ballasore*, the comet was observed in ♏ 11° 45′; and, therefore, at 5^h. morning at *London*, was in ♏ 13° nearly. And, by the theory, the comet was at that time in ♏ 13° 22′ 42″.

Nov. 25, before sun-rise, *Montenari* observed the comet in ♏ $17\frac{3}{4}$° nearly; and *Cellius* observed at the same time that the comet was in a right line between the bright star in the right thigh of Virgo and the southern scale of Libra; and this right line cuts the comet's way in ♏ 18° 36′. And, by the theory, the comet was in ♏ $18\frac{1}{3}$° nearly.

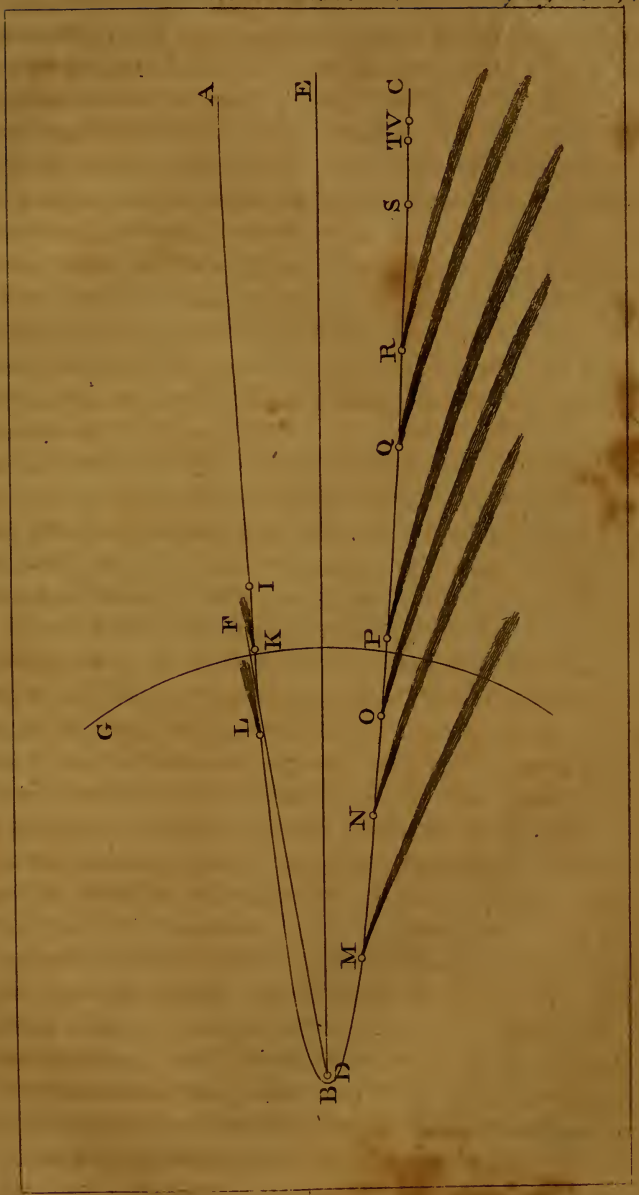
From all this it is plain that these observations agree with the theory, so far as they agree with one another; and by this agreement it is made clear that it was one and the same comet that appeared all the time from *Nov.* 4 to *Mar.* 9. The path of this comet did twice cut the plane of the ecliptic, and therefore was not a right line. It did cut the ecliptic not in opposite parts of the heavens, but in the end of Virgo and beginning of Capricorn, including an arc of about 98°; and therefore the way of the comet did very much deviate from the path of a great circle; for in the month of *Nov.* it declined at least 3° from the ecliptic towards the south; and in the month of *Dec.* following it declined 29° from the ecliptic towards the north; the two parts of the orbit in which the comet descended towards the sun, and ascended again from the sun, declining one from the other by an apparent angle of above 30°, as observed by *Montenari*. This comet travelled

over 9 ſigns, to wit, from the laſt deg. of ♌ to the beginning of ♈, beſide the ſign of ♌, through which it paſſed before it began to be ſeen; and there is no other theory by which a comet can go over ſo great a part of the heavens with a regular motion. The motion of this comet was very unequable; for about the 20th of *Nov.* it deſcribed about 5° a day. Then its motion being retarded between *Nov.* 26 and *Dec.* 12, to wit, in the ſpace of 15½ days, it deſcribed only 40°. But the motion thereof being afterwards accelerated, it deſcribed near 5° a day, till its motion began to be again retarded. And the theory which juſtly correſponds with a motion ſo unequable, and through ſo great a part of the heavens, which obſerves the ſame laws with the theory of the planets, and which accurately agrees with accurate aſtronomical obſervations, cannot be otherwiſe than true.

And, thinking it would not be improper, I have given (Pl. 18) a true repreſentation of the orbit which this comet deſcribed, and of the tail which it emitted in ſeveral places, in the annexed figure; protracted in the plane of the trajectory. In this ſcheme ABC repreſents the trajectory of the comet, D the ſun, DE the axis of the trajectory, DF the line of the nodes, GH the interſection of the ſphere of the *orbis magnus* with the plane of the trajectory, I the place of the comet *Nov.* 4, *Ann.* 1680; K the place of the ſame *Nov.* 11; L the place of the ſame *Nov.* 19; M its place *Dec.* 12; N its place *Dec.* 21; O its place *Dec.* 29; P its place *Jan.* 5 following; Q its place *Jan.* 25; R its place *Feb.* 5; S its place *Feb.* 25; T its place *March* 5; and V its place *March* 9. In determining the length of the tail, I made the following obſervations.

Nov. 4 and 6, the tail did not appear; *Nov.* 11, the tail juſt begun to ſhew itſelf, but did not appear above ½ deg. long through a 10 feet teleſcope; *Nov.* 17, the tail was ſeen by *Ponthæus* more than 15° long; *Nov.* 18, in *New England,* the tail appeared 30° long, and directly oppoſite to the ſun, extending itſelf to the planet Mars, which was then in ♍, 9° 54′; *Nov.* 19, in *Maryland,* the tail was found 15° or 20° long; *Dec.* 10 (by the obſervation of Mr. *Flamſted*), the

tail paſſed through the middle of the diſtance intercepted between the tail of the Serpent of *Ophiuchus* and the ſtar δ in the ſouth wing of *Aquila*, and did terminate near the ſtars A, ω, b, in *Bayer*'s tables. Therefore the end of the tail was in ♑ 19½°, with latitude about 34¼° north; *Dec.* 11, it aſcended to the head of *Sagitta* (*Bayer*'s α, β), terminating in ♑ 26° 43′, with latitude 38° 34′ north; *Dec.* 12, it paſſed through the middle of *Sagitta*, nor did it reach much farther; terminating in ♒ 4°, with latitude 42½° north nearly. But theſe things are to be underſtood of the length of the brighter part of the tail; for with a more faint light, obſerved, too, perhaps, in a ſerener ſky, at *Rome*, *Dec.* 12, 5ʰ. 40′, by the obſervation of *Ponthæus*, the tail aroſe to 10° above the rump of the Swan, and the ſide thereof towards the weſt and towards the north was 45′ diſtant from this ſtar. But about that time the tail was 3° broad towards the upper end; and therefore the middle thereof was 2° 15′ diſtant from that ſtar towards the ſouth, and the upper end was ♓ in 22°, with latitude 61° north; and thence the tail was about 70° long; *Dec.* 21, it extended almoſt to *Caſſiopeia*'s chair, equally diſtant from β and from *Schedir*, ſo as its diſtance from either of the two was equal to the diſtance of the one from the other, and therefore did terminate in ♈ 24°, with latitude 47½°; *Dec.* 29, it reached to a contact with *Scheat* on its left, and exactly filled up the ſpace between the two ſtars in the northern foot of *Andromeda*, being 54° in length; and therefore terminated in ♉ 19°, with 35° of latitude; *Jan.* 5, it touched the ſtar π in the breaſt of *Andromeda* on its right ſide, and the ſtar μ of the girdle on its left; and, according to our obſervations, was 40° long; but it was curved, and the convex ſide thereof lay to the ſouth; and near the head of the comet it made an angle of 4° with the circle which paſſed through the ſun and the comet's head; but towards the other end it was inclined to that circle in an angle of about 10° or 11°; and the chord of the tail contained with that circle an angle of 8°. *Jan.* 13, the tail terminated between *Alamech* and *Algol*, with a light that was ſenſible enough; but with a faint light it ended over againſt the ſtar κ in *Perſeus*'s ſide. The diſtance of the end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of the tail from the circle paſſing through the ſun and the comet was 3° 50′; and the inclination of the chord of the tail to that circle was 8½°. *Jan.* 25 and 26, it ſhone with a faint light to the length of 6° or 7°; and for a night or two after, when there was a very clear ſky, it extended to the length of 12°, or ſomething more, with a light that was very faint and very hardly to be ſeen; but the axis thereof was exactly directed to the bright ſtar in the eaſtern ſhoulder of *Auriga*, and therefore deviated from the oppoſition of the ſun towards the north by an angle of 10°. Laſtly, *Feb.* 10, with a teleſcope I obſerved the tail 2° long; for that fainter light which I ſpoke of did not appear through the glaſſes. But *Ponthæus* writes, that, on *Feb.* 7, he ſaw the tail 12° long. *Feb.* 25, the comet was without a tail, and ſo continued till it diſappeared.

Now if one reflects upon the orbit deſcribed, and duly conſiders the other appearances of this comet, he will be eaſily ſatisfied that the bodies of comets are ſolid, compact, fixed, and durable, like the bodies of the planets; for if they were nothing elſe but the vapours or exhalations of the earth, of the ſun, and other planets, this comet, in its paſſage by the neighbourhood of the ſun, would have been immediately diſſipated; for the heat of the ſun is as the denſity of its rays, that is, reciprocally as the ſquare of the diſtance of the places from the ſun. Therefore, ſince, on *Dec.* 8, when the comet was in its perihelion, the diſtance thereof from the centre of the ſun was to the diſtance of the earth from the ſame as about 6 to 1000, the ſun's heat on the comet was at that time to the heat of the ſummer-ſun with us as 1000000 to 36, or as 28000 to 1. But the heat of boiling water is about 3 times greater than the heat which dry earth acquires from the ſummer-ſun, as I have tried; and the heat of red-hot iron (if my conjecture is right) is about three or four times greater than the heat of boiling water. And therefore the heat which dry earth on the comet, while in its perihelion, might have conceived from the rays of the ſun, was about 2000 times greater than the heat of red-hot iron. But by ſo fierce a heat, va-

pours and exhalations, and every volatile matter, muſt have been immediately conſumed and diſſipated.

This comet, therefore, muſt have conceived an immenſe heat from the ſun, and retained that heat for an exceeding long time; for a globe of iron of an inch in diameter, expoſed red-hot to the open air, will ſcarcely loſe all its heat in an hour's time; but a greater globe would retain its heat longer in the proportion of its diameter, becauſe the ſurface (in proportion to which it is cooled by the contact of the ambient air) is in that proportion leſs in reſpect of the quantity of the included hot matter; and therefore a globe of red hot iron equal to our earth, that is, about 40000000 feet in diameter, would ſcarcely cool in an equal number of days, or in above 50000 years. But I ſuſpect that the duration of heat may, on account of ſome latent cauſes, increaſe in a yet leſs proportion than that of the diameter; and I ſhould be glad that the true proportion was inveſtigated by experiments.

It is farther to be obſerved, that the comet in the month of *December*, juſt after it had been heated by the ſun, did emit a much longer tail, and much more ſplendid, than in the month of *November* before, when it had not yet arrived at its perihelion; and, univerſally, the greateſt and moſt fulgent tails always ariſe from comets immediately after their paſſing by the neighbourhood of the ſun. Therefore the heat received by the comet conduces to the greatneſs of the tail: from whence, I think, I may infer, that the tail is nothing elſe but a very fine vapour, which the head or nucleus of the comet emits by its heat.

But we have had three ſeveral opinions about the tails of comets; for ſome will have it that they are nothing elſe but the beams of the ſun's light tranſmitted through the comets' heads, which they ſuppoſe to be tranſparent; others, that they proceed from the refraction which light ſuffers in paſſing from the comet's head to the earth; and, laſtly, others, that they are a ſort of clouds or vapour conſtantly riſing from the comets' heads, and tending towards the parts oppoſite to the ſun. The firſt is the opinion of ſuch as are yet unacquainted with optics; for the beams of the ſun are ſeen in a darkened

room only in conſequence of the light that is reflected from them by the little particles of duſt and ſmoke which are always flying about in the air; and, for that reaſon, in air impregnated with thick ſmoke, thoſe beams appear with great brightneſs, and move the ſenſe vigorouſly; in a yet finer air they appear more faint, and are leſs eaſily diſcerned; but in the heavens, where there is no matter to reflect the light, they can never be ſeen at all. Light is not ſeen as it is in the beam, but as it is thence reflected to our eyes; for viſion can be no otherwiſe produced than by rays falling upon the eyes; and, therefore, there muſt be ſome reflecting matter in thoſe parts where the tails of the comets are ſeen: for otherwiſe, ſince all the celeſtial ſpaces are equally illuminated by the ſun's light, no part of the heavens could appear with more ſplendor than another. The ſecond opinion is liable to many difficulties. The tails of comets are never ſeen variegated with thoſe colours which commonly are inſeparable from refraction; and the diſtinct tranſmiſſion of the light of the fixed ſtars and planets to us is a demonſtration that the æther or celeſtial medium is not endowed with any refractive power: for as to what is alledged, that the fixed ſtars have been ſometimes ſeen by the *Egyptians* environed with a *Coma*, or *Capitlitium*, becauſe that has but rarely happened, it is rather to be aſcribed to a caſual refraction of clouds; and ſo the radiation and ſcintillation of the fixed ſtars to the refractions both of the eyes and air; for upon laying a teleſcope to the eye, thoſe radiations and ſcintillations immediately diſappear. By the tremulous agitation of the air and aſcending vapours, it happens that the rays of light are alternately turned aſide from the narrow ſpace of the pupil of the eye; but no ſuch thing can have place in the much wider aperture of the object-glaſs of a teleſcope; and hence it is that a ſcintillation is occaſioned in the former caſe, which ceaſes in the latter; and this ceſſation in the latter caſe is a demonſtration of the regular tranſmiſſion of light through the heavens, without any ſenſible refraction. But, to obviate an objection that may be made from the appearing of no tail in ſuch comets as ſhine but with a faint light, as if the ſecondary rays were then too

weak to affect the eyes, and for that reason it is that the tails of the fixed stars do not appear, we are to consider, that by the means of telescopes the light of the fixed stars may be augmented above an hundred fold, and yet no tails are seen; that the light of the planets is yet more copious without any tail; but that comets are seen sometimes with huge tails, when the light of their heads is but faint and dull. For so it happened in the comet of the year 1680, when in the month of *December* it was scarcely equal in light to the stars of the second magnitude, and yet emitted a notable tail, extending to the length of 40°, 50°, 60°, or 70°, and upwards; and afterwards, on the 27th and 28th of *January*, when the head appeared but as a star of the 7th magnitude, yet the tail (as we said above), with a light that was sensible enough, though faint, was stretched out to 6 or 7 degrees in length, and with a languishing light that was more difficultly seen, even to 12°, and upwards. But on the 9th and 10th of *February*, when to the naked eye the head appeared no more, through a telescope I viewed the tail of 2° in length. But farther; if the tail was owing to the refraction of the celestial matter, and did deviate from the opposition of the sun, according to the figure of the heavens, that deviation in the same places of the heavens should be always directed towards the same parts. But the comet of the year 1680, *December* 28^{d}. $8\frac{1}{2}^{h}$. P. M. at *London*, was seen in ♓ 8° 41′, with latitude north 28° 6′; while the sun was in ♑ 18° 26′. And the comet of the year 1577, *December* 29^{d}. was in ♓ 8° 41′, with latitude north 28° 40′, and the sun, as before, in about ♑ 18° 26′. In both cases the situation of the earth was the same, and the comet appeared in the same place of the heavens; yet in the former case the tail of the comet (as well by my observations as by the observations of others) deviated from the opposition of the sun towards the north by an angle of $4\frac{1}{2}$ degrees; whereas in the latter there was (according to the observations of *Tycho*) a deviation of 21 degrees towards the south. The refraction, therefore, of the heavens being thus disproved, it remains that the *phænomena* of the tails of comets must be derived from some reflecting matter.

And that the tails of comets do ariſe from their heads, and tend towards the parts oppoſite to the ſun, is farther confirmed from the laws which the tails obſerve. As that, lying in the planes of the comets' orbits which paſs through the ſun, they conſtantly deviate from the oppoſition of the ſun towards the parts which the comets' heads in their progreſs along theſe orbits have left. That to a ſpectator, placed in thoſe planes, they appear in the parts directly oppoſite to the ſun; but, as the ſpectator recedes from thoſe planes, their deviation begins to appear, and daily becomes greater. That the deviation, *cæteris paribus*, appears leſs when the tail is more oblique to the orbit of the comet, as well as when the head of the comet approaches nearer to the ſun, eſpecially if the angle of deviation is eſtimated near the head of the comet. That the tails which have no deviation appear ſtraight, but the tails which deviate are likewiſe bended into a certain curvature. That this curvature is greater when the deviation is greater; and is more ſenſible when the tail, *cæteris paribus*, is longer; for in the ſhorter tails the curvature is hardly to be perceived. That the angle of deviation is leſs near the comet's head, but greater towards the other end of the tail; and that becauſe the convex ſide of the tail regards the parts from which the deviation is made, and which lie in a right line drawn out infinitely from the ſun through the comet's head. And that the tails that are long and broad, and ſhine with a ſtronger light, appear more reſplendent and more exactly defined on the convex than on the concave ſide. Upon which accounts it is plain that the *phænomena* of the tails of comets depend upon the motions of their heads, and by no means upon the places of the heavens in which their heads are ſeen; and that, therefore, the tails of comets do not proceed from the refraction of the heavens, but from their own heads, which furniſh the matter that forms the tail. For, as in our air, the ſmoke of a heated body aſcends either perpendicularly if the body is at reſt, or obliquely if the body is moved obliquely, ſo in the heavens, where all bodies gravitate towards the ſun, ſmoke and vapour muſt (as we have already ſaid) aſcend from the ſun, and either riſe

perpendicularly if the ſmoking body is at reſt, or obliquely if the body, in all the progreſs of its motion, is always leaving thoſe places from which the upper or higher parts of the vapour had riſen before; and that obliquity will be leaſt where the vapour aſcends with moſt velocity, to wit, near the ſmoking body, when that is near the ſun. But, becauſe the obliquity varies, the column of vapour will be incurvated; and becauſe the vapour in the preceding ſides is ſomething more recent, that is, has aſcended ſomething more late from the body, it will therefore be ſomething more denſe on that ſide, and muſt on that account reflect more light, as well as be better defined. I add nothing concerning the ſudden uncertain agitation of the tails of comets, and their irregular figures, which authors ſometimes deſcribe, becauſe they may ariſe from the mutations of our air, and the motions of our clouds, in part obſcuring thoſe tails; or, perhaps, from parts of the *Via Lactea*, which might have been confounded with and miſtaken for parts of the tails of the comets as they paſſed by.

But that the atmoſpheres of comets may furniſh a ſupply of vapour great enough to fill ſo immenſe ſpaces, we may eaſily underſtand from the rarity of our own air; for the air near the ſurface of our earth poſſeſſes a ſpace 850 times greater than water of the ſame weight; and therefore a cylinder of air 850 feet high is of equal weight with a cylinder of water of the ſame breadth, and but one foot high. But a cylinder of air reaching to the top of the atmoſphere is of equal weight with a cylinder of water about 33 feet high: and, therefore, if from the whole cylinder of air the lower part of 850 feet high is taken away, the remaining upper part will be of equal weight with a cylinder of water 32 feet high: and from thence (and by the hypotheſis, confirmed by many experiments, that the compreſſion of air is as the weight of the incumbent atmoſphere, and that the force of gravity is reciprocally as the ſquare of the diſtance from the centre of the earth) raiſing a calculus, by cor. prop. 22, book 2, I found, that, at the height of one ſemi-diameter of the earth, reckoned from the earth's ſurface, the air is more rare than with us in a far greater proportion than of the whole

ſpace within the orb of Saturn to a ſpherical ſpace of one inch in diameter; and therefore if a ſphere of our air of but one inch in thickneſs was equally rarefied with the air at the height of one ſemi-diameter of the earth from the earth's ſurface, it would fill all the regions of the planets to the orb of Saturn, and far beyond it. Wherefore ſince the air at greater diſtances is immenſely rarefied, and the *coma* or atmoſphere of comets is ordinarily about ten times higher, reckoning from their centres, than the ſurface of the nucleus, and the tails riſe yet higher, they muſt therefore be exceedingly rare: and though, on account of the much thicker atmoſpheres of comets, and the great gravitation of their bodies towards the ſun, as well as of the particles of their air and vapours mutually one towards another, it may happen that the air in the celeſtial ſpaces and in the tails of comets is not ſo vaſtly rarefied, yet from this computation it is plain that a very ſmall quantity of air and vapour is abundantly ſufficient to produce all the appearances of the tails of comets; for that they are, indeed, of a very notable rarity appears from the ſhining of the ſtars through them. The atmoſphere of the earth, illuminated by the ſun's light, though but of a few miles in thickneſs, quite obſcures and extinguiſhes the light not only of all the ſtars, but even of the moon itſelf; whereas the ſmalleſt ſtars are ſeen to ſhine through the immenſe thickneſs of the tails of comets, likewiſe illuminated by the ſun, without the leaſt diminution of their ſplendor. Nor is the brightneſs of the tails of moſt comets ordinarily greater than that of our air, an inch or two in thickneſs, reflecting in a darkened room the light of the ſun-beams let in by a hole of the window-ſhutter.

And we may pretty nearly determine the time ſpent during the aſcent of the vapour from the comet's head to the extremity of the tail, by drawing a right line from the extremity of the tail to the ſun, and marking the place where that right line interſects the comet's orbit; for the vapour that is now in the extremity of the tail, if it has aſcended in a right line from the ſun, muſt have begun to riſe from the head at the time when the head was in the point of interſection. It is

true, the vapour does not riſe in a right line from the ſun, but, retaining the motion which it had from the comet before its aſcent, and compounding that motion with its motion of aſcent, ariſes obliquely; and, therefore, the ſolution of the problem will be more exact, if we draw the line which interſects the orbit parallel to the length of the tail; or rather (becauſe of the curvilinear motion of the comet) diverging a little from the line or length of the tail. And by means of this principle I found that the vapour which, *January* 25, was in the extremity of the tail, had begun to riſe from the head before *December* 11, and therefore had ſpent in its whole aſcent 45 days; but that the whole tail which appeared on *December* 10 had finiſhed its aſcent in the ſpace of the two days then elapſed from the time of the comet's being in its perihelion. The vapour, therefore, about the beginning and in the neighbourhood of the ſun roſe with the greateſt velocity, and afterwards continued to aſcend with a motion conſtantly retarded by its own gravity; and the higher it aſcended, the more it added to the length of the tail; and while the tail continued to be ſeen, it was made up of almoſt all that vapour which had riſen ſince the time of the comet's being in its perihelion; nor did that part of the vapour which had riſen firſt, and which formed the extremity of the tail, ceaſe to appear, till its too great diſtance, as well from the ſun, from which it received its light, as from our eyes, rendered it inviſible. Whence alſo it is that the tails of other comets which are ſhort do not riſe from their heads with a ſwift and continual motion, and ſoon after diſappear, but are permanent and laſting columns of vapours and exhalations, which, aſcending from the heads with a ſlow motion of many days, and partaking of the motion of the heads which they had from the beginning, continue to go along together with them through the heavens. From whence again we have another argument proving the celeſtial ſpaces to be free, and without reſiſtance, ſince in them not only the ſolid bodies of the planets and comets, but alſo the extremely rare vapours of comets' tails, maintain their rapid motions with great freedom, and for an exceeding long time.

Kepler aſcribes the aſcent of the tails of the comets to the atmoſpheres of their heads; and their direction towards the parts oppoſite to the ſun to the action of the rays of light carrying along with them the matter of the comets' tails; and without any great incongruity we may ſuppoſe, that, in ſo free ſpaces, ſo fine a matter as that of the æther may yield to the action of the rays of the ſun's light, though thoſe rays are not able ſenſibly to move the groſs ſubſtances in our parts, which are clogged with ſo palpable a reſiſtance. Another author thinks that there may be a ſort of particles of matter endowed with a principle of levity, as well as others are with a power of gravity; that the matter of the tails of comets may be of the former ſort, and that its aſcent from the ſun may be owing to its levity; but, conſidering that the gravity of terreſtrial bodies is as the matter of the bodies, and therefore can be neither more nor leſs in the ſame quantity of matter, I am inclined to believe that this aſcent may rather proceed from the rarefaction of the matter of the comets' tails. The aſcent of ſmoke in a chimney is owing to the impulſe of the air with which it is entangled. The air rarefied by heat aſcends, becauſe its ſpecific gravity is diminiſhed, and in its aſcent carries along with it the ſmoke with which it is engaged; and why may not the tail of a comet riſe from the ſun after the ſame manner? For the ſun's rays do not act upon the mediums which they pervade otherwiſe than by reflection and refraction; and thoſe reflecting particles heated by this action, heat the matter of the æther which is involved with them. That matter is rarefied by the heat which it acquires, and becauſe, by this rarefaction, the ſpecific gravity with which it tended towards the ſun before is diminiſhed, it will aſcend therefrom, and carry along with it the reflecting particles of which the tail of the comet is compoſed. But the aſcent of the vapours is further promoted by their circumgyration about the ſun, in conſequence whereof they endeavour to recede from the ſun, while the ſun's atmoſphere and the other matter of the heavens are either altogether quieſcent, or are only moved with a ſlower circumgyration derived from the rotation of the ſun. And theſe are the cauſes of the

aſcent of the tails of the comets in the neighbourhood of the ſun, where their orbits are bent into a greater curvature, and the comets themſelves are plunged into the denſer and therefore heavier parts of the ſun's atmoſphere: upon which account they do then emit tails of an huge length; for the tails which then ariſe, retaining their own proper motion, and in the mean time gravitating towards the ſun, muſt be revolved in ellipſes about the ſun in like manner as the heads are, and by that motion muſt always accompany the heads, and freely adhere to them. For the gravitation of the vapours towards the ſun can no more force the tails to abandon the heads, and deſcend to the ſun, than the gravitation of the heads can oblige them to fall from the tails. They muſt by their common gravity either fall together towards the ſun, or be retarded together in their common aſcent therefrom; and, therefore (whether from the cauſes already deſcribed, or from any others), the tails and heads of comets may eaſily acquire and freely retain any poſition one to the other, without diſturbance or impediment from that common gravitation.

The tails, therefore, that riſe in the perihelion poſitions of the comets will go along with their heads into far remote parts, and together with the heads will either return again from thence to us, after a long courſe of years, or rather will be there rarefied, and by degrees quite vaniſh away; for afterwards, in the deſcent of the heads towards the ſun, new ſhort tails will be emitted from the heads with a ſlow motion; and thoſe tails by degrees will be augmented immenſely, eſpecially in ſuch comets as in their perihelion diſtances deſcend as low as the ſun's atmoſphere; for all vapour in thoſe free ſpaces is in a perpetual ſtate of rarefaction and dilatation; and from hence it is that the tails of all comets are broader at their upper extremity than near their heads. And it is not unlikely but that the vapour, thus perpetually rarefied and dilated, may be at laſt diſſipated and ſcattered through the whole heavens, and by little and little be attracted towards the planets by its gravity, and mixed with their atmoſphere; for as the ſeas are abſolutely neceſſary to the conſtitution of our earth, that

from them, the ſun, by its heat, may exhale a ſufficient quantity of vapours, which, being gathered together into clouds, may drop down in rain, for watering of the earth, and for the production and nouriſhment of vegetables; or, being condenſed with cold on the tops of mountains (as ſome philoſophers with reaſon judge), may run down in ſprings and rivers; ſo for the conſervation of the ſeas, and fluids of the planets, comets ſeem to be required, that, from their exhalations and vapours condenſed, the waſtes of the planetary fluids ſpent upon vegetation and putrefaction, and converted into dry earth, may be continually ſupplied and made up; for all vegetables entirely derive their growths from fluids, and afterwards, in great meaſure, are turned into dry earth by putrefaction; and a ſort of ſlime is always found to ſettle at the bottom of putrefied fluids; and hence it is that the bulk of the ſolid earth is continually increaſed; and the fluids, if they are not ſupplied from without, muſt be in a continual decreaſe, and quite fail at laſt. I ſuſpect, moreover, that it is chiefly from the comets that ſpirit comes, which is indeed the ſmalleſt but the moſt ſubtle and uſeful part of our air, and ſo much required to ſuſtain the life of all things with us.

The atmoſpheres of comets, in their deſcent towards the ſun, by running out into the tails, are ſpent and diminiſhed, and become narrower, at leaſt on that ſide which regards the ſun; and in receding from the ſun, when they leſs run out into the tails, they are again enlarged, if *Hevelius* has juſtly marked their appearances. But they are ſeen leaſt of all juſt after they have been moſt heated by the ſun, and on that account then emit the longeſt and moſt reſplendent tails; and, perhaps, at the ſame time, the nuclei are environed with a denſer and blacker ſmoke in the lowermoſt parts of their atmoſphere; for ſmoke that is raiſed by a great and intenſe heat is commonly the denſer and blacker. Thus the head of that comet which we have been deſcribing, at equal diſtances both from the ſun and from the earth, appeared darker after it had paſſed by its perihelion than it did before; for in the month of *December* it was commonly compared with the ſtars of the third magnitude, but in *November* with thoſe of the

firſt or ſecond; and ſuch as ſaw both appearances have deſcribed the firſt as of another and greater comet than the ſecond. For, *November* 19, this comet appeared to a young man at *Cambridge*, though with a pale and dull light, yet equal to *Spica Virginis*; and at that time it ſhone with greater brightneſs than it did afterwards. And *Montenari*, *November* 20, ſt. vet. obſerved it larger than the ſtars of the firſt magnitude, its tail being then 2 degrees long. And Mr. *Storer* (by letters which have come into my hands) writes, that in the month of *December*, when the tail appeared of the greateſt bulk and ſplendor, the head was but ſmall, and far leſs than that which was ſeen in the month of *November* before ſunriſing; and, conjecturing at the cauſe of the appearance, he judged it to proceed from there being a greater quantity of matter in the head at firſt, which was afterwards gradually ſpent.

And, which farther makes for the ſame purpoſe, I find, that the heads of other comets, which did put forth tails of the greateſt bulk and ſplendor, have appeared but obſcure and ſmall. For in *Braſil*, *March* 5, 1668, 7^{h}. P. M., *St. N. P. Valentinus Eſtancius* ſaw a comet near the horizon, and towards the ſouth weſt, with a head ſo ſmall as ſcarcely to be diſcerned, but with a tail above meaſure ſplendid, ſo that the reflection thereof from the ſea was eaſily ſeen by thoſe who ſtood upon the ſhore; and it looked like a fiery beam extended 23° in length from the weſt to ſouth, almoſt parallel to the horizon. But this exceſſive ſplendor continued only three days, decreaſing apace afterwards; and while the ſplendor was decreaſing, the bulk of the tail increaſed: whence in *Portugal* it is ſaid to have taken up one quarter of the heavens, that is, 45 degrees, extending from weſt to eaſt with a very notable ſplendor, though the whole tail was not ſeen in thoſe parts, becauſe the head was always hid under the horizon: and from the increaſe of the bulk and decreaſe of the ſplendor of the tail, it appears that the head was then in its receſs from the ſun, and had been very near to it in its perihelion, as the comet of 1680 was. And we read, in the *Saxon* Chronicle, of a like comet appearing in the year 1106,

the ſtar whereof was ſmall and obſcure (as that of 1680), *but the ſplendour of its tail was very bright, and like a huge fiery beam ſtretched out in a direction between the eaſt and north,* as *Hevelius* has it alſo from *Simeon,* the monk of *Durham.* This comet appeared in the beginning of *February,* about the evening, and towards the ſouth weſt part of heaven; from whence, and from the poſition of the tail, we infer that the head was near the ſun. *Matthew Paris* ſays, *It was diſtant from the ſun by about a cubit, from three of the clock* (rather ſix) *till nine, putting forth a long tail.* Such alſo was that moſt reſplendent comet deſcribed by *Ariſtotle,* lib. 1. *Meteor.* 6. *The head whereof could not be ſeen, becauſe it had ſet before the ſun, or at leaſt was hid under the ſun's rays; but next day it was ſeen as well as might be; for, having left the ſun but a very little way, it ſet immediately after it. And the ſcattered light of the head, obſcured by the too great ſplendour* (of the tail) *did not yet appear. But afterwards* (as *Ariſtotle* ſays) *when the ſplendour* (of the tail) *was now diminiſhed* (the head of), *the comet recovered its native brightneſs; and the ſplendor* (of its tail) *reached now to a third part of the heavens* (that is, to 60°). *This appearance was in the winter ſeaſon* (an. 4, Olymp. 101), *and, riſing to* Orion's *girdle, it there vaniſhed away.* It is true that the comet of 1618, which came out directly from under the ſun's rays with a very large tail, ſeemed to equal, if not to exceed, the ſtars of the firſt magnitude; but, then, abundance of other comets have appeared yet greater than this, that put forth ſhorter tails; ſome of which are ſaid to have appeared as big as Jupiter, others as big as Venus, or even as the moon.

We have ſaid, that comets are a ſort of planets revolved in very eccentric orbits about the ſun; and as, in the planets which are without tails, thoſe are commonly leſs which are revolved in leſſer orbits, and nearer to the ſun, ſo in comets it is probable that thoſe which in their perihelion approach nearer to the ſun are generally of leſs magnitude, that they may not agitate the ſun too much by their attractions. But as to the tranſverſe diameters of their orbits, and the periodic times of their revolutions, I leave them to be determined by

comparing comets together which after long intervals of time return again in the ſame orbit. In the mean time, the following propoſition may give ſome light in that enquiry.

PROPOSITION XLII. PROBLEM XXII.

To correct a comet's trajectory found as above.

Operation 1. Aſſume that poſition of the plane of the trajectory which was determined according to the preceding propoſition; and ſelect three places of the comet, deduced from very accurate obſervations, and at great diſtances one from the other. Then ſuppoſe A to repreſent the time between the firſt obſervation and the ſecond, and B the time between the ſecond and the third; but it will be convenient that in one of thoſe times the comet be in its perigeon, or at leaſt not far from it. From thoſe apparent places find, by trigonometric operations, the three true places of the comet in that aſſumed plane of the trajectory; then through the places found, and about the centre of the ſun as the focus, deſcribe a conic ſection by arithmetical operations, according to prop. 21, book 1. Let the areas of this figure which are terminated by radii drawn from the ſun to the places found be D and E; to wit, D the area between the firſt obſervation and the ſecond, and E the area between the ſecond and third; and let T repreſent the whole time in which the whole area D + E ſhould be deſcribed with the velocity of the comet found by prop. 16, book 1.

Oper. 2. Retaining the inclination of the plane of the trajectory to the plane of the ecliptic, let the longitude of the nodes of the plane of the trajectory be increaſed by the addition of 20 or 30 minutes, which call P. Then from the aforeſaid three obſerved places of the comet let the three true places be found (as before) in this new plane; as alſo the orbit paſſing through thoſe places, and the two areas of the ſame deſcribed between the two obſervations, which call d and e; and let t be the whole time in which the whole area d + e ſhould be deſcribed.

Oper. 3. Retaining the longitude of the nodes in the firſt operation, let the inclination of the plane of the trajectory to the plane of the ecliptic be increaſed by adding thereto 20′ or

30′, which call Q. Then from the aforeſaid three obſerved apparent places of the comet let the three true places be found in this new plane, as well as the orbit paſſing through them, and the two areas of the ſame deſcribed between the obſervation, which call δ and ε; and let τ be the whole time in which the whole area $\delta + \varepsilon$ ſhould be deſcribed.

Then taking C to 1 as A to B; and G to 1 as D to E; and g to 1 as d to e; and γ to 1 as δ to ε; let S be the true time between the firſt obſervation and the third; and, obſerving well the ſigns + and —, let ſuch numbers m and n be found out as will make $2G - 2C$, $= mG - mg + nG - n\gamma$; and $2T - 2S = mT - mt + nT - n\tau$. And if, in the firſt operation, I repreſents the inclination of the plane of the trajectory to the plane of the ecliptic, and K the longitude of either node, then $I + nQ$ will be the true inclination of the plane of the trajectory to the plane of the ecliptic, and $K + mP$ the true longitude of the node. And, laſtly, if in the firſt, ſecond, and third operations, the quantities R, r, and ϱ, repreſent the parameters of the trajectory, and the quantities $\frac{1}{L}$, $\frac{1}{l}$, $\frac{1}{\lambda}$, the tranſverſe diameters of the ſame, then $R + mr - mR + n\varrho - nR$ will be the true parameter, and $\frac{1}{L + ml - mL + n\lambda - nL}$ will be the true tranſverſe diameter of the trajectory which the comet deſcribes; and from the tranſverſe diameter given the periodic time of the comet is alſo given. Q.E.I. But the periodic times of the revolutions of comets, and the tranſverſe diameters of their orbits, cannot be accurately enough determined but by comparing comets together which appear at different times. If, after equal intervals of time, ſeveral comets are found to have deſcribed the ſame orbit, we may thence conclude that they are all but one and the ſame comet revolved in the ſame orbit; and then from the times of their revolutions the tranſverſe diameters of their orbits will be given, and from thoſe diameters the elliptic orbits themſelves will be determined.

To this purpoſe the trajectories of many comets ought to be computed, ſuppoſing thoſe trajectories to be parabolic;

for such trajectories will always nearly agree with the *phænomena*, as appears not only from the parabolic trajectory of the comet of the year 1680, which I compared above with the observations, but likewise from that of the notable comet which appeared in the years 1664 and 1665, and was observed by *Hevelius*, who, from his own observations, calculated the longitudes and latitudes thereof, though with little accuracy. But from the same observations Dr. *Halley* did again compute its places; and from those new places determined its trajectory, finding its ascending node in ♈ 21° 13′ 55″; the inclination of the orbit to the plane of the ecliptic 21° 18′ 40″; the distance of its perihelion from the node, estimated in the comet's orbit, 49° 27′ 30″, its perihelion in ♌ 8° 40′ 30″, with heliocentric latitude south 16° 01′ 45″; the comet to have been in its perihelion *November* 24^{d}. 11^{h}. 52′ P. M. equal time at *London*, or 13^{h}. 8′ at *Dantzick*, O. S.; and that the *latus rectum* of the parabola was 410286 such parts as the sun's mean distance from the earth is supposed to contain 100000. And how nearly the places of the comet computed in this orbit agree with the observations, will appear from the annexed table, calculated by Dr. *Halley*.

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Apparent Time at Dantzick.	The obſerved Diſtances of the Comet from	The obſerved Places.	The Places computed in the Orb.
December.	The Lion's heart.....46°. 24′. 20″	Long. ♎ 7ᵈ. 01′. 00″	♎ 7°. 01′. 29″
3ᵈ. 18ʰ. 29′½	The Virgin's ſpike.....22 . 52 . 10	Lat. S. 21 . 39 . 00	21 . 38 . 50
4 . 18 . 01 ½	The Lion's heart.....46 . 02 . 45	Long. ♎ 6 . 15 . 00	♎ 6 . 16 . 05
	The Virgin's ſpike.....23 . 52 . 40	Lat. S. 22 . 24 . 00	22 . 24 . 00
7 . 17 . 48	The Lion's heart .....44 . 48 . 00	Long. ♎ 3 . 06 . 00	♎ 3 . 07 . 33
	The Virgin's ſpike.....27 . 56 . 40	Lat. S. 25 . 22 . 00	25 . 21 . 40
17 . 14 . 43	The Lion's heart.....53 . 15 . 15	Long. ♌ 2 . 56 . 00	♌ 2 . 56 . 00
	Orion's right ſhoulder .....45 . 43 . 30	Lat. S. 49 . 25 . 00	49 . 25 . 00
19 . 09 . 25	Procyon.....35 . 13 . 50	Long. ♊ 28 . 40 . 30	♊ 28 . 43 . 00
	The bright ſtar of the Whale's jaw..52 . 56 . 00	Lat. S. 45 . 48 . 00	45 . 46 . 00
20 . 09 . 53 ½	Procyon.....40 . 49 . 00	Long. ♊ 13 . 03 . 00	♊ 13 . 05 . 00
	The bright ſtar of the Whale's jaw..40 . 04 . 00	Lat. S. 39 . 54 . 00	39 . 53 . 00
21 . 09 . 09 ½	Orion's right ſhoulder.....26 . 21 . 25	Long. ♊ 2 . 16 . 00	♊ 2 . 18 . 30
	The bright ſtar of the Whale's jaw..29 . 28 . 00	Lat. S. 33 . 41 . 00	33 . 39 . 40
22 . 09 . 00	Orion's right ſhoulder.....29 . 47 . 00	Long. ♉ 24 . 24 . 00	♉ 24 . 27 . 00
	The bright ſtar of the Whale's jaw..20 . 29 . 30	Lat. S. 27 . 45 . 00	27 . 46 . 00
26 . 07 . 58	The bright ſtar of Aries.....23 . 20 . 00	Long. ♉ 9 . 00 . 00	♉ 9 . 02 . 28
	Aldebaran .....26 . 44 . 00	Lat. S. 12 . 36 . 00	12 . 34 . 13
27 . 06 . 45	The bright ſtar of Aries .....20 . 45 . 00	Long. ♉ 7 . 05 . 40	♉ 7 . 08 . 45
	Aldebaran .....28 . 10 . 00	Lat. S. 10 . 23 . 00	10 . 23 . 13
28 . 07 . 39	The bright ſtar of Aries.....18 . 29 . 00	Long. ♉ 5 . 24 . 45	♉ 5 . 27 . 52
	Palilicium.....29 . 37 . 00	Lat. S. 8 . 22 . 50	8 . 23 . 37
31 . 06 . 45	Andromeda's girdle.....30 . 48 . 10	Long. ♉ 2 . 07 . 40	♉ 2 . 08 . 20
	Palilicium.....32 . 53 . 30	Lat. S. 4 . 13 . 00	4 . 16 . 25
Jan. 1665.	Andromeda's girdle.....25 . 11 . 00	Long. ♈ 28 . 24 . 47	♈ 28 . 24 . 00
7 . 07 . 37 ½	Palilicium.....37 . 12 . 25	Lat. N. 0 . 54 . 00	0 . 53 . 00
13 . 07 . 00	Andromeda's head .....28 . 07 . 10	Long. ♈ 27 . 06 . 54	♈ 27 . 06 . 39
	Palilicium.....38 . 55 . 20	Lat. N. 3 . 06 . 50	3 . 07 . 40
24 . 7 . 29	Andromeda's girdle.....20 . 32 . 15	Long. ♈ 26 . 29 . 15	♈ 26 . 28 . 50
	Palilicium.. .....40 . 05 . 00	Lat. N. 5 . 25 . 50	5 . 26 . 00
Feb.		Long. ♈ 27 . 04 . 46	♈ 27 . 24 . 55
7 . 08 . 37		Lat. N. 7 . 03 . 29	7 . 03 . 15
22 . 08 . 46		Long. ♈ 28 . 29 . 46	♈ 28 . 29 . 58
		Lat. N. 8 . 12 . 56	8 . 10 . 25
March.		Long. ♈ 29 . 18 . 15	♈ 29 . 18 . 20
1 . 08 . 16		Lat. N. 8 . 36 . 26	8 . 36 . 12
7 . 08 . 37		Long. ♉ 0 . 02 . 48	♉ 0 . 2 . 42
		Lat. N. 8 . 56 . 30	8 . 56 . 56

In *February*, the beginning of the year 1665, the firſt ſtar of Aries, which I ſhall hereafter call γ, was in Υ 28° 30′ 15″, with 7° 8′ 58″ north lat.; the ſecond ſtar of Aries was in Υ 29° 17′ 18″, with 8° 28′ 16″ north lat.; and another ſtar of the ſeventh magnitude, which I call A, was in Υ 28° 24′ 45″, with 8° 28′ 33″ north lat. The comet *Feb.* 7^{d}. 7^{h}. 30′ at *Paris* (that is, *Feb.* 7^{d}. 8^{h}. 37′ at *Dantzick)* O. S. made a triangle with thoſe ſtars γ and A, which was right-angled in γ; and the diſtance of the comet from the ſtar γ was equal to the diſtance of the ſtars γ and A, that is, 1° 19′ 46″ of a great circle; and therefore in the parallel of the latitude of the ſtar γ it was 1° 20′ 26″. Therefore if from the longitude of the ſtar γ there be ſubducted the longitude 1° 20′ 26″, there will remain the longitude of the comet Υ 27° 9′ 49″. M. *Auzout*, from this obſervation of his, placed the comet in Υ 27° 0′, nearly; and, by the ſcheme in which Dr. *Hooke* delineated its motion, it was then in Υ 26° 59′ 24″. I place it in Υ 27° 4′ 46″, taking the middle between the two extremes.

From the ſame obſervation, M. *Auzout* made the latitude of the comet at that time 7° and 4′ or 5′ to the north; but he had done better to have made it 7° 3′ 29″, the difference of the latitudes of the comet and the ſtar γ being equal to the difference of the longitude of the ſtars γ and A.

February 22^{d}. 7^{h}. 30′ at *London*, that is, *February* 22^{d}. 8^{h}. 46′ at *Dantzick*, the diſtance of the comet from the ſtar A, according to Dr. *Hooke*'s obſervation, as was delineated by himſelf in a ſcheme, and alſo by the obſervations of M. *Auzout*, delineated in like manner by M. *Petit*, was a fifth part of the diſtance between the ſtar A and the firſt ſtar of Aries, or 15′ 57″; and the diſtance of the comet from a right line joining the ſtar A and the firſt of Aries was a fourth part of the ſame fifth part, that is, 4′; and therefore the comet was in Υ 28° 29′ 46″, with 8° 12′ 36″ north lat.

March 1, 7^{h}. 0′ at *London*, that is, *March* 1, 8^{h}. 16′ at *Dantzick*, the comet was obſerved near the ſecond ſtar in Aries, the diſtance between them being to the diſtance between the firſt and ſecond ſtars in Aries, that is, to 1° 33′, as 4 to 45

according to Dr. *Hooke*, or as 2 to 23 according to M. *Gottignies*. And, therefore, the diſtance of the comet from the ſecond ſtar in Aries was 8′ 10″ according to Dr. *Hooke*, or 8′ 5″ according to M. *Gottignies*; or, taking a mean between both, 8′ 10″. But, according to M. *Gottignies*, the comet had gone beyond the ſecond ſtar of Aries about a fourth or a fifth part of the ſpace that it commonly went over in a day, to wit, about 1′ 35″ (in which he agrees very well with M. *Auzout*); or, according to Dr. *Hooke*, not quite ſo much, as perhaps only 1′. Wherefore if to the longitude of the firſt ſtar in Aries we add 1′, and 8′ 10″ to its latitude, we ſhall have the longitude of the comet ♈ 29° 18′, with 8° 36′ 26″ north lat.

March 7, 7ʰ. 30′ at *Paris* (that is, *March* 7, 8ʰ. 37′ at *Dantzick*), from the obſervations of M. *Auzout*, the diſtance of the comet from the ſecond ſtar in Aries was equal to the diſtance of that ſtar from the ſtar A, that is, 52′ 29″; and the difference of the longitude of the comet and the ſecond ſtar in Aries was 45′ or 46′, or, taking a mean quantity, 45′ 30″; and therefore the comet was in ♉ 0° 2′ 48″. From the ſcheme of the obſervations of M. *Auzout*, conſtructed by M. *Petit*, *Hevelius* collected the latitude of the comet 8° 54′. But the engraver did not rightly trace the curvature of the comet's way toward the end of the motion; and *Hevelius*, in the ſcheme of M. *Auzout*'s obſervations which he conſtructed himſelf, corrected this irregular curvature, and ſo made the latitude of the comet 8° 55′ 30″. And, by farther correcting this irregularity, the latitude may become 8° 56′, or 8° 57′.

This comet was alſo ſeen *March* 9, and at that time its place muſt have been in ♉ 0° 18′, with 9° 3½′ north lat. nearly.

This comet appeared three months together, in which ſpace of time it travelled over almoſt ſix ſigns, and in one of the days thereof deſcribed almoſt 20 deg. Its courſe did very much deviate from a great circle, bending towards the north, and its motion towards the end from retrograde became direct; and, notwithſtanding its courſe was ſo uncommon, yet by the table it appears that the theory, from beginning to end, agrees with the obſervations no leſs accurately than the

theories of the planets uſually do with the obſervations of them; but we are to ſubduct about 2′ when the comet was ſwifteſt, which we may effect by taking off 12″ from the angle between the aſcending node and the perihelion, or by making that angle 49° 27′ 18″. The annual parallax of both theſe comets (this and the preceding) was very conſpicuous, and by its quantity demonſtrates the annual motion of the earth in the *orbis magnus*.

This theory is likewiſe confirmed by the motion of that comet, which in the year 1683 appeared retrograde, in an orbit whoſe plane contained almoſt a right angle with the plane of the ecliptic, and whoſe aſcending node (by the computation of Dr. *Halley*) was in ♏ 23° 23′; the inclination of its orbit to the ecliptic 83° 11′; its perihelion in ♊ 25° 29′ 30″; its perihelion diſtance from the ſun 56020 of ſuch parts as the radius of the *orbis magnus* contains 100000; and the time of its perihelion *July* 2ᵈ. 3ʰ. 50′. And the places thereof, computed by Dr. *Halley* in this orbit, are compared with the places of the ſame obſerved by Mr. *Flamſted*, in the following table.

1683 Eq. time.	Sun's place.	Comet's Long. comp	Lat. Nor. comput.	Comet's Long. obser.	Lat. Nor. observed.	Diff. Long.	Diff. Lat.
d. h.	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	′ ″	′ ″
July 13. 12. 55	☌ 1. 02. 30	♋ 13. 05. 42	29. 28. 13	♋ 13. 06. 42	29. 28. 20	+ 1. 00	+ 0. 07
15. 11. 15	2. 53. 12	11. 37. 48	29. 34. 00	11. 39. 43	29. 34. 50	+ 1. 55	+ 0. 50
17. 10. 20	4. 45. 45	10. 07. 06	29. 33. 30	10. 08. 40	29. 34. 06	+ 1. 34	+ 0. 30
23. 13. 40	10. 38. 21	5. 10. 27	28. 51. 42	5. 11. 30	28. 50. 28	+ 1. 03	— 1. 14
25. 14. 5	12. 35. 28	3. 27. 53	24. 24. 47	3. 27. 00	28. 23. 40	— 0. 53	— 1. 07
31. 9. 42	18. 09. 22	♊ 27. 55. 03	26. 22. 52	♊ 27. 54. 24	26. 22. 25	— 0. 39	— 0. 27
31. 14. 55	18. 21. 53	27. 41. 07	26. 16. 57	27. 41. 08	26. 14. 50	+ 0. 01	— 2. 07
Aug. 2. 14. 56	20. 17. 16	25. 29. 32	25. 16. 19	25. 28. 46	25. 17. 28	— 0. 46	+ 1. 09
4. 10. 49	22. 02. 50	23. 18. 20	24. 10. 49	23. 16. 55	24. 12. 19	— 1. 25	+ 1. 30
6. 10. 9	23. 56. 45	20. 42. 23	22. 47. 05	20. 40. 32	22. 49. 05	— 1. 51	+ 2. 00
9. 10. 26	26. 50. 52	16. 07. 57	20. 06. 37	16. 05. 55	20. 06. 10	— 2. 02	— 0. 27
15. 14. 1	♍ 2. 47. 13	3. 30. 48	11. 37. 33	3. 26. 18	11. 32. 01	— 4. 30	— 5. 32
16. 15. 10	3. 48. 02	0. 43. 07	9. 34. 16	0. 41. 55	9. 34. 13	— 1. 12	— 0. 03
18. 15. 44	5. 45. 33	♉ 24. 52. 53	5. 11. 15	♉ 24. 49. 05	5. 09. 11	— 3. 48	— 2. 04
			South		South		
22. 14. 44	9. 35. 49	11. 07. 14	5. 16. 58	11. 07. 12	5. 16. 58	— 0. 02	— 0. 03
23. 15. 52	10. 36. 48	7. 02. 18	8. 17. 09	7. 01. 17	8. 16. 41	— 1. 01	— 0. 28
26. 16. 2	13. 31. 10	♈ 24. 45. 31	16. 38. 00	♈ 24. 44. 00	16. 38. 20	— 1. 31	+ 0. 20

This theory is yet farther confirmed by the motion of that retrograde comet which appeared in the year 1682. The afcending node of this (by Dr. *Halley*'s computation) was in ♉ 21° 16′ 30″; the inclination of its orbit to the plane of the ecliptic 17° 56′ 00″; its perihelion in ♒ 2° 52′ 50″; its perihelion diftance from the fun 58328 parts, of which the radius of the *orbis-magnus* contains 100000; the equal time of the comet's being in its perihelion *Sept.* 4ᵈ. 7ʰ. 39′. And its places, collected from Mr. *Flamfted*'s obfervations, are compared with its places computed from our theory in the following table.

1682 App. time.		Sun's place.	Comet's Lon. comp.	Lat. Nor. comp.	Com. Long. observed.	Lat. Nor. observed.	Diff. Long.	Diff. Lat.
d	h	′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	′ ″	′ ″
Aug. 19.	16.38	♍ 7.00.07	♌ 18.14.28	25.50.07	♌ 18.14.40	25.49.55	—0.12	+ 0.12
20.	15.38	7.55.52	24.46.23	26.14.42	24.46.22	26.12.52	+ 0.01	+ 1.50
21.	08.21	8.36.14	29.37.15	26.20.03	29.38.02	26.17.37	—0.47	+ 2.26
22.	08.08	9.33.55	♍ 6.29.53	26.08.42	♍ 6.30.03	26.07.12	—0.10	+ 1.30
29.	08.20	16.22.40	♎ 12.37.54	18.37.47	♎ 12.37.49	18.34.05	+ 0.05	+ 3.42
30.	07.45	17.19.41	15.36.01	17.26.43	15.35.18	17.27.17	+ 0.43	—0.34
Sept. 1.	07.33	19.16.09	20.30.53	15.13.00	20.27.04	15.09.49	+ 3.49	+ 3.11
4.	07.22	22.11.28	25 42.00	12.23 48	25.40.58	12.22.00	+ 1.02	+ 1.48
5.	07.32	23.10.29	27.00.46	11.33.08	26.59 24	11.33.51	+ 1.22	—0.43
8.	07.16	26.05 58	29.58.44	9.26.46	29.58.45	9.26.43	—0.01	+ 0.03
9.	07.26	27.05.09	♏ 0.44.10	8.49.10	♏ 0.44.04	8 48.25	+ 0.06	+ 0.45

This theory is alſo confirmed by the retrograde motion of the comet that appeared in the year 1723. The aſcending node of this comet (according to the computation of Mr. *Bradley,* Savilian Profeſſor of Aſtronomy at *Oxford)* was in ♈ 14° 16′. The inclination of the orbit to the plane of the ecliptic 49° 59′. Its perihelion was in ♉ 12° 15′ 20″. Its perihelion diſtance from the ſun 998651 parts, of which the radius of the *orbis magnus* contains 1000000, and the equal time of its perihelion *September* 16ᵈ. 16ʰ. 10′. The places of this comet computed in this orbit by Mr. *Bradley,* and compared with the places obſerved by himſelf, his uncle Mr.

Pound, and Dr. *Halley,* may be seen in the following table.

1723. Eq. time.	Comet's Long. obs.	Lat. Nor. obs.	Comet's Lon. com.	Lat. Nor. comp.	Diff. Lon.	Diff. Lat.
d h ′	° ′ ″	° ′ ″	° ′ ″	° ′ ″	″	″
Oct. 9.8. 5	♒7.22.15	5. 2. 0	♒7.21.26	5. 2.47	+49	—47
10.6.21	6.41.12	7.44.13	6.41.42	7.43.18	—50	+55
12.7.22	5.39.58	11.55. 0	5.40.19	11.54.55	—21	+ 5
14.8.57	4.59.49	14.43.50	5. 0.37	14.44. 1	—48	—11
15.6.35	4.47.41	15.40.51	4.47.45	15.40.55	— 4	— 4
21.6.22	4. 2.32	19.41.49	4. 2.21	19.42. 3	+11	—14
22.6.24	3.59. 2	20. 8.12	3.59.10	20. 8.17	— 8	— 5
24.8. 2	3.55.29	20.55.18	3.55.11	20.55. 9	+18	+ 9
29.8.56	3.56.17	22.20.27	3.56.42	22.20.10	—25	+17
30.6.20	3.58. 9	22.32.28	3.58.17	22.32.12	— 8	+16
Nov. 5.5.53	4.16.30	23.38.33	4.16.23	23.38. 7	+ 7	+26
8.7. 6	4.29.36	24. 4.30	4.29.54	24. 4.40	—18	—10
14.6.20	5. 2.16	24.48.46	5. 2.51	24.48.16	—35	+30
20.7.45	5.42.20	25.24.45	5.43.13	25.25.17	—53	—32
Dec. 7.6.45	8. 4.13	26.54.18	8. 3.55	26.53.42	+18	+36

From these examples it is abundantly evident that the motions of comets are no less accurately represented by our theory than the motions of the planets commonly are by the theories of them; and, therefore, by means of this theory, we may enumerate the orbits of comets, and so discover the periodic time of a comet's revolution in any orbit; whence, at last, we shall have the transverse diameters of their elliptic orbits and their aphelion distances.

That retrograde comet which appeared in the year 1607 described an orbit whose ascending node (according to Dr. *Halley*'s computation) was in ♉ 20° 21′; and the inclination of the plane of the orbit to the plane of the ecliptic 17° 2′; whose perihelion was in ♒ 2° 16′; and its perihelion distance from the sun 58680 of such parts as the radius of the *orbis magnus* contains 100000; and the comet was in its perihelion *October* 16ᵈ. 3ʰ. 50′; which orbit agrees very nearly with the orbit of the comet which was seen in 1682. If these were not two different comets, but one and the same, that comet will finish one revolution in the space of 75 years; and the greater axis of its orbit will be to the greater axis of the *orbis*

magnus as $\sqrt[3]{75 \times 75}$ to 1, or as 1778 to 100, nearly. And the aphelion diſtance of this comet from the ſun will be to the mean diſtance of the earth from the ſun as about 35 to 1; from which data it will be no hard matter to determine the elliptic orbit of this comet. But theſe things are to be ſuppoſed on condition, that, after the ſpace of 75 years, the ſame comet ſhall return again in the ſame orbit. The other comets ſeem to aſcend to greater heights, and to require a longer time to perform their revolutions.

But, becauſe of the great number of comets, of the great diſtance of their aphelions from the ſun, and of the ſlowneſs of their motions in the aphelions, they will, by their mutual gravitations, diſturb each other; ſo that their eccentricities and the times of their revolutions will be ſometimes a little increaſed, and ſometimes diminiſhed. Therefore we are not to expect that the ſame comet will return exactly in the ſame orbit, and in the ſame periodic times: it will be ſufficient if we find the changes no greater than may ariſe from the cauſes juſt ſpoken of.

And hence a reaſon may be aſſigned why comets are not comprehended within the limits of a zodiac, as the planets are; but, being confined to no bounds, are with various motions diſperſed all over the heavens; namely, to this purpoſe, that in their aphelions, where their motions are exceedingly ſlow, receding to greater diſtances one from another, they may ſuffer leſs diſturbance from their mutual gravitations: and hence it is that the comets which deſcend the loweſt, and therefore move the ſloweſt in their aphelions, ought alſo to aſcend the higheſt.

The comet which appeared in the year 1680 was in its perihelion leſs diſtant from the ſun than by a ſixth part of the ſun's diameter; and becauſe of its extreme velocity in that proximity to the ſun, and ſome denſity of the ſun's atmoſphere, it muſt have ſuffered ſome reſiſtance and retardation; and therefore, being attracted ſomething nearer to the ſun in every revolution, will at laſt fall down upon the body of the ſun. Nay, in its aphelion, where it moves the ſloweſt, it may ſometimes happen to be yet farther retarded by the attractions of

other comets, and in conſequence of this retardation deſcend to the ſun. So fixed ſtars, that have been gradually waſted by the light and vapours emitted from them for a long time, may be recruited by comets that fall upon them; and from this freſh ſupply of new fuel thoſe old ſtars, acquiring new ſplendor, may paſs for new ſtars. Of this kind are ſuch fixed ſtars as appear on a ſudden, and ſhine with a wonderful brightneſs at firſt, and afterwards vaniſh by little and little. Such was that ſtar which appeared in *Caſſiopeia*'s chair; which *Cornelius Gemma* did not ſee upon the 8th of *November*, 1572, though he was obſerving that part of the heavens upon that very night, and the ſky was perfectly ſerene; but the next night (*November* 9) he ſaw it ſhining much brighter than any of the fixed ſtars, and ſcarcely inferior to *Venus* in ſplendor. *Tycho Brahe* ſaw it upon the 11th of the ſame month, when it ſhone with the greateſt luſtre; and from that time he obſerved it to decay by little and little; and in 16 months' time it entirely diſappeared. In the month of *November*, when it firſt appeared, its light was equal to that of *Venus*. In the month of *December* its light was a little diminiſhed, and was now become equal to that of *Jupiter*. In *January* 1573 it was leſs than *Jupiter*, and greater than *Sirius*; and about the end of *February* and the beginning of *March* became equal to that ſtar. In the months of *April* and *May* it was equal to a ſtar of the ſecond magnitude; in *June*, *July*, and *Auguſt*, to a ſtar of the third magnitude; in *September*, *October*, and *November*, to thoſe of the fourth magnitude; in *December* and *January* 1574 to thoſe of the fifth; in *February* to thoſe of the ſixth magnitude; and in *March* it entirely vaniſhed. Its colour at the beginning was clear, bright, and inclining to white; afterwards it turned a little yellow; and in *March* 1573 it became ruddy, like *Mars* or *Aldebaran*: in *May* it turned to a kind of duſky whiteneſs, like that we obſerve in *Saturn*; and that colour it retained ever after, but growing always more and more obſcure. Such alſo was the ſtar in the right foot of *Serpentarius*, which *Kepler*'s ſcholars firſt obſerved *September* 30, O.S. 1604, with a light exceeding that of *Jupiter*

though the night before it was not to be ſeen; and from that time it decreaſed by little and little, and in 15 or 16 months entirely diſappeared. Such a new ſtar appearing with an unuſal ſplendor is ſaid to have moved *Hipparchus* to obſerve, and make a catalogue of, the fixed ſtars. As to thoſe fixed ſtars that appear and diſappear by turns, and increaſe ſlowly and by degrees, and ſcarcely ever exceed the ſtars of the third magnitude, they ſeem to be of another kind, which revolve about their axes, and, having a light and a dark ſide, ſhew thoſe two different ſides by turns. The vapours which ariſe from the ſun, the fixed ſtars, and the tails of the comets, may meet at laſt with, and fall into, the atmoſpheres of the planets by their gravity, and there be condenſed and turned into water and humid ſpirits; and from thence, by a ſlow heat, paſs gradually into the form of ſalts, and ſulphurs, and tinctures, and mud, and clay, and ſand, and ſtones, and coral, and other terreſtrial ſubſtances.

GENERAL SCHOLIUM.

The hypotheſis of vortices is preſſed with many difficulties. That every planet by a radius drawn to the ſun may deſcribe areas proportional to the times of deſcription, the periodic times of the ſeveral parts of the vortices ſhould obſerve the duplicate proportion of their diſtances from the ſun; but that the periodic times of the planets may obtain the ſeſquiplicate proportion of their diſtances from the ſun, the periodic times of the parts of the vortex ought to be in the ſeſquiplicate proportion of their diſtances. That the ſmaller vortices may maintain their leſſer revolutions about *Saturn*, *Jupiter*, and other planets, and ſwim quietly and undiſturbed in the greater vortex of the ſun, the periodic times of the parts of the ſun's vortex ſhould be equal; but the rotation of the ſun and planets about their axes, which ought to correſpond with the motions of their vortices, recede far from all theſe proportions. The motions of the comets are exceedingly regular, are governed by the ſame laws with the motions of the planets, and can by no means be accounted for by the hypotheſis of vortices; for comets are carried with very eccentric motions

through all parts of the heavens indifferently, with a freedom that is incompatible with the notion of a vortex.

Bodies projected in our air ſuffer no reſiſtance but from the air. Withdraw the air, as is done in Mr. *Boyle*'s vacuum, and the reſiſtance ceaſes; for in this void a bit of fine down and a piece of ſolid gold deſcend with equal velocity. And the parity of reaſon muſt take place in the celeſtial ſpaces above the earth's atmoſphere; in which ſpaces, where there is no air to reſiſt their motions, all bodies will move with the greateſt freedom; and the planets and comets will conſtantly purſue their revolutions in orbits given in kind and poſition, according to the laws above explained; but though theſe bodies may, indeed, perſevere in their orbits by the mere laws of gravity, yet they could by no means have at firſt derived the regular poſition of the orbits themſelves from thoſe laws.

The ſix primary planets are revolved about the ſun in circles concentric with the ſun, and with motions directed towards the ſame parts, and almoſt in the ſame plane. Ten moons are revolved about the earth, Jupiter and Saturn, in circles concentric with them, with the ſame direction of motion, and nearly in the planes of the orbits of thoſe planets: but it is not to be conceived that mere mechanical cauſes could give birth to ſo many regular motions, ſince the comets range over all parts of the heavens in very eccentric orbits; for by that kind of motion they paſs eaſily through the orbs of the planets, and with great rapidity; and in their aphelions, where they move the ſloweſt, and are detained the longeſt, they recede to the greateſt diſtances from each other, and thence ſuffer the leaſt diſturbance from their mutual attractions. This moſt beautiful ſyſtem of the ſun, planets, and comets, could only proceed from the counſel and dominion of an intelligent and powerful Being. And if the fixed ſtars are the centres of other like ſyſtems, theſe, being formed by the like wiſe counſel, muſt be all ſubject to the dominion of One; eſpecially ſince the light of the fixed ſtars is of the ſame nature with the light of the ſun, and from every ſyſtem light paſſes into all the other ſyſtems: and leſt the ſyſtems of the fixed ſtars ſhould, by their gravity, fall on each other

mutually, he hath placed thoſe ſyſtems at immenſe diſtances one from another.

This Being governs all things, not as the ſoul of the world, but as Lord over all; and on account of his dominion he is wont to be called *Lord God* παντοκράτωρ, or *Univerſal Ruler;* for *God* is a relative word, and has a reſpect to ſervants; and *Deity* is the dominion of God not over his own body, as thoſe imagine who fancy God to be the ſoul of the world, but over ſervants. The Supreme God is a Being eternal, infinite, abſolutely perfect; but a being, however perfect, without dominion, cannot be ſaid to be Lord God; for we ſay, my God, your God, the God of *Iſrael,* the God of Gods, and Lord of Lords; but we do not ſay, my Eternal, your Eternal, the Eternal of *Iſrael,* the Eternal of gods; we do not ſay, my Infinite, or my Perfect: theſe are titles which have no reſpect to ſervants. The word *God** uſually ſignifies *Lord;* but every lord is not a God. It is the dominion of a ſpiritual being which conſtitutes a God: a true, ſupreme, or imaginary dominion makes a true, ſupreme, or imaginary God. And from his true dominion it follows that the true God is a living, intelligent, and powerful Being; and, from his other perfections, that he is ſupreme, or moſt perfect. He is eternal and infinite, omnipotent and omniſcient; that is, his duration reaches from eternity to eternity; his preſence from infinity to infinity; he governs all things, and knows all things that are or can be done. He is not eternity or infinity, but eternal and infinite; he is not duration or ſpace, but he endures and is preſent. He endures for ever, and is every where preſent; and by exiſting always and every where, he conſtitutes duration and ſpace. Since every particle of ſpace is *always,* and every indiviſible moment of duration is *every where,* certainly the Maker and Lord of all things cannot be

* Dr. *Pocock* derives the Latin word *Deus* from the *Arabic du* (in the oblique caſe *di*), which ſignifies *Lord*. And in this ſenſe princes are called *gods*, *Pſal.* lxxxii. ver. 6; and *John* x. ver. 35. And *Moſes* is called a *god* to his brother *Aaron*, and a *god* to *Pharaoh* (*Exod.* iv. ver. 16; and vii. ver. 8). And in the ſame ſenſe the ſouls of dead princes were formerly, by the Heathens, called *gods*, but falſely, becauſe of their want of dominion.

never and *no where*. Every ſoul that has perception is, though in different times and in different organs of ſenſe and motion, ſtill the ſame indiviſible perſon. There are given ſucceſſive parts in duration, co-exiſtent parts in ſpace, but neither the one nor the other in the perſon of a man, or his thinking principle; and much leſs can they be found in the thinking ſubſtance of God. Every man, ſo far as he is a thing that has perception, is one and the ſame man during his whole life, in all and each of his organs of ſenſe. God is the ſame God, always and every where. He is omnipreſent not *virtually* only, but alſo *ſubſtantially*; for virtue cannot ſubſiſt without ſubſtance. In him * are all things contained and moved; yet neither affects the other: God ſuffers nothing from the motion of bodies; bodies find no reſiſtance from the omnipreſence of God. It is allowed by all that the Supreme God exiſts neceſſarily; and by the ſame neceſſity he exiſts *always* and *every where*. Whence alſo he is all ſimilar, all eye, all ear, all brain, all arm, all power to perceive, to underſtand, and to act; but in a manner not at all human, in a manner not at all corporeal, in a manner utterly unknown to us. As a blind man has no idea of colours, ſo have we no idea of the manner by which the all-wiſe God perceives and underſtands all things. He is utterly void of all body and bodily figure, and can therefore neither be ſeen, nor heard, nor touched; nor ought he to be worſhipped under the repreſentation of any corporeal thing. We have ideas of his attributes, but what the real ſubſtance of any thing is we know not. In bodies, we ſee only their figures and colours, we hear only the ſounds, we touch only their outward ſurfaces, we ſmell only the ſmells, and taſte the ſavours; but their in-

* This was the opinion of the Antients. So *Pythagoras*, in *Cicer. de Nat. Deor.* lib. i. *Thales Anaxagoras*, *Virgil*, Georg. lib. iv. ver. 220; and Æneid, lib. vi. ver. 721. *Philo Allegor.* at the beginning of lib. i. *Aratus*, in his Phænom. at the beginning. So alſo the ſacred writers; as St. *Paul*, *Acts*, xvii. ver. 27, 28. St. *John*'s Goſp. chap. xiv. ver. 2. *Moſes*, in *Deut.* iv. ver. 39; and x. ver. 14. *David*, *Pſal.* cxxxix. ver. 7, 8, 9. *Solomon*, 1 *Kings*, viii. ver. 27. *Job* xxii. ver. 12, 13, 14. *Jeremiah* xxiii. ver. 23, 24. The Idolaters ſuppoſed the ſun, moon, and ſtars, the ſouls of men, and other parts of the world, to be parts of the Supreme God, and therefore to be worſhipped; but erroneouſly.

ward ſubſtances are not to be known either by our ſenſes, or by any reflex act of our minds: much leſs, then, have we any idea of the ſubſtance of God. We know him only by his moſt wiſe and excellent contrivances of things, and final cauſes; we admire him for his perfections; but we reverence and adore him on account of his dominion: for we adore him as his ſervants; and a god without dominion, providence, and final cauſes, is nothing elſe but Fate and Nature. Blind metaphyſical neceſſity, which is certainly the ſame always and every where, could produce no variety of things. All that diverſity of natural things which we find ſuited to different times and places could ariſe from nothing but the ideas and will of a Being neceſſarily exiſting. But, by way of allegory, God is ſaid to ſee, to ſpeak, to laugh, to love, to hate, to deſire, to give, to receive, to rejoice, to be angry, to fight, to frame, to work, to build; for all our notions of God are taken from the ways of mankind by a certain ſimilitude, which, though not perfect, has ſome likeneſs, however. And thus much concerning God; to diſcourſe of whom from the appearances of things does certainly belong to Natural Philoſophy.

Hitherto we have explained the phænomena of the heavens and of our ſea by the power of gravity, but have not yet aſſigned the cauſe of this power. This is certain, that it muſt proceed from a cauſe that penetrates to the very centres of the ſun and planets, without ſuffering the leaſt diminution of its force; that operates not according to the quantity of the ſurfaces of the particles upon which it acts (as mechanical cauſes uſe to do), but according to the quantity of the ſolid matter which they contain, and propagates its virtue on all ſides to immenſe diſtances, decreaſing always in the duplicate proportion of the diſtances. Gravitation towards the ſun is made up out of the gravitations towards the ſeveral particles of which the body of the ſun is compoſed; and in receding from the ſun decreaſes accurately in the duplicate proportion of the diſtances as far as the orb of Saturn, as evidently appears from the quieſcence of the aphelions of the planets; nay, and even to the remoteſt aphelions of the co-

mets, if thoſe aphelions are alſo quieſcent. But hitherto I have not been able to diſcover the cauſe of thoſe properties of gravity from phænomena, and I frame no hypotheſes; for whatever is not deduced from the phænomena is to be called an hypotheſis; and hypotheſes, whether metaphyſical or phyſical, whether of occult qualities or mechanical, have no place in experimental philoſophy. In this philoſophy particular propoſitions are inferred from the phænomena, and afterwards rendered general by induction. Thus it was that the impenetrability, the mobility, and the impulſive force of bodies, and the laws of motion and of gravitation, were diſcovered. And to us it is enough that gravity does really exiſt, and act according to the laws which we have explained, and abundantly ſerves to account for all the motions of the celeſtial bodies, and of our ſea.

And now we might add ſomething concerning a certain moſt ſubtle Spirit which pervades and lies hid in all groſs bodies; by the force and action of which Spirit the particles of bodies mutually attract one another at near diſtances, and cohere, if contiguous; and electric bodies operate to greater diſtances, as well repelling as attracting the neighbouring corpuſcles; and light is emitted, reflected, refracted, inflected, and heats bodies; and all ſenſation is excited, and the members of animal bodies move at the command of the will, namely, by the vibrations of this Spirit, mutually propagated along the ſolid filaments of the nerves, from the outward organs of ſenſe to the brain, and from the brain into the muſcles. But theſe are things that cannot be explained in few words, nor are we furniſhed with that ſufficiency of experiments which is required to an accurate determination and demonſtration of the laws by which this electric and elaſtic Spirit operates.

End of the Mathematical Principles.

APPENDIX.

Among the explications (given by a friend) of ſome propoſitions in this work not demonſtrated by the author, the editor finding the following, has thought it proper to annex them. Thus,

To cor. 2, prop. 91, book 1, page 199.

TO find the force whereby a ſphere (AdBg) on the diameter AB, attracts the body P. (Pl. 19, Fig. 1.)

Let SA = SB = r, PS = d, PE = x, PB = a = d + r, PA = α = d — r: therefore aα = dd — rr; alſo a + α = 2d, a — α = 2r; therefore aa — $\alpha\alpha$ = 4dr; and SE = d — x, AE = x — α, BE = a — x.

Now the force whereby the circle, whoſe radius is Ed, attracts the body P, is as $1 - \frac{PE}{Pd}$ (by cor. 1, prop. 90.).

And $\overline{Ed}^2$ = (AE × EB = $\overline{SA}^2 - \overline{SE}^2$ = rr — dd + 2dx — xx =) — aα + 2dx — xx. Alſo $\overline{Pd}^2$ = ($\overline{Ed}^2$ + $\overline{EP}^2$ = 2dx — aα — xx + xx =) 2dx — aα: therefore $\frac{PE}{Pd}$ = $\frac{x}{\sqrt{-a\alpha + 2dx}}$. Therefore $1 - \frac{x}{\sqrt{-a\alpha + 2dx}}$, or $\dot{x} - \frac{\dot{x}x}{\sqrt{-a\alpha + 2dx}}$ is the fluxion of the attractive force of the ſphere on the body P, or the ordinate of a curve whoſe area repreſents that force.

But the fluent of $\dot{x}$ is x; and the fluent of $\frac{\dot{x}x}{\sqrt{-a\alpha + 2dx}}$ is $\frac{a\alpha + dx}{3dd}\sqrt{-a\alpha + 2dx}$ (by *Tab.* 1, *Form* 4, *Caſ.* 2, *Quadr. of Curv.).*

Therefore $x - \frac{a\alpha + dx}{3dd}\sqrt{-a\alpha + 2dx}$ is the general expreſſion of the area of the curve.

Now let $x = a$, then area $= (a - \frac{a\alpha + da}{3dd}\sqrt{-a\alpha + 2da}$ $=) \frac{d^3 + r^3}{3dd} = A.$

Alſo let $x = \alpha$, then area $= (\alpha - \frac{a\alpha + d\alpha}{3dd}\sqrt{-a\alpha + 2d\alpha}$ $=) \frac{d^3 - r^3}{3dd} = B.$

And the force whereby the ſphere attracts the body P is as $(A - B$, or as $\frac{2r^3}{3d^2} =) \frac{2\overline{SA}^3}{3\overline{PS}^2}$.

2. The force whereby the ſpheroid ADBG attracts the body P, may, in the ſame manner, be found thus. Let $SC = c$.

The force of a circle whoſe radius is ED to attract P is as $1 - \frac{PE}{PD}$ (by cor. 1, prop. 90). Now $\overline{ED}^2 = \frac{\overline{SC}^2}{\overline{SA}^2} \times AEB =$ $\frac{cc}{rr} \times \overline{-a\alpha + 2dx - xx}$ (by the conics); and $\overline{PD}^2 = (\overline{ER}^2$ $= \overline{ED}^2 + \overline{EP}^2 = \frac{-a\alpha cc + 2dccx - ccxx}{rr} + xx =)$ $\frac{-a\alpha cc + 2dccx + \overline{rr - cc} \times x2}{rr}$. Therefore $(1 - \frac{PE}{PD} = 1 - \frac{x}{\sqrt{-\frac{a\alpha cc}{rr} + \frac{2dcc}{rr}x + \frac{rr - cc}{rr}xx}}$, or) $\dot{x}$ $- \frac{\dot{x}x}{\sqrt{-\frac{a\alpha cc}{rr} + \frac{2dcc}{rr}x + \frac{rr - cc}{rr}xx}}$ is the fluxion of the attractive force of the ſpheroid on the body P, or the ordinate of a curve whoſe area is the meaſure of that force.

Now the fluent of $\dot{x}$ is x; and (by *Caſ.* 2, *Form* 8, *Tab.* 2, *Quad. Cur.*) the fluent of $\frac{xx}{\sqrt{-\frac{a\alpha cc}{rr}+\frac{2dcc}{rr}x+\frac{rr-cc}{rr}xx}}$ is $\left(\frac{-\frac{8dcc}{rr}s+\frac{4dcc}{rr}xv-\frac{4a\alpha cc}{rr}v}{-\frac{4a\alpha cc}{rr}\times\frac{rr-cc}{rr}-\frac{4ddc^4}{r^4}}=\frac{-2drrs+drrxv-a\alpha rrv}{a\alpha\times cc-rr-ddcc}\right.$

$=\frac{-2ds+dxv-a\alpha v}{-cc-dd+rr}=)\ \frac{2ds-dxv+a\alpha v}{cc+dd-rr}$. Therefore $x+\frac{dxv-a\alpha v-2ds}{cc+dd-rr}$ is the general expreſſion for the area of the curve.

But $v=PD=ER=\sqrt{-\frac{a\alpha cc}{rr}+\frac{2dcc}{rr}x-\frac{cc-rr}{rr}xx}$ is an ordinate to a conic ſection whoſe abſciſſa is x; and s, σ, the areas NMB, NKA, adjacent to the ordinates BM, AK: put $D=s-\sigma$.

Let $x=a$, or $PE=PB=BM$; then $v=a$, or $PD=PB=BM$, and the area $=a+\frac{daa-\alpha aa-2ds}{cc+dd-rr}=A$. And let $x=\alpha$, or $PE=PA=AK$; then $v=\alpha$, or $PD=PA=AK$, and the area $=\alpha+\frac{d\alpha\alpha-a\alpha\alpha-2d\sigma}{cc+dd-rr}=B$.

And the attractive force of the ſpheroid on P is as $(A-B=a-\alpha+\frac{d\times\overline{aa-\alpha\alpha}-a\alpha\times\overline{a-\alpha}-2d\times\overline{s-\sigma}}{cc+dd-rr}$

$=2r+\frac{2ddr+2r^3-2dD}{cc+dd-rr}=)\ \frac{2rcc+2d\times\overline{2dr-D}}{cc+dd-rr}$.

But $2d=(a+\alpha=)$ BM + AK, therefore $2dr=$ trapezium ABMK; and $D=(s-\sigma=)$ area AKRMB; therefore $D-2dr=$ mixtilinear area KRMLK $=C$; conſequently $2dr-D=-C$; therefore $2d\times\overline{2dr-D}=-2dC$; therefore the attractive force of the ſpheroid on P is as $\frac{2rcc-2dC}{cc+dd-rr}=\frac{2AS\times\overline{SC}^3-2PS\times KRMK}{\overline{SC}^3+\overline{PS}^3-\overline{AS}^3}$.

Conſequently the attractive force of the ſpheroid upon the

body P will be to the attractive force of a ſphere whoſe diameter is AB upon the ſame body P as $\frac{rcc - dC}{cc + dd - rr}$ to $\frac{r^3}{3dd}$, or as $\frac{AS \times \overline{SC}^2 - PS \times KRMK}{\overline{SC}^2 + \overline{PS}^2 - \overline{AS}^2}$ to $\frac{\overline{AS}^3}{3\overline{PS}^2}$.

To Schol. prop. 34, *book* 2, *p.* 94.

For let it be propoſed to find the vertex of the cone, a fruſtum of which has the deſcribed property.

Let CFGB be the fruſtum, and S the vertex required. (Pl. 19, Fig. 2.)

Now conceive the medium to conſiſt of particles which ſtrike the ſurface of a body (moving in it) in a direction oppoſite to that of the motion; then the reſiſtance will be the force which is made up of the efficacy of the forces of all the ſtrokes.

In any line Pp, parallel to the axis of the cone, and meeting its ſurface in p, take pm of a given length, for the ſpace deſcribed by each point of the cone in a given time: draw mq perpendicular to the ſide (CF) of the cone, and qn perpendicular to pm.

Therefore the line pm will repreſent the velocity, or force, with which a particle of the medium ſtrikes the ſurface of the cone obliquely in p.

But the force mp is equivalent to two forces, the one (mq) perpendicular, the other (pq) parallel to the ſide of the cone; which laſt is therefore of no effect.

And the perpendicular force mq is equivalent to two forces, the one (mn) parallel to the axis of the cone, the other (qn) perpendicular to it; which alſo is deſtroyed by the contrary action of another particle on the oppoſite ſide of the cone.

There remains only the force mn, which has any effect in reſiſting or moving the cone in the direction of its axis.

Therefore the whole force of a ſingle particle, or the effect of the perpendicular ſtroke of a particle, upon the baſe of a circumſcribing cylinder, is to the effect of the oblique ſtroke

upon the surface of the cone (in p) as mp to mn, or as $\overline{mp}^2$ to $(mp \times mn =) \overline{mq}^2$, or as $\overline{CF}^2$ to $\overline{CH}^2$.

Now the number of particles striking in a parallel direction on any surface is as the area of a plane figure perpendicular to that direction, and that would just receive those strokes.

Therefore the number of particles striking against the frustum, that is, against the surfaces described by the rotation of FD and CF, each particle with the forces mp and mn respectively, is as the circle described by (FD or) OH, and the annulus described by CH, that is, as $\overline{OH}^2$ to $\overline{CO}^2 - \overline{OH}^2$.

But the whole force of the medium in resisting is the sum of the forces of the several particles.

Therefore the resistance of the medium, or the whole efficacy of the force of all the strokes against the end FG of the frustum, is to the resistance against the convex surface thereof as $(mp \times \overline{OH}^2$ to $mn \times \overline{CO}^2 - \overline{OH}^2$, or as $\overline{CF}^2 \times \overline{OH}^2$ to $\overline{CH}^2 \times \overline{CO}^2 - \overline{OH}^2$, or as) $\overline{OH}^2$ to $\frac{\overline{CH}^2 \times \overline{CO}^2 - \overline{OH}^2}{\overline{CF}^2}$.

Therefore the whole resistance of the medium against the frustum may be represented by $(\overline{OH}^2 + \frac{\overline{CH}^2 \times \overline{CO}^2 - \overline{OH}^2}{\overline{CF}^2} = \frac{\overline{CF}^2 \times \overline{OH}^2 - \overline{CH}^2 \times \overline{OH}^2 + \overline{CH}^2 \times \overline{OC}^2}{CF^2} =) \frac{\overline{HF}^2 \times \overline{OH}^2 + \overline{CH}^2 \times \overline{OC}^2}{\overline{CF}^2}$, which call z; that is, putting $OC = r$, $OD = 2a$, $OS = y$, then $CH = (\frac{OC \times FH}{OS} =) \frac{2ar}{y}$, and $OH = \frac{ry - 2ar}{y}$) $z = \frac{r^4 + r^2y^2 - 4ar^2y + 4a^2r^2}{r^2 + y^2}$; therefore $r^4 + r^2y^2 - 4ar^2y + 4a^2r^2 = r^2z + y^2z$. Consequently $2r^2\dot{y}\dot{y} - 4ar^2\dot{y} = 2yz\dot{y} + y^2\dot{z} + r^2\dot{z}$. But z is a mi-

nimum; therefore $ny = 2arr = zy$; consequently $(z =) \frac{r^2y - 2ar^2}{y} = \frac{r^4 + r^2y^2 - 4ar^2y + 4a^2r^2}{rr + yy}$.

Hence $yy - 2ay = rr$; and, making $OQ = QD = a$, then $(y - a =) QS = (\sqrt{rr + aa} =) QC$.

To the same Schol. p. 94.

On the right line BC (Pl. 19, Fig 3) suppose the parallelograms $BG\gamma b$, $MN\nu m$, of the least breadth, to be erected, whose heights BG, MN, their distance Mb, and half the sum of their bases $\frac{1}{2}Mm + \frac{1}{2}Bb = a$, are given: let half the difference of the bases $\frac{1}{2}Mm - \frac{1}{2}Bb$ be called x, let G and N be points in the curve GND; and producing $b\gamma$ and $m\nu$ to g and n (so that $\gamma g = \nu n = b$), the points g and n may also be in the same curve.

Now if the figure CDNGB, revolving about the axis BC, generates a solid, and that solid moves forwards in a rare and elastic medium from C towards B (the position of the right line BC remaining the same), then will the sum of the resistances against the surfaces generated by the lineolæ Gg, Nn, be the least possible, when $\overline{Gg}^4$ is to $\overline{Nn}^4$ as $BG \times Bb$ to $MN \times Mm$.

For the force of a particle on Gg and Nn to move them in the direction BC is as $\frac{1}{\overline{Gg}^2}$ and $\frac{1}{\overline{Nn}^2}$, and the number of particles that strike in the same time on the surfaces generated by Gg and Nn are as (the annuli described by $g\gamma$ and $n\nu$, that is, as $BG \times g\gamma$ and $MN \times n\nu$, or as) BG and MN; therefore the resistances against those surfaces are as $\frac{BG}{\overline{Gg}^2}$ to $\frac{MN}{\overline{Nn}^2}$, that is (putting y for $\overline{Gg}^2$, and z for $\overline{Nn}^2$), as $\frac{BG}{y}$ to $\frac{MN}{Z}$.

But the sum of these resistances $\left(\frac{BG}{y} + \frac{MN}{Z}\right)$ is a minimum. Therefore $-BG \times \frac{\dot{y}}{yy} - MN \times \frac{\dot{z}}{zz} = 0$, or $MN \times \frac{\dot{z}}{zz} = -BG \times \frac{\dot{y}}{yy}$. But $y = (\overline{Gg}^2 = \overline{Bb}^2 + \gamma g^2 =) aa$

Fig. 1.

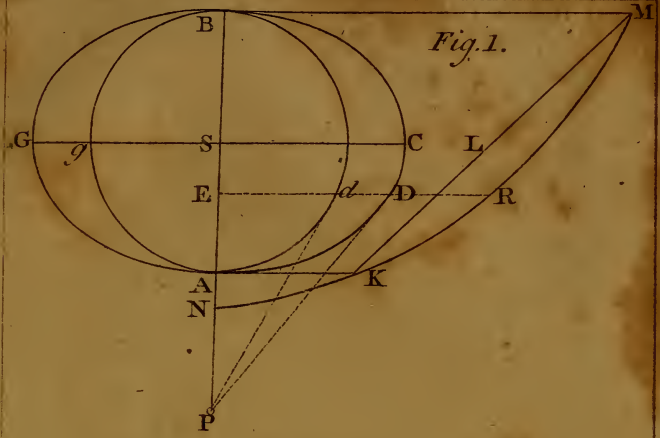


Fig 2.

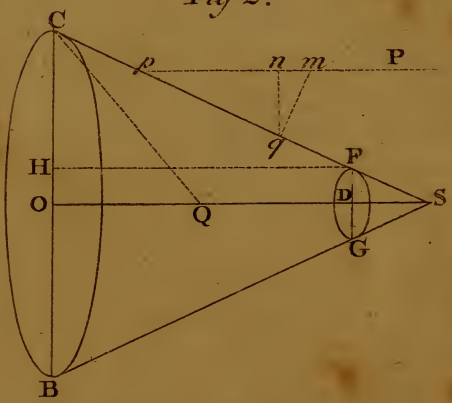
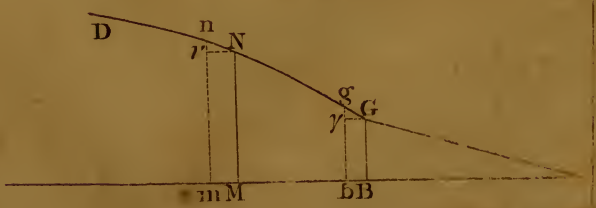


Fig 3.



$-2ax + xx + bb$; and $z = (\overline{Nn}^2 = \overline{Mm}^2 + \overline{\nu n}^2 =)$ $aa + 2ax + xx + bb$; therefore $\dot{y} = 2x\dot{x} - 2a\dot{x}$, and $\dot{z} = 2a\dot{x} + 2x\dot{x}$: consequently $\frac{MN}{zz} \times 2\dot{x} \times \overline{a + x} = \frac{BG}{yy} \times 2\dot{x} \times \overline{a - x}$; or $(\frac{MN}{zz} \times \overline{a + x} =) \frac{MN}{zz} \times Mm = (\frac{BG}{yy} \times \overline{a - x} =) \frac{BG}{yy} \times Bb$. Therefore $(yy)\ \overline{Gg}^4 : (zz)\ \overline{Nn}^4 :: BG \times Bb : MN \times Mm$.

Consequently, that the sum of the resistances against the surfaces generated by the lineolæ Gg and Nn may be the least possible, $\overline{Gg}^4$ must be to $\overline{Nn}^4$ as GBb to NMm.

Wherefore, if γg be made equal to γG, so that the angle γGg may be 45°, and the angle BGg 135°; also $\overline{Gg}^2 = \overline{2\gamma g}^2$, and $\overline{Gg}^4 = \overline{4\gamma g}^4$; then $\overline{4\gamma g}^4 : \overline{Nn}^4 :: GBb : NMm$; and since GR is parallel to Nn, and BG, BR parallel to $n\nu$, $N\nu$; also $n\nu = g\gamma = \gamma G$; it follows that $(n\nu = \gamma G =)$ Bb : $(N\nu =)$ Mm :: BG : BR, therefore $Bb = \frac{BG \times Mm}{BR}$; also $(n\nu =)\ \gamma G : Nn :: BG : GR$. Consequently $\frac{\overline{4\gamma g}^4}{\overline{Nn}^4} =) \frac{\overline{4BG}^4}{\overline{GR}^4} = (\frac{GBb}{NMm} =) \frac{\overline{BG}^2}{\overline{MN \times BR}}$. Therefore $\overline{4BG}^2 \times BR$ is to $\overline{GR}^3$ as GR to MN.

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